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Pole-Zero Diagram Approach to the Design of Coupled Ring Resonator-Arrays for Photonic Applications

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ABSTRACT

Pole-zero diagrams are often employed in electronic filter design because of their simple and direct visualization of spectral characteristics. This paper proposes the use of pole-zero diagrams for tailoring the spectral response of ring resonator array filters for photonic applications. We show that there exist close relations between the pole-zero diagram features of the resonator array and its wavelength response characteristics, and demonstrate that the pole-zero diagram approach could be a very useful tool in the design of photonic devices based on resonator filters. In particular, we employ the pole-zero diagram approach for the case of a parallel-coupled ring resonator array for interleaving, and use this method to produce a new low crosstalk design.

Keywords: Ring resonators, optical waveguide filters, integrated optics

1. INTRODUCTION

Optical filters are an essential component in today's wavelength division multiplexed fibre-optic networks. The growing complexity of these networks is driving demand for components with lower cost, smaller footprint, and lower power dissipation. Thus photonic devices are sought with desirable properties such as small chip area, low insertion loss, high fabrication tolerances, and integrability. One possible means of achieving these goals is the use of ring resonator filters. Ring resonators usually range in size from several micrometers to hundreds of micrometers, and can be cascaded to tailor filter passband shape and other characteristics. Highly compact filters using ring resonators have been fabricated for applications such as add-drop multiplexing.¹

Ring resonators filters have been analyzed using Z-transforms to calculate filter spectral response.² Algorithms for optimizing the spectral response of series-coupled ring resonator arrays and MZI-based filters were presented, and ring resonator filters designed for applications such gain equalization and dispersion compensation.² The transfer function describing the spectral response of ring resonators filters generally has a very complex dependence on device parameters such as resonance frequencies and coupling coefficients. Inspection of this function usually provides no clues for optimizing the filter design. Also, the algorithms that have been developed for certain filter configurations² generally cannot be applied to other filter designs. However, Z-transforms can be graphically represented using pole-zero diagrams.^{2,3} Pole-zero diagrams allow complex parametrical dependencies to be clearly visualized, and thus it may be possible to optimize a filter's performance by adjusting pole and zero positions. This paper explains the use of pole-zero diagrams for modifying spectral response to optimize crosstalk and passband shape.

Previously, pole-zero diagrams have appeared in a number of papers in the area of photonics. For example, pole-zero diagrams have been used in the design of narrow bandpass digital and notch filters based on fibre double-coupler resonators.⁴ Device parameters were varied to manipulate pole and zero positions. The parameter adjustments were quite large, resulting in filters with significantly different pole-zero diagrams and spectra for different applications. In our paper, however, we use fine parameter adjustments to adjust pole and zero positions in order to optimize passband shape and crosstalk.

To illustrate the use of pole-zero diagrams, we use them to design wavelength interleavers. Interleavers combine two separate streams of channels into one stream with half the channel spacing. Deinterleavers perform the reverse function,

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separating the odd and even channels. Interleavers allow the number of channels in a WDM network to be doubled while allowing for the use of inexpensive multiplexers with wide channel spacing. They can therefore result in considerable cost reductions. Commercial interleaving devices commonly employ cascaded Mach-Zehnder interferometers (MZIs). State-of-the-art MZI interleavers have been produced that occupy chip areas as small as 4.2 cm x 2 cm.⁵ Interleavers based on ring resonators have been the subject of several theoretical and experimental studies.^{6,7,8,9} There are several basic configurations employing ring resonators that can be used to construct an interleaver. Parallel-coupled ring resonator arrays have been proposed.⁶ Filters comprising of MZIs with ring resonators in one of the arms were fabricated using various material technologies.^{7,8,9} The best configuration depends on a number of factors, and in this paper we chose to look at designs that do not employ MZIs, but use only ring resonators.

