802.11 positioning using signal strength fingerprinting

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802.11 Positioning using Signal Strength Fingerprinting

Master of Science (MSc) Thesis
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ORIGINALITY STATEMENT

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Abstract

The effectiveness of location aware applications is dependent on the accuracy of the supporting positioning system. This work evaluates the accuracy of an indoors 802.11 positioning system based on signal strength fingerprinting. The system relies on an empirical survey of signal strength prior to positioning. During this survey, signal strength recordings are made at a set of positions across the environment. These recordings are used as training data for the system during positioning. In this thesis, two surveying methods, five positioning algorithms, and two spatial output averaging methods are trialled. Accuracy is determined by empirical testing in two separate environments: a 100m^2 domestic house and the 1,333m^2 third floor of the University of New South Wales Computer Science and Engineering building. In the two environments, the lowest mean distance errors are 1.25m and 2.86m respectively.

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List of Publications

I am the author or co-author of the following publications that have resulted from research performed during the course of the degree:

- James Salter, Binghao Li, Daniel Woo, Andrew G. Dempster and Chris Rizos. *802.11 Positioning In The Home.* In proceedings of 5th IEEE Consumer Communications And Networking Conference, pages 598–602, Las Vegas, Nevada, 10-12 January 2008.

- Daniel Woo, Nick Mariette, James Salter, Chris Rizos and Nigel Helyer. *Audio Nomad.* In proceedings of 19th International Technical Meeting of the Satellite Division of the U.S. Institute of Navigation (ION GNSS), pages 3117–3123, Fort Worth, TX, USA, 26-29 September 2006.

- Binghao Li, James Salter, Andrew Dempster and Chris Rizos. *Indoor Positioning Techniques Based on Wireless LAN.* In proceedings of 1st IEEE International Conference on Wireless Broadband And Ultra Wideband Communications, Paper 113, CD-ROM proceedings, Sydney, Australia, 13-16 March 2006.

Contents

1 Introduction **8**

1.1 Research Areas . . . 8

1.2 Chapter Contents . . . 10

2 Literature Review **12**

2.1 Outdoor Wireless Positioning Systems . . . 12

2.1.1 Global Positioning System (GPS) . . . 12

2.1.2 Cell-phone localization . . . 13

2.1.3 Outdoors 802.11 . . . 15

2.2 Indoor Wireless Positioning Systems . . . 16

2.2.1 Ultra-Wideband . . . 18

2.2.2 Standards . . . 19

2.3 802.11 positioning . . . 20

2.4 Other consumer wireless technologies . . . 32

2.4.1 RFID . . . 32

2.4.2 Bluetooth . . . 32

2.4.3 ZigBee . . . 33

2.5 Summary of Positioning Systems . . . 33

2.6 Conclusions . . . 34

3 802.11 Positioning Methods **37**

3.1 802.11 Networks . . . 37

3.2 RF Signal propagation . . . 38

3.2.1 Free space loss . . . 38

3.2.2 Shadow Fading . . . 39

3.2.3 Multipath Fading . . . 40

3.3 Inputs and Outputs . . . 41

3.3.1 Input . . . 41

3.3.2 Output . . . 42

3.4 Centroid method . . . 43

3.5 Location Fingerprinting . . . 43

3.5.1 Signal Strength Maps 43
3.5.2 Indoor RF Signal Propagation 44
3.5.3 Point and Sector Surveying 45
3.5.4 Density of Fingerprints 46
3.6 Positioning Algorithms 48
3.6.1 Nearest Neighbour method 48
3.6.2 Output averaging 51
3.6.3 Bayesian methods 53
3.6.4 Distribution Difference method 57
3.7 Conclusions 65

4 Test Environments and Procedures 67
4.1 Test Environments 67
4.1.1 House Environment 67
4.1.2 CSE Level 3 Environment 69
4.2 Test Procedure 72
4.3 Sampling Rates and Channel Hopping 73
4.4 Input Averaging 75
4.5 Conclusions 76

5 Comparison of Algorithms and Sampling Methods 77
5.1 Summary of Results 77
5.2 Algorithm Specific Parameters 79
5.2.1 Nearest Neighbour 80
5.2.2 Bayesian Histogram 80
5.2.3 p-norm Distance 82
5.2.4 Match Distance 82
5.2.5 Best Parameter Ranges 84
5.3 Spatial Output Averaging 84
5.3.1 Bayesian methods and Weighted Output Averaging 87
5.3.2 Overall Effect of Output Averaging 90
5.4 Sector versus Point Surveying 91
5.4.1 Nearest Neighbour 92
5.4.2 Bayesian Histogram 93
5.4.3 Bayesian Gaussian 94
5.4.4 p-norm Distance 95
5.4.5 Match Distance 95
5.5 House Versus CSE Environment 95
5.6 Comparisons with other studies 97
5.7 Conclusions 98

6 Access Point Layout **100**
6.1 Simulation of Layouts . . . 100
6.2 Results . . . 104
6.2.1 House . . . 104
6.2.2 CSE . . . 104
6.3 Investigation of variations in accuracy . . . 105
6.4 Conclusions . . . 112

7 Infrastructure-side positioning **114**
7.1 Motivation . . . 114
7.2 Implementation . . . 116
7.2.1 Network Layout . . . 116
7.3 Compensating for Transmission Power . . . 118
7.3.1 Power offsets . . . 118
7.3.2 Determining the Transmission Power . . . 118
7.4 Experimental Setup . . . 119
7.5 Results . . . 121
7.5.1 Mean Offsets . . . 121
7.5.2 Fixed Offset . . . 122
7.5.3 Adaptive Offset . . . 123
7.5.4 Summary . . . 124
7.6 Conclusions . . . 125

8 Conclusions and Future Research **126**
8.1 Conclusions . . . 126
8.2 Suggested Future Research . . . 127
8.2.1 Reference Implementations and Test Data . . . 127
8.2.2 Temporal Tracking . . . 128
8.3 Final Conclusions . . . 128

A Implementation of Experimental Software **129**
A.1 Linux . . . 129
A.1.1 Kismet Implementation . . . 130
A.1.2 Alternatives . . . 131
A.2 Windows . . . 131

B Comprehensive Results from the Comparison of Algorithms **134**
B.1 Best Parameter Values . . . 134
B.2 Mean Distance Errors for All Tests . . . 137

C Signal Strength Maps **178**
C.1 House Environment 178
C.2 CSE Environment 188
C.2.1 Photos 188
C.2.2 Signal Strength Maps 188

Chapter 1

Introduction

The effectiveness of location aware applications, such as navigation and locational search, is dependent on the accuracy of the supporting positioning system. In outdoor environments, the NAVSTAR Global Positioning System (GPS) [111] is the first choice solution due to its high level of accuracy, reliability, and relatively low cost. However, GPS accuracy and availability suffers indoors or when the signal path is blocked by buildings, as is often the case in urban areas. Recently, consumer wireless technologies such as 802.11 wireless networking and mobile phone networks have been used for indoor and outdoor positioning in addition to their conventional data transport function [4, 15, 63]. These systems are attractive compared to purpose built positioning systems because there are many devices already equipped with wireless technology, such as mobile phones, laptops, and personal digital assistants (PDAs).

The focus of this thesis is indoor 802.11-based positioning using a technique called *signal strength fingerprinting*, which is currently the most accurate and practical positioning method using 802.11 signals as a positioning medium [4, 90, 43]. The technique is named such because prior to on-line positioning, the strength of the wireless signal is empirically surveyed at a set of locations; this association between signal strength information and location constitutes a set of unique location *fingerprints*. To calculate position, the current input signal strength is compared to the information in the fingerprints, and the fingerprints are ranked according to their similarity to the input. The positions of the most similar fingerprints are spatially averaged to produce the final output.

1.1 Research Areas

Several areas of the fingerprinting approach are investigated with the ultimate goal of maximizing accuracy. An empirical experimental approach is used to generate results, drawing on test data gathered in two separate environments: a 100m^2 domestic house, and the 1,333m^2 third floor of the University of New South Wales Computer Science and

Engineering building. The areas investigated are as follows:

- **Five positioning algorithms are compared.** The Nearest Neighbour [4], Bayesian Histogram [14, 90], and Bayesian Gaussian [43] methods are compared, as well as two novel algorithms introduced by this work: the Distribution Distance based p-norm Distance and Match Distance methods. Our two novel methods aim to take more information into account than the other methods by considering the frequency distribution of signal strength values of the input, over a sliding window of time. Our experiments show that the choice of algorithm and surveying method has only a small effect on overall accuracy, assuming that optimal parameters to the algorithm are used. For the two environments tested, the difference in distance errors between the best and worst classes of algorithm, using optimal parameters, is 14% and 32% of the mean distance error for each environment respectively.

- ***Arithmetic* and *weighted* methods of spatial output averaging are compared.** Spatial output averaging is a technique where the locations of a fixed number of best ranked fingerprints (based on their similarity the output) are averaged to produce the final result. In the *arithmetic* output averaging method, the locations are simply averaged, with each contributing equally to the result; in the *weighted* output averaging method, the weighted averaged is used based on weights generated by the positioning algorithm. It is shown that both arithmetic and weighted output averaging methods consistently produce a reduction in distance error of approximately 20% when using the point surveying method, and 10% using the sector surveying method, with small variations in results depending on the positioning algorithm used.

- ***Point* and *sector* signal strength surveying methods are compared.** In the *point* surveying method [4, 90], signal strength is recorded at several specific positions in the environment, facing four directions at each position to allow for signal strength being affected by the body of the surveyor. In the *sector* surveying method [14, 61, 43], signal strength is recorded while the the surveyor moves about a small area. Although both methods have been used separately by other researchers, our work is the first explicit comparison of the two. The results of our comparison vary for the two environments: in one environment sector surveying improves accuracy by up to 20%; in the other environment it decreases accuracy by up to 15%.

- **The effects of access point layout on accuracy are demonstrated.** Access point layout refers to both the *number* of access points and their *positions*. Data is recorded for one access point layout in each test environment and several different layouts are simulated by removing selected access points from the data set. It is shown that accuracy improves with a greater number of access points, and that the positions of access points can have an effect on accuracy.

- **Description of modifications made to an entry level consumer access point so that it can be used to record signal strength for positioning.** A network of these access points allows any 802.11 device to be accurately positioned, without any device specific calibration or installation of software on the device.

In the house and CSE building environments, the lowest mean distance errors (achieved using the best combination of surveying method, positioning algorithm, and spatial averaging method) were 1.25m and 2.86m respectively. The difference in accuracy can be attributed to the differences between the two environments, such as construction materials, size, levels of interference, and access point layout. This level of accuracy is roughly comparable with other work using a similar quantitative approach to accuracy testing [4, 90, 43, 66], although it is difficult to compare closely due to differences in testing procedures and between test environments.

1.2 Chapter Contents

Chapter 2 contains a detailed literature review of the state of the art in wireless positioning. Outdoor and indoor approaches using various wireless mediums are discussed, followed by a comprehensive review of 802.11 positioning techniques.

Chapter 3 describes the signal strength fingerprinting approach in detail. The technique is placed in the context of signal propagation modelling approaches. Different approaches to surveying are discussed, and the five algorithms used in experiments are explained.

Chapter 4 describes the two test environments and the methodology used to conduct the experiments that produced the empirical data used in subsequent chapters. The signal strength information in each environment is also characterized based on aggregate statistics of sampling rate, signal strength gradient and fingerprint variance.

Chapter 5 contains an empirical comparison of positioning algorithms and surveying methods. Accuracy testing is performed using two surveying methods and each algorithm and spatial output averaging method. A comprehensive range of parameters is tested for each algorithm and the parameters that produced the best results are isolated.

Chapter 6 analyses the effects of access point density and layout. It is shown how these factors affect the signal strength information used by the positioning system to estimate accuracy, and that some access point layouts produce substantially higher errors than other layouts with an identical number of access points in the same area. This is due to the different levels of fingerprint uniqueness in their respective fingerprint maps.

Chapter 7 describes the implementation of an infrastructure-side positioning system using access points to detect signal strength. An adaptive method for compensating for variations in transmission power of client devices using this system is also described.

Our conclusions and suggestions for future research are described in Chapter 8.

Chapter 2

Literature Review

This chapter contains a review of work related to wireless positioning systems. Firstly, outdoor systems employed in long range scenarios are discussed, designed for city-wide, national or global coverage, and indoor systems designed for use in smaller areas, usually inside buildings. Wireless positioning systems based on consumer wireless communication hardware are then reviewed, in particular systems based on 802.11 wireless networking.

2.1 Outdoor Wireless Positioning Systems

This section describes wireless positioning systems that operate outdoors and over city or country-wide areas. Until recently, these systems were purpose built, critical to military operations, and very expensive. More recently, cheaper civilian oriented systems have been designed that compute position based on existing wireless infrastructure, such as cell phone networks and 802.11 access points. As well as the profit incentive for telecommunications companies, who can use them as a platform for location based services, these efforts are driven by ”Enhanced 911” legislation in the United States, which requires that the position of cellular 911 callers must be able to be determined to within 50 to 300 metres depending on the technology used [30, 31].

2.1.1 Global Positioning System (GPS)

The NAVSTAR Global Positioning System (GPS) is a time of arrival (TOA) based satellite navigation system designed, financed, deployed and operated by the U.S. Department of Defence [111]. GPS is also extensively used for civilian purposes, despite the fact that it is first and foremost a military system. A basic off the shelf unit is typically accurate to within 5-10 metres, with more advanced techniques such as differential GPS and carrier-phase positioning able to boost accuracy to the metre and centimetre level respectively. GPS is the first choice positioning system for many users due to its relatively high accuracy, availability, and reliability [89].

Unfortunately, the TOA approach used in GPS is less accurate, or non-functional, when no direct line of sight to the satellite constellation is available. If the signal path is reflected off any obstacles, calculations assuming a direct path are invalid, and accuracy is degraded. The reflected signals also experience some attenuation due to multipath fading (see section 3.2.3). Hence, GPS is usually unavailable or inaccurate inside buildings.

In a small study, LaMarca et al. [63] found that, because most humans spend most of their time indoors, the average person is in a location where GPS is available for only 4.5% of the day. Nevertheless, GPS is widely used by consumers for navigation, particularly in vehicles, which are more likely to be outdoors where the system is available.

2.1.2 Cell-phone localization

Cell-phone localization methods fall into several major categories: assisted GPS, time based approaches such as TOA and TDOA, using cell-IDs of nearby base stations, and signal strength fingerprinting.

Assisted GPS handsets contain a GPS receiver that the cellular network assists by performing the actual position calculations when supplied with raw pseudorange information by the phone. Depending on the implementation, the network may be able to improve the estimate through knowledge of local ionospheric interference patterns. It can also reduce the time taken by the handset to search for satellites by providing it with current satellite constellation information [103].

Time difference of arrival (TDOA) techniques estimate location using the differences in the time of arrival across multiple base stations of a signal from the handset. Three receiving base stations are required to calculate a two-dimensional position, four for a three-dimensional position (including height). For two-dimensional calculation, each TDOA measurement equates to a hyperbolic line in space along which the handset could be located; multiple measurements produce multiple hyperbolic lines and their intersection is used to calculate the actual location of the handset. Similarly to the TOA approach used in GPS, TDOA systems suffer accuracy degradation in non-line-of-sight (NLOS) conditions. Accuracy of TDOA systems varies between 50 and 500 metres depending on conditions [103, 24, 94].

Several TDOA implementations exist. Existing observed time distance (E-OTD) systems use existing time difference measurements of GSM cellular networks to provide a positional estimate. In E-OTD, the handsets measure the time differences and report them to the network. This approach requires both new infrastructure and modifications to handset software [26, 13].

TruePosition's Uplink Time Difference of Arrival (U-TDOA) is an alternative technique to E-OTD that works with any existing handset without modification, requiring only supporting infrastructure. In U-TDOA, the network, rather than the handset, performs TDOA measurements [109]. TruePosition's system is also capable of switching to an Angle of Arrival (AOA) technique when an insufficient number of base stations causes TDOA to be ineffective.

The type of network used may also affect the operation of TDOA localization. In CDMA networks, base stations all share the same downlink frequency, which means that in-band interference from the carrier network frequently inhibits reception from more than one base station simultaneously. This is particularly true when the handset is close to one base station and far away from another. GSM base stations can operate on multiple frequency bands and do not suffer from this problem [103, 13].

If high accuracy is not required (such as in countries without E-911 requirements), the location of the cellular sector the handset is connected to may be sufficient to provide a reasonable estimate of the handset's location. This is referred to as the cell-ID approach. Cell sectors range in size between 3 and 20 kilometres [103], and possible accuracy is of a similar order of magnitude.

Several hybrid solutions combining AGPS, TDOA, and Cell-ID methods have also been proposed. These methods combine location information using the different approaches using a technique called sensor fusion, which takes into account the availability and quality of the positional estimate of each technique to provide a final result. These approaches can provide more robust estimates than using one technique alone [103, 101, 63, 88]. For example, when GPS is totally unavailable while indoors, TDOA can still provide an estimation of position.