In this paper we propose the use of pole-zero diagrams for designing and optimizing optical ring resonator filters, focusing on parallel-coupled ring resonator array for wavelength interleaving. We demonstrate that the pole-zero diagrams provide information that can assist in filter design, information that is not apparent from simply inspecting a filter's spectrum or its transfer function. In addition, we outline how pole-zero diagrams can be used for optimizing spectral characteristics. Finally, we present a new design of wavelength interleaver with low crosstalk and good passband shape. This filter comprises a four-ring parallel-coupled array with two ring resonators coupled to one of the output ports.

2. POLE-ZERO DIAGRAM DERIVATION FOR RING RESONATOR FILTERS

In this section we will explain how to derive a filter's spectral response and pole-zero diagram. The time response of a filter is completely characterized by a discrete impulse response function, $h(n)$.³ The frequency or wavelength response is obtained from the Z-transform of this impulse function:

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n} \quad (1)$$

Here, z is a complex variable. Each term in this equation represent a delay, with z^{-1} corresponding to a unit delay, z^{-2} corresponding to twice this, and so on. In an actual filter, these delays correspond to path length differences, such as the path difference between the two arms of an MZI or the path corresponding to a ring resonator.

Following various manipulations, equation (1) can be put into the following form:

$$H(z) = \frac{\Gamma z^{N-M} \prod_{m=1}^M (z - z_m)}{\prod_{p=1}^N (z - z_p)} \quad (2)$$

The poles z_p and zeros z_m can be read from this equation. A pole-zero diagram is a graph of pole and zero values on the complex z -plane. The position of the poles and zeros determine the frequency response. A zero positioned on the unit circle results in zero transmission at the frequency corresponding with the angle of that zero. Likewise, a pole on the unit circle will give unity transmission at its corresponding frequency. As poles and zeros move away from the unit circle, their effect on the magnitude spectrum decreases. Arbitrary magnitude spectra can be synthesized by adding poles and zeros at various locations on the complex plane.^{2,3} A complete circuit of the unit circle corresponds to the free spectral range (FSR) of the filter. Thus pole-zero diagrams are suitable for filters with periodic response.

An MZI introduces a single zero in the complex plane for both the through and cross ports, and cascading MZI stages introduces additional zeros.² MZIs are FIR (finite impulse response) filters, as they have no poles, which thus limits their crosstalk and passband flatness. Ring resonators have feedback paths and therefore poles. They are IIR (infinite impulse response) filters, which usually have better crosstalk and passband flatness than FIR filters.

To illustrate pole-zero diagram derivation for ring resonator filters, we derive the Z-transform for a single ring resonator (see Fig. 1), following the approach of Madsen and Zhao.² The circumference of the ring resonator corresponds to a unit

delay length, which is equal to $z^{-1} = e^{-i\omega}$. Here, ω is the normalized frequency $(\Omega - \Omega_c)2\pi R/c$, Ω is the optical frequency, Ω_c the centre frequency, n the ring effective index, and R the ring radius. Dissipation in the ring is included by multiplying z^{-1} by γ , where $(1 - \gamma^2)$ is the power attenuation per cycle. The delays corresponding to half the ring circumference are $\sqrt{z^{-1}\gamma}$, as shown in Fig. 1.

Fig. 1. A ring resonator coupled to input and output buses by directional couplers K_1 and K_2 .

The signal amplitudes at the various ports must now be determined. If r_1 is the straight-through amplitude coupling factor and s_1 the crossover coupling factor of directional coupler K_1 , and similarly for K_2 , then the signal at the drop port is:

$$S_{drop} = -s_1 s_2 \sqrt{z^{-1}\gamma} \{1 + r_1 r_2 z^{-1}\gamma + \dots\} S_{input} \quad (3)$$

This equation can be simplified to give the drop port transfer function:

$$\frac{S_{drop}}{S_{input}} = \frac{-s_1 s_2 \sqrt{z^{-1}\gamma}}{1 - r_1 r_2 z^{-1}\gamma} \quad (4)$$

It is evident from this equation that there is a single pole at $r_1 r_2 \gamma$. Fig. 2 shows the magnitude spectrum and pole-zero diagram for a ring with power coupling factor of 0.7 (corresponding to $s_1 = s_2 = 0.84$) at both couplers and power dissipation in the ring of 10% per cycle.