Chen et al. [15] achieve an average GSM positioning accuracy of 94 metres outdoors, and 5 metres indoors, using a signal strength fingerprinting approach and special hardware. However, this approach suffers from the need to perform a site survey prior to positioning, making coverage inherently limited. It also requires a special GSM modem to access the signal strength information in sufficient detail. This is due to the limited amount of signal strength information consumer cell phones provide to user applications, rather than any inherent technical limitation. Varshavsky et al. [113] speculate that these limitations reflect the desires of both mobile phone manufacturers and service providers, who have an interest in limiting the accessibility of positioning information in order to sell their own location based services. This factor could prevent the GSM fingerprinting technique from becoming widespread on consumer mobile phones.

Laitinen et al. [62] also use location fingerprinting of GSM signal strength to achieve accuracy of better than 90 metres in 90% of cases, performing a site survey using a moving vehicle and co-ordinates manually entered by a surveyor.

In the United States, the development of cell-phone localization is driven by E-911 wireless standards, which stipulate that the position of a handset making an emergency call must be able to be determined by cellular infrastructure. This is implemented by the major carriers using either Assisted GPS (A-GPS), which requires handset support, or time difference of arrival (TDOA) techniques [29], which require new infrastructure but can be implemented to work with any handset [103].

Both techniques have been verified to provide accuracy that meets E-911 requirements, which are:

- For network-based technologies, within 100 metres for 67 percent of calls, and within 300 metres for 95 percent of calls
- For handset-based technologies, within 50 metres for 67 percent of calls, and 150 metres for 95 percent of calls [29].

In 2007, regulators fined three wireless carriers a total of US $2.83 million for failing to comply with the above requirements [22].

In order to meet E-911 requirements, carriers are advised by the FCC (the regulatory body) to consider the geography of the regions they serve when choosing between TDOA and AGPS approaches [29]. TDOA approaches are generally more attractive than AGPS as they do not require more expensive AGPS handsets, but TDOA may not function well in rural areas where less than 3 base stations are available.

2.1.3 Outdoors 802.11

Place Lab [63, 16, 100] uses 802.11, GSM and Bluetooth to perform positioning outdoors, with a view to eventually achieving global coverage. The project's authors consider it a direct competitor to both GPS and E911 compliant cell phone positioning systems, which are comparable in terms of accuracy but generally operate under fees charged by service providers. While Place Lab offers a low cost solution in that it can operate using any WiFi chipset, it mostly relies on third party WiFi location databases generated by volunteers such as WiGLE [117]. Unfortunately, these databases require constant refreshing due to their reliance on access points that can be moved or switched off at any time, and no coverage exists in areas that have not been surveyed. In testing, using 802.11 beacons only, Place Lab had an average accuracy of 20.5 metres in an urban area, 13.5 metres in a residential area, and 22.6 metres in a suburban area, with 100%, 90.6%, and 42% availability respectively. Using GSM, accuracy was 107.2, 161.4, and 216 metres, and availability was 100%, 100%, and 99.7% respectively. Combining GSM with 802.11 failed to improve accuracy over 802.11 alone.

Skyhook Wireless [98] is a commercial 802.11 positioning system using a similar approach to Place Lab, with coverage in many cities all over the world. The base station maps are recorded and maintained by the company, with gives a greater guarantee of their integrity than the community based Place Lab. The company claims average accuracy of 10-20 metres in urban areas.

2.2 Indoor Wireless Positioning Systems

This section reviews indoor wireless positioning systems, designed for use inside buildings. These systems tend to be more accurate and less expensive than outdoor systems, although they still require some deployment of infrastructure.

Infrared

Active Badge [115] is an IR (infrared) based positioning system designed to augment office paging systems. Each user carries a badge that broadcasts an identifying pulse only once every 15 seconds, which reduces power consumption and prevents signal collisions. IR was selected as the medium because it is relatively low cost, and an IR broadcast is able to be restricted to within a room, as the signal cannot penetrate walls. This makes room localization easy: if one sensor is placed in each room, the location of a particular badge is the room associated with the single sensor that can see that badge. Although simple and low cost, by its nature the system's accuracy is limited to reporting accuracy the nearest room. It also requires a significant amount of infrastructure in the form of a badge for every user and a sensor in each room, making it somewhat impractical for large installations and casual use.

Ultrasonic

The Cricket system [84, 85] is a hybrid ultrasound and radio frequency (RF) based positioning system. It uses custom low-cost hardware in an infrastructure of listeners and beacons. Mobile beacons send an RF and ultrasound pulse simultaneously, at random intervals, and fixed listeners measure the difference in the time of arrival of each. This difference is used to infer the distance of the user from the beacon. The system estimates position to within 25cm, and heading to within 3 degrees. While the authors argue that the system is relatively low cost, for building-wide deployment a large number of beacons are required as ultrasonic signals are blocked by walls. Moreover, as ultrasound receivers share a common frequency, interference between neighbouring beacons places an upper limit on the density of beacons.

Hazas and Ward [44; 45] use wide-band ultrasonic equipment to eliminate the interference between transmitters that occur in the Cricket system, and to allow information from multiple nodes to be considered in the positional estimate. Similarly to GPS, each transmitter's signal is modulated by a unique code to create a CDMA signal. Position is then determined using two alternative methods. For the asynchronous method, the receiver is informed of the time at which the transmitters sent the last ultrasonic pulse via a wireless communications channel. The receiver then trilateral its position using the time of flight and an estimate of the speed of the signal. In the asynchronous method, the receiver is not

Figure 2.1: *Prototype Locata receiver [6]*

aware of the time at which the pulse was sent, and pseudo-range positioning is performed similar to GPS. The system has comparable average accuracy to the Cricket system, but has better performance in noisy situations and is more scalable due to the utilization of a wider spectrum.

Dijk et al. [23] propose a method for using a single ultrasonic base station to provide accurate room level positioning. The authors argue that for use in homes, the extensive infrastructure and installation effort of existing indoor positioning systems make them unsuitable for home use. Their system consists of three ultrasound transmitters inside a single unit and a receiver on the mobile device (although in theory the transmitter/receiver configuration would work vice versa as well). The unfavourable dilution of precision and increased obstruction of line of sight brought about through the use of a single base station are shown to reduce accuracy compared to a typical multi-station ultrasonic installation, but not greatly enough to reduce the usefulness of the system. The median distance error ranges from between 20 centimetres for a direct line of sight scenario to 50 centimetres when the path was blocked (although the error for this scenario was bounded very high, at 2.5 metres for the 90th percentile).

Radio Frequency

Locata [6, 8] (figure 2.1) is a pseudolite based positioning technology with sub-centimetre accuracy, designed for ubiquitous indoor and outdoor use. It performs positioning similarly to GPS (that is, using TOA), except the constellation is made up of pseudolites (LocataLites) rather than satellites. Locata also uses carrier-phase positioning, a technique prohibitively expensive in most GPS systems due to the need for more complex electronics. The cost of the system makes it mostly appropriate for surveying applications or commercial enterprises such as mining.

Multipath fading is the main source of error in the carrier-phase technique used by Locata [87]. Satellite based navigation is less susceptible to this type of error as the elevation angles of the direct path between receiver and transmitter are greater, and therefore signals reflected from the earth's surface in particular can be attenuated using directional antennas. Ground based systems such as Locata must work with very low angles of elevation and therefore must use alternate methods to deal with multipath fading caused by signals reflected from the earth's surface.

In Barnes et al. [7], multiple transmission antennas separated in space by up to 1.5m are used to help successfully mitigate multipath fading. It is also suggested that future Locata systems will include the ability to transmit concurrently in a second frequency, which is expected to further reduce multipath fading.

Nerguizian et al. [74] propose a localization technique for use in mining, where NLOS situations are common. Rather than use a time or angle of arrival approach, both of which are affected badly by NLOS conditions and multipath fading, or signal strength, which fluctuates over time, the locations are characterized by features of the channel's impulse response for LOS and NLOS situations. This is a form of location fingerprinting similar to that used in signal strength based approaches to 802.11 positioning. They use a neural network to compare incoming signals with previously recorded training data. Empirical testing in an underground site proved the system could produce accuracy of less than 2 metres in 80% of cases.

MeshNetworks' Positioning System (MPS) [71] uses the time of flight of 2.4 GHz radio signals to triangulate the position of wireless nodes. The system uses Quad Channel Military Radio (QCMR), a radio technology designed for wide area systems. QMCR radios have a range of 1 mile with direct line of sight and can function in moving vehicles at speeds of up to 250mph. Position accuracy is stated as better than 10m.

A large number of very high accuracy systems have also been developed specifically for augmented reality, e.g. [36, 35, 64, 50]. Typically these rely on inertial sensors, ultrasound, and/or image recognition. These systems can be accurate to the millimetre level but require expensive and elaborate infrastructure and hardware. By contrast, our research is concerned with providing a relatively accurate solution that runs on existing consumer hardware for a very low cost. Therefore, such systems will not be reviewed in detail.

2.2.1 Ultra-Wideband

Ultra-wideband (UWB) radios spread signal power over a wider range than conventional narrow-band radio transmitters, which affords several advantages. The larger number of frequency components within the signal increases the chance that the signal can pass through objects, and the increased spread of signal power means that UWB systems can share the spectrum with other radios with reduced interference. AOA, TDOA/TOA, and signal strength based approaches are possible [69, 39].

Time based approaches to positioning (that is, TDOA or TOA) can yield accuracy less than an inch under ideal conditions. Similarly to GPS, clock jitter accounts for a large amount of inaccuracy and must be taken into account in position calculations. Other sources of error are multipath propagation (resulting in a scrambling of the signal), multiple access interference from other radios, and NLOS conditions [39].

Fontana et al. [32] describes the commercialisation of the first commercial UWB asset tracking system, deployed for use in hospitals and industrial facilities. The system uses TDOA and requires a minimum number of three receivers to have direct, or attenuated, line of sight to a transmitting tag. The sensors have a range of greater than 180 metres in free air, and were tested to have a range of 60 metres through as many as 10 commercial-grade walls. The stated accuracy of the system is within 30 centimetres.

The commercial Ubisense UWB based asset tracking system [110] provides 15cm 3D accuracy using Ubisense Tags that attach to critical assets and Ubisense Sensors that use a combination of TDOA and AOA to estimate the position of the tags. Sensors are grouped into cells which are typically rectangular in shape. In each cell a master sensor co-ordinates the activities of the other sensors and communicates with all the tags whose location is detected within the cell. The Ubisense system has an update rate of 160Hz and the sensors are claimed to have a range of up to 160m. Details on how the system degrades under multipath and NLOS conditions are not provided. The company claims to have several major corporate clients including BMW, GM, and DHL.

Time Domain's PLUS RTLS is another commercial UWB system with stated accuracy of less than one metre [107].

2.2.2 Standards

ISO/IEC 24730 [51, 52] defines two air interface protocols and a single application programming interface (API) for real-time location systems to use in asset management. The standard does not define the localization method, only communications protocols for accessing position information. The air interference specifies a scheme whereby tags transmit periodic time stamped 'blinks' which are used by the infrastructure to estimate position using time based approaches, amplitude triangulation, or angle of arrival. Whichever method is used is up to the implementor. The standard is supported by WhereNet's Where-Tag [116] and G2 Microsystems' system-on-chip [37].

Standards for localization in cellular networks are released on an ongoing basis by the 3rd Generation Partnership Project (3GPP) and the Open Mobile Alliance (OMA) standards bodies [1, 78]. These standards primarily deal with protocols for exchanging location information between handsets, across networks, and between providers (to support roaming handsets). A detailed inventory of these standards is out of the scope of this work.

2.3 802.11 positioning

Signal Strength Fingerprinting

Most research in the 802.11 positioning field builds on the basic ideas presented in the pioneering RADAR system [4]. RADAR is a radio frequency based system for positioning and tracking, using only commercially available RF receivers and access points. The system repeatedly interrogates a consumer model chipset for the Received Signal Strength Indicator (RSSI), an indicator of received signal strength power, expressed in units specific to the RF hardware. RSSI varies consistently according to position, and is made readily available by the hardware and operating system. Notably, RADAR is implemented using pre-802.11 standard wireless hardware (Lucent WaveLan technology), but the techniques it employs are readily applicable to any 802.11 system.

An off-line calibration phase takes place prior to on-line positioning use. During this phase, a signal strength map of the area is generated, which maps signal strength data to position. During the on-line phase, positioning is performed by matching the observed RSSI to the map, and taking the corresponding position as the output.

Two techniques are presented for generating this signal strength map. In the empirical approach, the positioning site is manually surveyed for the signal strength at discrete set of evenly spaced positions. A user equipped with a laptop computer or PDA makes entries in the signal strength map by walking the area and indicating their current position on a graphical map. For each of these positions, the signal strength data is recorded while facing north, south, east and west. Separate recordings for each heading are necessary as the observed signal strength varies significantly when the direct signal path is obscured by the mobile user's body.

The second approach taken in RADAR is predictive modelling. This technique aims to be cost effective compared to the fingerprint method by eliminating the need for a lengthy surveying phase (potentially several hours for a single office floor). Given that the access point locations are known, the predicted signal strength is calculated taking into account signal attenuation caused by distance and obstructions caused by walls. The model is empirically parameterised using both the rate of decay and the attenuation caused by each wall.

Figure 2.2 shows a map of the fingerprint points used in RADAR. The distance between each fingerprint varies, but it is on average 1.5 meters.

During the on-line phase, position is calculated using the Nearest Neighbour algorithm (described in detail in Chapter 3). This algorithm ranks each fingerprint according to the magnitude of the Euclidean distance between the current input and each fingerprint. The final output is either the position of the fingerprint with the closest distance to the input, or positions from a set of the nearest fingerprints can be spatially averaged (Section 3.6.2) to give a more robust result.

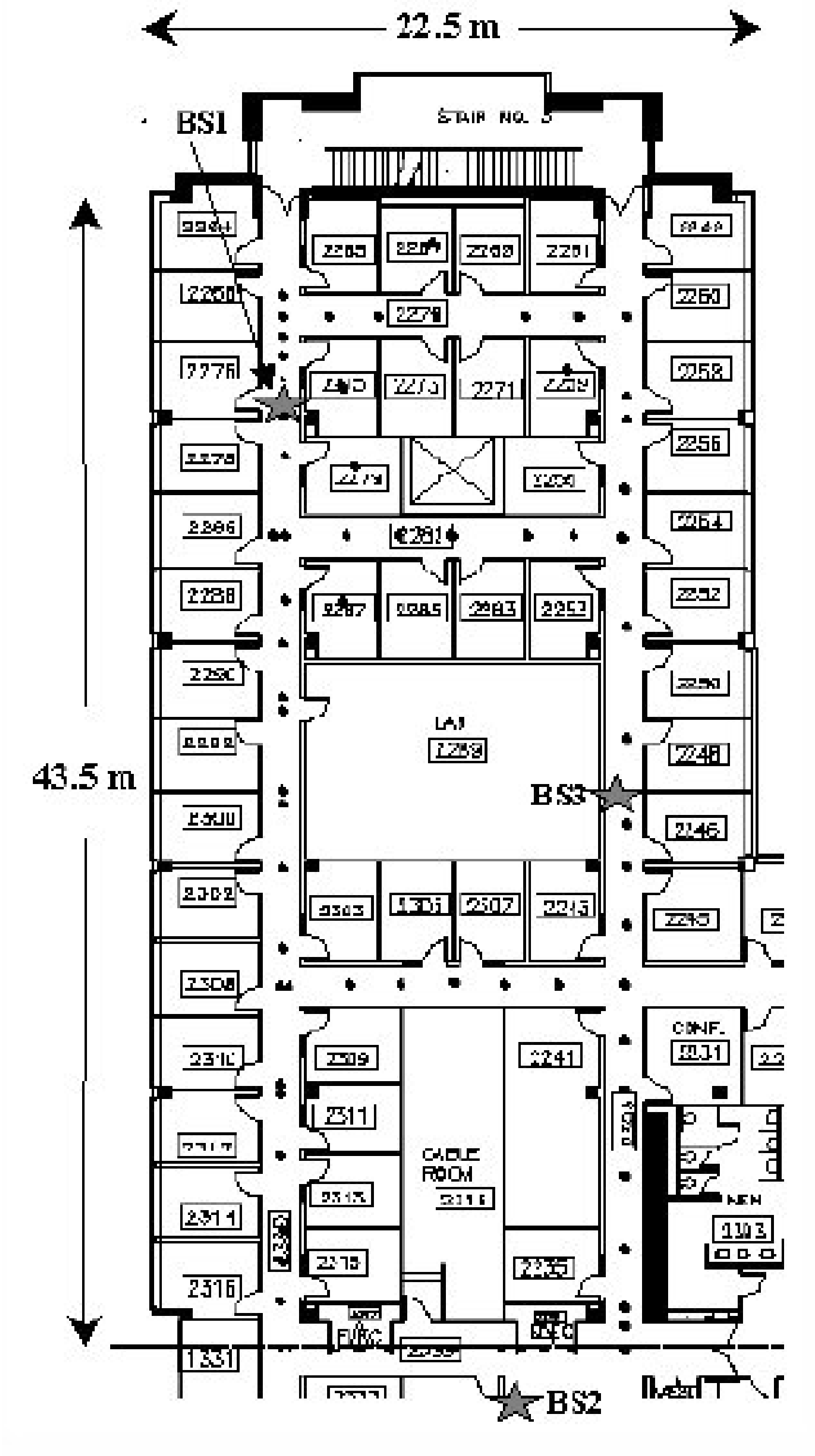


Figure 2.2: *Map of the floor in RADAR where the experiments were conducted. The black dots denote locations were empirical signal strength information was collected. The large stars show the locations of the 3 base stations [4].*

To test accuracy, a mobile user is simulated by removing a fingerprint at random from the signal strength map and using its values to simulate the observed signal strength as input. The positional output is then compared to the actual location at which the fingerprint was recorded, and the error distance calculated as the Euclidean distance (in metres) between actual and predicted locations. Removing the test point from the map before testing ensures that the test data is separate from the map, but it also means that the minimum distance error is the distance between the test fingerprint and it's neighbours; hence the results are somewhat conservative.