Fig. 2. (a) The magnitude spectrum at the drop port of the single ring-resonator filter shown in Fig. 1. The power coupling factor is 0.7 for both couplers and the power dissipation in the ring is 10% per cycle. (b) The corresponding pole-zero diagram. Since the signal is periodic and symmetrical, only half of the FSR is plotted. The frequency scale is normalized, with 0 corresponding to the resonance frequency and 1 corresponding to half the FSR. The outermost circle for all pole-zero diagrams in this paper corresponds to the unit circle.

For the through port there is a pole at $r_1 r_2 \gamma$, as for the drop port, and a zero at $r_2 \gamma / r_1$, as shown in Fig. 3.

Fig. 3. (a) Magnitude spectrum and (b) pole-zero diagram for the through port of the single ring-resonator filter shown in Fig. 1. The power coupling factor is 0.7 at both couplers and the power dissipation in the ring is 10% per cycle.

We shall now derive the Z-transform for parallel-coupled ring-resonator arrays. Fig. 4 shows a schematic of this filter. The first step is to assign delay terms to the various optical waveguide paths in the filter. The circumference of the rings corresponds to a delay of z^{-1} , and the spacing between the rings, L , is chosen such that $z^{-1/2} = e^{-\beta L}$, where β is the bus propagation constant. This ensures a periodic spectral response suitable for a wavelength interleaver.¹⁰

Fig. 4. Schematic of a parallel-coupled ring resonator filter. This filter can operate as a deinterleaver, sending odd channels to the drop port and even channels to the through port. It can also operate as an interleaver, combining channels from the input and drop ports and sending the combined signal to the through port.

Relations between the signal amplitudes at various points in the filter can now be determined. The transfer matrix method^{10,11} was used to obtain the transfer equations for the drop and through port signals, however we substitute z^{-1} for $e^{-i\omega}$. Once the transfer equations are found, they can be put into the form of equation (2) to find the zeros and poles. Due to the length and complexity of the equation, we will not show them here, but will simply present pole-zero diagrams in subsequent sections.

3. POLE-ZERO DIAGRAMS FOR WAVELENGTH INTERLEAVER FILTERS

3.1 OPTIMAL INTERLEAVER FILTER RESPONSE

In order to design a filter, it is useful to first produce a pole-zero diagram for an ideal classical filter, such as a Chebyshev filter. The purpose of this is to determine the best performance that is achievable with a filter of a given order. Minimization of filter order is important since it simplifies filter design, allowing fewer MZI and ring resonator stages to be used. The pole-zero diagram of this ideal filter can be later compared to the pole-zero diagram of an actual filter to assist in optimizing the filter design.

We designed an interleaving filter with the lowest possible crosstalk for a given filter order using the Filter Design and Analysis Tool in Matlab. An IIR Chebyshev Type II was chosen since it is free of passband ripple. The -3 dB passband width was set as 50% of the FSR, and the stopband frequency was set as 80% of the FSR. Crosstalk was defined as the

maximum value of the filter magnitude response in the stopband. We varied the crosstalk, and Matlab produced minimum order filter designs that satisfied these crosstalk levels.

It was found that for crosstalk of higher than -26 dB, the resulting filter design was second order. For between -26 dB and -40 dB, the filter was third order. Interleaving filters require crosstalk of better than -23 dB to be of commercial interest.¹⁰ Since a second order filter can barely satisfy the crosstalk requirements of an interleaver, a third order filter was chosen. Fig. 5 shows the spectrum of a third order filter with minimum crosstalk (-40 dB). This can be regarded as the perfect spectrum for the drop port of an interleaver filter. An interleaver ideally has symmetrical response in the drop and through ports, and the corresponding pole-zero plots are likewise symmetrical. The through port spectrum is identical to the drop port's but shifted by half the free-spectral range. The poles and zeros for the through port are obtained by reflecting the poles and zeros of the drop port through the imaginary axis.