Tests took place over a series of hallways covering an area of 22.5 metres by 43.5 metres, covered by 3 base stations. The system has a median error of between 2 to 3 metres using the fingerprinting method, and 4 to 5 metres using the predictive model.

Kotanen et al. [58] and Prasithsangaree et al. [83] independently produced similar results using the same Nearest Neighbour algorithm and testing procedures.

Smailagic and Kogan [99] compared several signal strength map based approaches: using the location of the nearest station, two vector-based approaches similar to that used in Bahl and Padmanabhan [4], and a novel approach called CMU-TMI (Carnegie Mellon University's Triangulation, Mapping and Interpolation algorithm). CMU-TMI uses a mapping scheme between signal space (based on a predictive model of signal strength) and physical space (the actual signal strength map) to produce a continuous interpolation of the signal strength map. In order words, the algorithm can estimate signal strength positions at locations between fingerprints. This can make the positional output more robust; during online positioning, input signal strength vectors that fall outside the sampled range of the actual training data can still produce a reasonable estimate. This is particularly useful for reducing the number of fingerprints required to produce an accurate estimate, which could make the system more practical to use over a wide area.

Castro et al. [14] retains the RADAR approach of building a signal strength map in the off-line phase and performing positioning using that data as a reference in the on-line phase, but use a Bayesian approach to compute the best match between the input and the map. The data recorded during the off-line phase is used to generate empirical probability distributions that describe the probability of subsequent signal strength observations during the on-line phase. During the on-line phase, Bayes' rule is used on the input and the map to infer the probability of being at each of the fingerprint locations.

The advantage of the Bayesian method is that it utilizes the frequency distribution of all signal strength values, a greater amount of information than the Nearest Neighbour approach, which only considers the average. I describe the Bayesian approach in detail in Chapter 3.

They determine the likelihood function through empirical sampling of the signal-to-noise-ratio (SNR), discretised as a histogram into 4 possible values at each location. The prior probability is modelled by manually specifying the possible transitions in location, i.e. which rooms it is possible to walk between. It is also suggested that the prior probability could take into account observed user movement patterns (as opposed to a motion model), but the authors do not explore this approach. The use of SNR could be contrasted to the use of RSSI, but it appears the authors simply chose it because it was the metric made most readily available by the chipset's API.

In contrast to the co-ordinate system used in RADAR, signal strength information is gathered over designated cells (such as rooms or sections of hallways). This seems to be because the authors desired a room location service rather than a co-ordinate based output, but no explicit reason is given.

Few quantitative results are presented describing the accuracy of the system. An accuracy of 97% correctness was achieved during one test in a series of hallways, although the meaning of this is unclear as the distance between test points and fingerprints is not stated. Later, the authors suggest that the system can distinguish between a cluster of cubicles, but not a cubicle within a cluster; this suggests similar performance to RADAR.

Roos et al. [90] also uses Bayesian techniques to perform positioning, but uses a co-ordinate system based approach and evaluate accuracy using a quantitative error distance analysis similar to RADAR's. The empirical signal strength values for each fingerprint is used as training data and the result is calculated using Bayes' rule. The parameter used for positioning is the Received Signal Strength Indicator (RSSI).

The signal strength map is arranged similarly to RADAR, with the local co-ordinate and heading of each fingerprint mapped to signal strength data. The signal strength data was represented as either a normalized frequency histogram of signal strength values, or as a sum of Gaussian kernels about each signal strength value. Only the empirical method of sampling is considered for generating the signal strength map, which is gathered facing north, south, east and west as in RADAR, except with lower granularity (on a 2 metre grid, as opposed to 1.5 metres).

The testing method is also similar, except that separate test data, gathered at points in between the fingerprint locations, is used. The experimental results show that the Bayesian technique performs slightly better than the Nearest Neighbour. The median error is approximately 1.5 metres, and the 90th percentile error is approximately 3 metres.

In work similar to Roos et al. [90], Ladd et al. [61] also use a Bayesian approach (using a histogram to represent the signal strength distribution, as shown in figure 2.3), but use both RSSI and the access point's intermittency to generate the likelihood function, and apply a motion model for tracking mobile users to achieve a small improvement in accuracy. This is modelled by adjusting the prior probability of each location when applying Bayes' rule, based on the restrictions that a walking person cannot move too fast or change directions too quickly. They also use an unspecified adaptive sampling method based on the convergence to a stable distribution in order to reduce the sampling time.

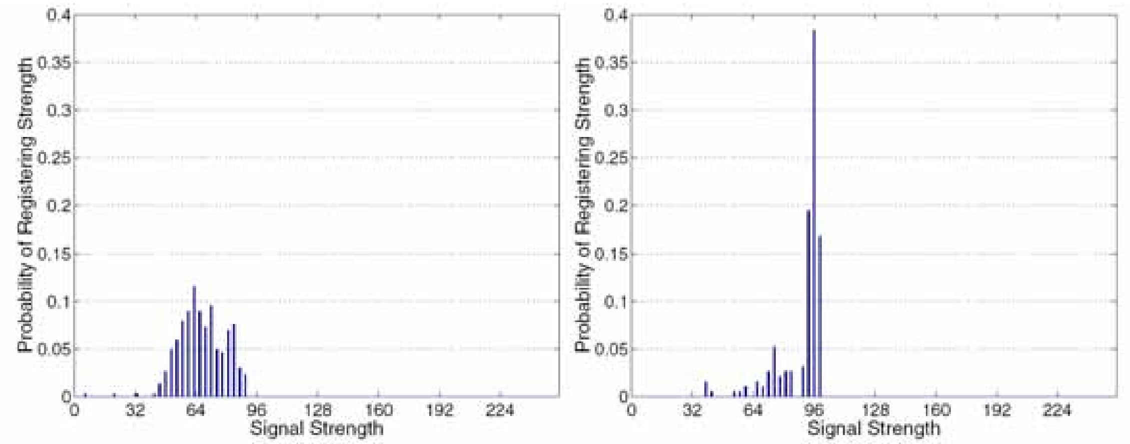


Figure 2.3: *Probability histograms for the same position facing two different directions, from [61]. The x-axis represents signal strength units from 0 to 255 (a hardware-specific unit, rather than dBm). The y axis represents the empirical probability of observation of each signal strength value.*

There is little detail given about the methodology of their accuracy testing, but their results state that a large number of tests (almost 50%) had a distance error of 0. This is impossible with the testing procedures used in prior research, which have test points at separate locations from the fingerprint locations (which the output is in terms of). I suspect the test results were at the same location as the fingerprints, although it is unclear whether the tests were drawn from the map data. This makes it difficult to compare the results to other work. There is also no comparison in accuracy with the Nearest Neighbour method.

Youssef et al. [122] also uses the Bayesian approach, but with clustering of the data according to the access points. The approach differs from the prior Bayesian methods in that it uses joint probability distributions across the data from several access points, clustered according to the access points with the largest signal strength values in each location. The prior approaches maintain separate distributions for each access point.

Testing was carried out using a cell based approach, with cells placed 1.5 metres apart, over an area of 67.5 by 25.5 metres. Their results demonstrate reduced computational complexity and improved accuracy compared to the Nearest Neighbour method (1.2 metres median, 90% of tests less than 1.8 metres error), but the analysis lacks a comparison to other Bayesian methods and their Nearest Neighbour implementation generates very poor results (median 4.5 metres) when compared to both our own and other research. They use separate training data in their analysis, but the locations of test points relative to the fingerprints is unclear.

In Youssef and Agrawala [120] the same authors propose a modification to their previous technique that slightly improves accuracy. The system detects when the locational estimate has changed over time by an unrealistic amount; in this case, greater than a threshold of 7.5 feet per second, for a walking user. When this occurs, the system adds an offset to the input data and re-estimates the position. While this was found to slightly improve their accuracy, the offset value used is highly specific to their data set and the technique may not generalise. More sophisticated motion model techniques such as those used in Haeberlen et al. [43] are likely to be more generally applicable.

Further improving the same system, Youssef and Agrawala [121] describes a technique for using multiple signal strength values in the online positioning phase to estimate a position, compared to most approaches which simply use an instantaneous snapshot of signal strength. Time series techniques are used to determine a probability distribution for input samples gathered over a time period and this is compared to the distributions associated with each fingerprint. Using this technique the distance error was reduced by over 50% compared to the system described in previous work (approximately 0.6 metres). It appears that the data set used is the same as in Youssef et al. [122], Youssef and Agrawala [120] although this is not explicitly confirmed. In this paper it is stated that the fingerprints were recorded in corridors only, rather than inside rooms of the building.

Haeberlen et al. [43] present the results of a Bayesian system for a system working over several stories of a large university building with total area of 12,558.4 square metres and 33 base stations. They use an empirical cell based map for localization, asserting that the cell approach proved to be more accurate than a point based map, although no results are presented to demonstrate this.

The RSSI distribution for each point is summarized as a unimodal Gaussian. The authors ignore the fact that at some points the signal strength can be multimodal as a result of multipath reflections, arguing that the technique negates the effect of transient minor modes corrupting the signal strength map, and that the space required to store a single Gaussian (mean and standard deviation) is much lower than a histogram approach. They remark that during their experiments, a bimodal Gaussian distribution model was also tested, but accuracy was not significantly improved (although they do not present the results), despite the existence of several locations with sparse (wide standard distribution) and bimodal distributions.

They also observe that signal strength varies significantly over the course of a day, mainly due to varying population of the area positioning is being performed in, as the presence of people affects the signal propagation path. Additionally, 802.11 hardware varies in the way signal strength values are reported, making it difficult to transfer their positioning system to a wide range of devices. Several methods for compensating for this distortion are proposed. Manual calibration involves recording data at a set of known locations and computing the least squares fit between the observed values and the corresponding values from the signal strength map. Quasi-automatic and automatic calibration methods use sophisticated machine learning techniques based on the Bayesian confidence to adjust the calibration parameters during on-line positioning.

The results presented are difficult to compare to other work. Most of the results are presented in terms of cell accuracy, rather than in metres, which is affected by variation in cell size and highly specific to their particular map. One set of results is presented in terms of metres, but the distance error is reported in terms of the distance between the correct and chosen cell and is therefore a fairly imprecise measure. This graph shows the correct cell was chosen in slightly less than 90% of cases, and less than 5 metres in 95% of cases. Most cells were office rooms of approximately 2.7 by 4.9 metres. Their Gaussian summarization performs better than a histogram approach, but no comparison to the Nearest Neighbour method is provided.

Siddiqi et al. [97] propose Monte-Carlo Localization (MCL) for 802.11 positioning. MCL uses a Bayesian approach similar to Ladd et al. [61] to infer position from training data and sensor measurements [33], and has been previously applied mostly to robot localization. Here, the scheme is adapted to signal strength fingerprinting and used to localizing a roaming robot. The robots rotary encoder (which measures relative movement) was used in conjunction with a probabilistic 802.11 signal strength positioning method and it was found that MCL produced superior accuracy to using only signal strength positioning in isolation.

It is noted that positioning a robot is easier than positioning humans as it is easier to build a motion model for robots and equip them with motion sensors. Experiments were carried out across four adjoining corridors forming a rectangle. The average distance error in the x-axis was 4 metres, while the average distance error in the y axis was 2 metres. The difference in accuracy between the two axes is attributed to the y-error being limited by hallway width.

Battiti et al. [9; 10] propose a neural network approach to localization and compare its accuracy to the Nearest Neighbour approach. Their experiment took place over a single office floor using three access points. It is not stated whether the test data was gathered separately from the training data. The neural network approach was found to have almost identical accuracy to the Nearest Neighbour method; each had average distance errors of 1.82 metres and 1.78 metres respectively. They suggest that given the results, the complexities of signal strength are not easily modelled with the neural network used.

Saha et al. [93] compare the Bayesian histogram method, the Nearest Neighbour method, and a neural network based method. The neural network based method slightly outperformed the other two methods in their experiments. They found that increasing the number of access points substantially improves accuracy as more information is available for the estimator to worth with. They also found that increasing the sampling time did not greatly improve accuracy.

Accuracy testing results are presented in two groups: when the test locations are the same as the fingerprint locations, and when they are not. All algorithms performed better in the first case than the second. The density of fingerprint locations is not stated, nor the exact size of the area considered, which makes it difficult to compare results with other work. The results produced very good accuracy regardless of the algorithm used, which suggested quite favourable conditions. When the test locations did not coincide with the fingerprint locations, the neural network algorithm had an average distance error of approximately 1.2 metres, with the Nearest Neighbour and Bayesian methods having average distance errors of 1.4 and 1.5 metres respectively.

A separate data set was gathered to consider how the number of access points affects accuracy. In this case, the authors describe data being gathered in a single corridor - probably too small an area to draw general conclusions. In the experiment using eight access points (compared to three, in the previous experiment) they were able to achieve an average distance error of less than 1 metre.

Gwon and Jain [42] propose a signal strength based localization method that does not require training or a fingerprint map, called triangular interpolation and extrapolation (TIX). TIX uses real-time measurements between access points to inform the model of signal strength decay across the area, which is then used to estimate the mobile user's location. Empirical results demonstrate that the algorithm produced similar results to the Nearest Neighbour algorithm (average distance errors of 5.4 metres and 4.7 metres respectively). These are relatively high distance errors, using the Nearest Neighbour results as a reference, indicating unfavourable conditions might have existed in their test domain. They gathered training data on an approximately 1.5m grid using test points at the same location as the fingerprints.

Niculescu and Nath [77] propose a system that uses the angle of arrival (AOA) from several base stations to estimate location. The technique uses custom base stations equipped with a rotating directional antenna. Unlike other high accuracy systems, no signal strength map is required, reducing deployment costs, and removing the need to recalibrate the system when the topology of the area changes. Accuracy is comparable to other 802.11 positioning systems, with a median error of approximately 2 metres. However, during location estimation the mobile user has to stop moving and face north, south, east, and west, waiting each time for the antenna to complete three rotations. The authors claim that this step is essential to determining an accurate AOA, as without it, the peak in signal strength recorded as the antenna faces the user is difficult to distinguish from peaks caused by reflections. This makes mobile tracking impossible.

Li et al. [65] use a hybrid method of predictive modelling and empirical surveying. Based on the principle that the signal is more predictable within a single room than globally, fingerprinting is used to isolate the correct room, then a predictive model of signal strength is used to determine the position of the user within the room. However, the results were similar to those available using the empirical method only.

Kaemarungsi [53] describes approaches for predicting the accuracy of a fingerprint based positioning system given the parameters of the system such as fingerprint density and number of access points used. It is determined that the orientation of the user is significant, that the mean and variance of the signal strength can drift over time, that the distribution of the signal strength is not log normal or Gaussian, that it is left-skewed, and that the standard deviation varies according to the signal level. Furthermore, signals from multiple access points are mostly independent.

A theoretical framework is then developed for establishing bounds on possible accuracy based on key parameters of the signal strength map and the algorithm used. Finally, a system design is proposed for estimating the performance of the system prior to carrying out the training phase. This system can potentially save time in the surveying phase by accurately estimating the fingerprint spacing required, and the number of access points required, to achieve a particular accuracy.

Kitasuka et al. [56; 55] propose WiPS, a signal strength based approach to 802.11 positioning that uses signal strength between mobile users in addition to the signal strength between mobile users and access points. No signal strength map is required. Simulation rather than empirical results are presented, the validity of which are questionable given the impossibility of simulating a room crowded with people (the conditions required for WiPS to function accurately).

Kushki et al. [60] propose a signal strength map based system using the Nadaraya-Watson estimator, a kernel based approach to estimating locational probability. The estimator has the advantages that it is non-parametric (that is, it represents the data without making assumptions of the shape of its distribution) and is capable of producing a locational output as the weighted average of the fingerprints, with weights determined by the associated probabilities. In this results, the approach is similar to the the k-weighting schemed proposed by Bahl and Padmanabhan [4]. The authors also propose a scheme for separating any multiple modes within each fingerprint into separate fingerprints representing the same location in order to deal with temporal variance of signal strength.

One problem with the proposed scheme is that key parameters such as smoothing factors had to be chosen manually according to the particular data set. The authors state that future work will attempt to determine general optimal values or a scheme for automatically determining them from the data.

Their empirical testing takes place over a small testing area of only one room and a single corridor, containing 10 test points and 19 fingerprints. Furthermore, orientation of the user was ignored, with all data gathered facing in a single direction. The results are also given in root mean squared error. These factors make the results difficult to compare to other work (which almost universally uses mean or percentile error). Nonetheless, their algorithm was shown to usually produce lower distance errors than the Nearest Neighbour method.