Fig. 5. (a) Magnitude response and (b) pole-zero diagram for a third order Chebyshev type II filter, with -3 dB passband width equal to 50% of the FSR, and stopband frequency equal to 80% of the FSR. Crosstalk is -40 dB.

In order to characterize the passband flatness we defined a shape factor as the -1 dB bandwidth divided by the -10 dB bandwidth. For the filter in Fig. 5(a) this is 0.78. Higher shape factor implies a more box-like spectrum, which is desirable for interleaver filters.

Note that the Chebyshev spectral response and pole-zero diagram in Fig. 5 can be attained using a perfect (ie. lossless, exact parameter values) MZI filter with one arm coupled to a ring resonator.²

3.2 FOUR-RING PARALLEL-COUPLED RING ARRAY

Pole-zero diagrams were found for parallel-coupled ring resonator arrays with various numbers of rings. It was found that the pole-zero diagram for a four-ring array best approximates the diagram in Fig. 5(b).

The procedure used to optimize the coupling coefficients between the rings and the straight waveguides has been described previously.¹⁰ The coupling is exponentially apodised, according to $A \text{Exp}[-a_p (N - N_c)^2]$, where N is the ring number, N_c is half of the total number of rings in the array, A is the coupling prefactor, and a_p is the apodization coefficient. For minimal equal crosstalk in the drop and through ports of a four-ring array the coupling prefactor equals 0.7 and the apodisation equals 0.125. The coupling coefficients between the rings and the input and output straight waveguides are equal at each ring.

Fig. 6(b) shows the drop port pole-zero diagram for the parallel-coupled filter. As in Fig. 5(b), there is a zero at 180° and zeros at 148° and 212° . However, there are three poles in the negative real plane, whereas in Fig. 5(b) they are positioned on or near the imaginary axis. Secondly, there are three poles and three zeros overlapping near $z = 1$. This overlap is close and so the poles and zeros cancel, resulting in a magnitude spectrum that is flat at the centre frequency.

Fig. 6. (a) Magnitude spectrum and (b) pole zero diagram for the drop port of the four-ring parallel-coupled array illustrated in Fig. 4. The coupling prefactor is 0.7 and the apodisation 0.125.

The pole-zero diagram of the through port differs considerably from the optimized Chebyshev filter, as Fig. 7 shows. There are four zeros and three poles near $z = 1$, and so there is one excess zero at this point. There are no zeros at 32° or 328° , and so the crosstalk is significantly higher for this port than the Chebyshev case. Also, the pole positions are the same as for the drop port. However, since the poles have moved away from the imaginary axis, the shape factor is degraded. The usefulness of pole-zero diagrams is apparent here. Since the number of zeros does not correspond to that of the optimized Chebyshev filter, it is impossible to attain the crosstalk levels and passband shape required by simply adjusting parameters. This is not evident from simply looking at the magnitude spectrum.

Fig. 7. (a) Magnitude spectrum and (b) pole zero diagram for the through port of the four-ring parallel-coupled array illustrated in Fig. 4. The coupling prefactor is 0.7 and the apodisation 0.125.

We now look at the effect of adjusting parameters on the pole and zero positions to illustrate how pole-zero diagrams may be used to optimize filter performance. In Fig. 8, the apodisation is varied. It is clear that the pole and zero positions cannot be adjusted independently, as both move when this parameter is varied. For large apodisation, the sidelobe is small but the width of the main peak limits crosstalk. For small apodisation, crosstalk is high due to a large sidelobe. The pole positions change slightly as apodisation varied, but have only a small effect on the drop port spectrum. Similar diagrams to Fig. 7 can be obtained for the through port. The crosstalk in the through port spectrum is determined mainly by the poles, which are in the same location as for the drop port. As apodisation increases, the poles move away from the unit circle and towards the origin so that crosstalk increases. When varying parameters, the

crosstalk in both the through and drop ports must be considered, as they are interrelated by their dependence on apodisation and other parameters.