Lin and Lin [68] performed a comparison of Nearest Neighbour, probabilistic and neural network methods, implemented as described in the literature, across a small set of corridors (24.6 x 17.6 metres) and found that the Nearest Neighbour method was the most accurate, followed by the probabilistic method and then the neural network method.

Brunato and Battiti [12] proposes a positioning algorithm for signal strength fingerprinting based on statistical learning theory, named the Support Vector Machine approach. In empirical testing, the approach was found to produce similar accuracy to the Nearest Neighbour, Bayesian and neural network algorithms when considering outright distance error, but produced superior results when classifying signal strength input into different rooms. The median distance error produced by the SVM approach was 2.75m.

Notably, their testing procedure avoided testing facing different directions at each fingerprint; all data was recorded facing north. This may have affected the result as multi-directional fingerprints can produce a greater amount of ambiguity in the map and reduce accuracy. It is unclear whether separate test and training data was used, which would have also affected the results.

Evennou et al. [28] propose the use of a particle filter to constrain the output of a 802.11 positioning system based on a model of human motion, and in empirical testing this was shown to improve accuracy. However, their testing data was limited to a single walk around their area. This makes it difficult to be certain that the technique is generally accurate for humans moving in different ways (walking, running, browsing, standing still and so on).

In related work the same authors fused sensor information from an 802.11 fingerprint based positioning system and an intertial sensor using both Kalman and particle filters [27]. The inertial sensor contained an accelerometer, which was used to measure the footsteps of the user, and a gyroscope, which measured changes in heading. The inertial sensor guides the movements of particles within the system. The combined system was able to achieve a mean error of 1.53m using a particle filter, compared to 1.86m using a human motion model and 2.88m without any filtering.

Seshadri et al. [96] also used a particle filter and a human motion model for 802.11 positioning and reported reasonably accurate results, but they present no comparison with the accuracy of the system without the particle filter, which makes it difficult to guage its effectiveness. Their test data also consists of only one walk across the area.

Although the human motion model approach has not been exhaustively tested, the technique is promising and we discuss it as a potential avenue for future research in chapter 8.

Time-based approaches

Gunther and Hoene [41] investigate the use of RTT (round-trip-time) TOA to determine the range of the user from an access point. They use a software approach and a consumer 802.11 chipset to measure timestamps of incoming packets to estimate the round trip time and infer distance. Because the approach relies on software, the underlying data is only as accurate as the 802.11 hardware's clock (which is not designed for this purpose) and the timestamps can be distorted by variable chipset processing times and a multi-tasking operating system if the chipset heavily relies on software drivers rather than hardware processing.

Although actual positioning experiments are not performed in their work, trilateration could be performed using the ranges to calculate positions. They argue that the time of flight has a greater proportionality to range than RSSI, as they saw only minimal decay over a range of 40 metres in free air. This observation might be due to the fact that they observed the data over free air only, which disregards the usual effect of attenuation through walls. It also might have something to do with the chipset and method of signal extraction being used, as our equipment and that of others seems to be more sensitive to distance.

The ranging accuracy of the technique was rarely better than within 10 metres of the actual range, which compares unfavourably to previous fingerprint based methods. Furthermore, their experiments were conducted only in direct line of sight situations; it is highly likely that when the signal path between transmitter and receiver involves reflection (as in most indoor positioning scenarios) the effectiveness of the technique will be greatly reduced [81, 86]

They improved the accuracy of their approach in Hoene and Willmann [47], Vorst et al. [114], by measuring different packet sequences. Positioning accuracy is improved to being within approximately 4 metres. However, their empirical experiments again tested only direct LOS conditions, so the ultimate usefulness of their technique remains unclear.

Ciurana et al. [19; 20; 21] use a similar RTT TOA approach to calculate position based on arrival times of 802.11 frames. Their approach required a custom hardware timing module which observed transmissions on the communications bus of a consumer 802.11 card chipset. This eliminates some of the timing variation encountered by Gunther and Hoene [41]. In theory, a manufacturer could incorporate this functionality into their hardware if a commercial product was viable. Experimental results under direct line-of-sight conditions are promising, yielding on average 2.63m distance error. However, no results are provided in NLOS conditions, where the TOA approach can be expected to perform poorly [81, 86].

Yamasaki et al. [119] used a TDOA approach requiring time-synchronized access points and acheived accuracy within 2.4m in empirical experiments. However, similar to other time-based approaches, they did not perform experiments in NLOS conditions.

Manodham et al. [70] proposes a hybrid signal strength and TDOA approach which also aims to avoid interruption the network connection to scan channels; although accuracy results look promising they are generated by simulation only, with no physical implementation or testing.

Signal strength map interpolation

Li et al. [66] describe a method for generating denser signal strength map fingerprints by interpolating an existing map. As well as reducing surveying time, the technique can improve accuracy by increasing the resolution of the map. They tested their method using the Nearest Neighbour algorithm, and showed an increase in average accuracy from 2.5 to 1.5 metres using a very sparse signal strength map. Gains when the signal strength map was already fairly dense were smaller.

Commercial vendors

Commercial vendors such as Ekahau [25] and Aeroscout [2] provide 802.11 positioning using purpose-built WiFi tags as well as COTS devices by using custom access points which report position. Neither provide many details of their positioning methods, although Ekahau appears to use a signal strength based method while Aeroscout claims to use a combined TDOA and signal strength method. Aeroscout provides custom access points which are presumably capable of time synchronization with their custom tags to enable TDOA. Aeroscout in particular markets their system as a replacement for traditional RFID systems which use low range, inexpensive tags with relative expensive RFID reader infrastructure. Their tags also incorporate a traditional short range RFID which can be read by an RFID reader if more precise accuracy is necessary when passing through a critical area.

Cisco's Wireless Location Appliance provides signal strength based location-aware infrastructure for 802.11 networks using Cisco access points. Accuracy is stated as 'within a few metres' [18]. Newbury Networks provide a similar location appliance working with 802.11 infrastructure. They claim to be able to 'locate devices to room level with proven accuracy at greater than 99% in under 30 seconds' [75].

WhereNet's WhereTag IV is an asset tracking tag capable of operating in either 802.11b mode, compatible with Cisco's Wireless Location Appliance, or in ISO/IEC 24730-2 mode (see section 2.2.2), where it uses time of flight measurements to estimate position accurate to within 2 metres [116].

G2 Microsystems has developed a system-on-chip solution for asset tags containing a low powered 802.11 chipset with positioning technology built in. They claim that the tags can provide location based on signal strength or using TDOA, but do not provide accuracy specifications [37].

There are also a large number of commercial vendors supplying RFID tags and infrastructure used for asset tracking (for example WiseTrack [118]). The position of the asset is derived from its current connectivity to one or more tags, or by the tags it was most recently connected to (for example, if it passes by them on a conveyor belt). While this is a relatively crude form of location tracking, it is out of the scope of this work.

2.4 Other consumer wireless technologies

This section describes positioning systems built on consumer wireless technologies other than 802.11.

2.4.1 RFID

Ni et al. [76] propose the LANDMARC system for positioning using RFID tags. RFID readers were placed in the environment, and the mobile user wears RFID tags. In order to nullify dynamic environmental effects causing signal fluctuation, reference tags are also placed in the environment. The authors argued that the reference tags are subject to the same environmental interference effects as those attached to the mobile user, and therefore help to offset them. The similarity between the signal strength information from the mobile user's tags and the reference tags are then used to infer position.

Line of sight issues caused by temporary obstructions were found to reduce the systems accuracy as they unpredictably attenuated or altogether blocked the signals from the tags. This issue was dealt with by increasing the tag density, thereby making it more likely that enough tags would be visible to make an accurate estimate. LANDMARC's latency was quite high, taking up to a minute between scans, due to the signal strength information being made available only through active scanning of each channel, rather than being delivered as the tags were scanned. The authors claim this could be easily resolved by the manufacturer. Collision avoidance between the 500 tags used in their experiments also caused long scan times (7.5 seconds) when reading tag presences. Again, this might not be a problem with all RFID solutions, and might be resolved by the manufacturers. In terms of accuracy, the systems typical median error was around 1 metre. However, the system is likely to be impractical over an area larger than a room due to the large amount of effort involved in deploying and calibrating the tags.

2.4.2 Bluetooth

Pandya et al. [82] compare the accuracy of Bluetooth and 802.11 positioning using the same data set, as well as fusing the output between both. The authors expected, but did not obtain, better results from Bluetooth on the basis that the service radius of Bluetooth access points is smaller than that of 802.11 access points. This is an erroneous assumption: both are 2.4Ghz RF standards and therefore both experience a similar attenuation gradient, so fingerprints are equally distinguishable with either. The smaller service radius is simply due to the initial broadcast power being lower.