Fig. 8. (a) Magnitude spectrum and (b) pole zero diagram for the drop port of the four-ring parallel-coupled array filter for variable apodisation. Apodisation is increased from 0.075 to 0.15 in steps of 0.025, with coupling prefactor fixed at 0.7, and the loss in the ring is 10%. Arrows indicate the direction of increasing apodisation.

It can be seen that the zeros are at angles of 148° and 212° for an apodisation of 0.0125. This corresponds to a minimum in the crosstalk versus apodisation, and also corresponds to the zero positions in Fig. 5(b) for the optimized Chebyshev filter.

It is interesting to note that crosstalk is lowest when the zeros are closest to their positions for the Chebyshev case. Therefore, optimization of the coupling ratio can be done by varying parameters and comparing the positions of the zeros with those for the Chebyshev filter, while observing the pole-zero diagrams and magnitude spectra. Optimal crosstalk is obtained when the pole and zero positions most closely approximate their positions in Fig. 5(b). Note that the parameter values for which the zero positions correspond to their optimal positions are not unique, however the values for which crosstalk in the through and drop ports are equal are unique in this case. Together with the zero positions, they specify the optimum parameter values. This example above suggests a useful optimization technique of adjusting parameters to graphically locate zeros and poles at their optimal positions.

3.3 SIX-RING INTERLEAVER FILTER

A filter consisting only of ring resonators may have advantages over a filter with MZIs, for instance it would avoid the problem of requiring precise splitting coefficients at the Y-junction branches of the MZI. The parallel filter described in the previous section has a drop port spectrum and pole-zero diagrams that are a reasonable match to the optimized Chebyshev filter, as can be seen by comparing Fig. 5 and Fig. 6. The through port is far from optimal, as Fig. 7 shows. However, the output from this port can be modified by adding two zeros, one at 32° and another at 328° . This can be done by adding two rings to the through port waveguide. One ring is detuned from the centre frequency to produce a zero at 32° , and the other is detuned to produce a zero at 328° . Detuning is done simply by adjusting the ring radii. A schematic of such a filter is shown in Fig. 9. Note that the wavelength shift due to detuning is slightly different for each channel, but for forty ITU channels this is only a few percent.

Fig. 9. Schematic of a six-ring filter design for deinterleaving. The two rings on the through port have slightly different radii to the rings in the parallel-coupled array in order to produce transmission nulls away from the central resonance frequency.

The pole-zero diagram and magnitude spectrum for the through port are shown in Fig. 10. The two poles enclosed in the ellipse are associated with the detuned rings. It is clear that this filter provides close to ideal transfer characteristics in this port. The drop port is not shown, however its spectrum is similar. The crosstalk in the through port is -39 dB, and the crosstalk in the drop port is -36 dB. The performance is better than that obtained simply by using six rings coupled in parallel, which had crosstalk levels of up to -23 dB.¹⁰ This filter was obtained by modifying the pole-zero diagrams for the parallel-coupled through port to obtain a pole-zero diagram close to that for an optimized Chebyshev filter. This design would not have been apparent without pole-zero diagrams to visualize the spectral characteristics of the filters, and illustrates clearly the usefulness of pole-zero diagrams.

Fig. 10. (a) Magnitude spectrum and (b) pole-zero diagram for the through port of the filter illustrated in Fig. 9. For the parallel stage, the optimal coupling prefactor was 0.5 and the optimal apodisation was 0.176. For the series stage, the ring-straight waveguide coupling was 0.8 and the ring-ring coupling was 0.39.

4. CONCLUSION

In this paper we outlined novel methods of applying pole-zero diagrams to the design of ring resonator array filters for photonics applications. We showed that pole-zero diagrams provide information that cannot be obtained by simply studying spectral characteristics. This information can be used for assessing the usefulness of a filter structure for an application, for optimizing filter parameters, and for producing new filter designs. Using the pole-zero method, we designed a new low cross-talk filter structure for wavelength interleaving and deinterleaving consisting of a parallel-coupled filter array with two detuned rings on the through port.

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