Tadlys' Topaz Location Tracking System [104] is a commercial tracking system based on Bluetooth tags with accuracy claimed to be up to 2-3 metres on average. The positioning delay is approximately 15-30 seconds.

Bluelon [11] provides a commercial asset tracking system based on Bluetooth tags. Their BodyTag optionally contains an accelerometer which could possibly be used in combination with an RF positioning method to provide a more solid positional estimate. Although the company lists position tracking as one of their capabilities, this is not emphasised and no accuracy figures are given.

Kotanen et al. [57] performed signal strength based Bluetooth positioning using a predictive model of signal strength and an extended Kalman filter to estimate new position data based on the current location. Measurements were only considered within a single room with direct line of sight conditions. This makes the real-world effectiveness of their approach difficult to evaluate as accuracy is likely to severely degrade under NLOS conditions where signal strength is much harder to predict than through free air. However, it is likely that the approach could be combined with more sophisticated signal strength modelling approaches such as those used in Bahl and Padmanabhan [4], Li et al. [65] to provide a working system.

2.4.3 ZigBee

AwarePoint's Realtime Awareness Solution [3] is an asset-tracking system using ZigBee (IEEE 802.15.4) technology. ZigBee is a wireless standard that has lower power requirements, lower data rate, and lower cost than 802.11 or Bluetooth, and comparable range to 802.11 (1-100m). AwarePoint claim that ZigBee is superior to 802.11 tracking systems in terms of accuracy, interference, network security concerns and cost. The company argues that their bespoke system will always be superior to attempting to leverage existing 802.11 infrastructure. Accuracy of the system is stated to be within 1.5 metres or to room level. The exact mechanism used for localization is not specified.

Cho et al. [17] describes the implementation of a ZigBee location system using time of flight measurements. The network uses a bespoke system of sensors and readers. Basic testing in LOS conditions showed the system to have accuracy within 3 metres.

2.5 Summary of Positioning Systems

Tables 2.1 and 2.2 summarize the positioning systems from the literature in terms of positioning method and accuracy. The former summarizes some wide-area systems and non-fingerprint-based 802.11 positioning systems, while the latter table specifically describes 802.11 fingerprint-based systems. Unfortunately there is no standard method of describing accuracy for positioning systems. Mean distance error is often used, but particularly in the 802.11 positioning field researchers use several different methods to describe their system's accuracy, such as the median, 90th, and 95th percentile errors. This makes summarization and comparison difficult. More details on respective errors can be found in the references themselves.

Table 2.2 also includes information on the size of the area in which experiments were performed, the number of access points used, and the number of access points per square metre. Unfortunately, this information is somewhat unreliable as some authors do not include the exact numbers of access points in the entire area, instead stating the average numbers contained in range of each fingerprint. The number of access points per metre square is also an extremely rough measure as some fingerprint maps do not cover the entire area of the locations used.

For the sake of providing a meaningful comparison, table 2.2 is restricted to containing only systems that have been empirically tested using a reasonably similar methodology to the RADAR system over a reasonably sized area (at least a single office floor).

2.6 Conclusions

In this chapter the state of the art in wireless positioning systems was described, with specific attention to indoor and 802.11-based systems. In the following chapter, the operation of 802.11 fingerprint-based systems is described in comprehensive detail, including techniques from the literature mentioned in this chapter and novel methods of our own design.

Method	Accuracy
GNSS (Multilateral RF TDOA from orbital transmitters)	mean 5-10m (unassisted), 1-2m (differential), 1mm (carrier-phase)
802.11 beacon database [63, 98]	mean 10-100m
Cellular Phone TDOA/TOA [103, 24, 94]	50-500m
Cellular Phone Location Fingerprinting [15, 62]	mean 5-90m (depends on fingerprint granularity)
Infra-Red Connectivity [115]	Current Room
Ultrasound [85, 45]	mean 25cm
Pseudolites (Multilateral RF TDOA from ground based transmitters) [6]	$<$ 1cm
UWB [32, 110]	mean 15-30cm
RTT TOA 802.11 [19, 20, 21]	mean 2-3m
802.11 signal strength AOA [77]	median 2m

Table 2.1: *Summary of non-802.11 fingerprint-based positioning systems.*

System	Algorithms Evaluated (most accurate first)	Map Resolution (m)	Test Procedure	Accuracy	Access Points	Area (m^2)	APs / m^2
RADAR [4]	Nearest Neighbour	1.5	Tests drawn from map, point excluded	Median 2.1m	3	978.75*	0.0031
Prasithsangaree et al. [83]	Nearest Neighbour	1.5	Unclear whether tests separate from map	Average 5.7m	10	425.25	0.0235
Smailagic and Kogan [99]	CMU-TMI, CMU-PM, Nearest Neighbour	14	Separate test points	Median 2m, 90% $<$ 4.5m	4-6 in range	500	unknown
Roos et al. [90]	Bayesian Histogram, Kernel	2	Separate tests at different locations	Median 1.45m, 90% $<$ 2.76m	10	640	0.0156
Youssef and Agrawala [121]	Probabilistic Clustering, Nearest Neighbour	1.5	Separate data from map, unclear locations	Average 0.6m	12	1,721.25*	0.0069
Haeberlen et al. [43]	Bayesian Gaussian	Avg 2.7x4.9 cells	Separate tests	90% cell correctness	Mean 14.86 in range	12,558.4	0.0026
Battiti et al. [10]	Nearest Neighbour, neural network	1.5	Unclear whether tests separate from map	Average 1.78m	3	624.75	0.0048
Gwon and Jain [42]	Nearest Neighbour, Centroid, Smallest Polygon	1.5	Separate test points at fingerprint locations	average 4.7m	4	1020	0.0039
Li et al. [66]	Nearest Neighbour	1.5-2	Separate tests	Median 1.2m, 95% $<$ 2.5	5	402.5	0.0124
Brunato and Battiti [12]	SVM, Bayesian, Nearest Neighbour	1.5	unclear whether tests separate from map	Median 2.75m, 95% $<$ 6.09m	6	750	0.008
Pandya et al. [82]	Smallest Polygon, Nearest Neighbour, Centroid	2	Separate tests at different locations	Average 4m	3	827.54	0.0036

Table 2.2: *Summary of 802.11 fingerprint-based positioning systems. * indicates that the data was gathered in corridors, rather than over the entire area.*

Chapter 3

802.11 Positioning Methods

This chapter describes the methods used in our work to convert signal strength data to a positional estimate. The fundamentals of 802.11 networking are reviewed, followed by an overview of indoor signal propagation and a discussion of the inputs and outputs of 802.11 signal strength based positioning systems. The fingerprinting technique is explained and discussed in the context of indoor RF signal strength propagation. Finally, three classes of algorithms are discussed that are experimentally compared in Chapter 5. The algorithms are the Nearest Neighbour [4] and Bayesian methods [14, 90, 43] and our novel Distribution Distance algorithm.

3.1 802.11 Networks

802.11 or ”WiFi” is a radio frequency wireless networking protocol widely used by mobile devices. There are three officially defined variants: 802.11a, 802.11b, and 802.11g. The 802.11b and g standards operate in a 2.4GHz ISM (Industrial, Scientific and Medical) band, chosen due to the fact that it is unlicensed in most countries. 802.11a operates in channels ranging between 4.9GHz and 5.8GHz, depending on the regulations of the country of operation. 802.11b has a maximum data rate of 11MBit/second while b, g and a run at up to 54Mbit/second. All the standards have the ability to throttle data rates down to maintain the link in adverse conditions. 802.11g is backwards compatible with b, and therefore most new 802.11 hardware supports both. 802.11b and 802.11g support up to 14 channels, depending on the regulations of the area. However, the spectrum bandwidth occupied by 802.11 communications is enough to cause these channels to overlap, which largely restricts installations to deploying no more than three access points within range of each other, to minimize interference [48, 38].

802.11a can achieve up to 54Mbit/second speeds, the same as 802.11g. The channels it operates in face more restrictions worldwide, and 802.11a equipment has a shorter range than 802.11g using omnidirectional antennas, which has largely stifled its popularity, despite preceding 802.11g by several years [38]. Operation in the 5GHz band does however mean that it is less susceptible to interference than 802.11b/g. 802.11 devices using the 2.4GHz band must compete with Bluetooth devices, cordless telephones, microwave ovens, and other appliances. 802.11a also has 12 non-overlapping channels (compared to 3 for 802.11b/g), which allows a larger number of access points to be closely positioned without interference [38].

In our research 802.11b and g access points are used. There should be no difference in positioning performance between these two standards, because they both use 2.4Ghz RF signals which propagate identically, and as such this is not addressed by this thesis.

3.2 RF Signal propagation

In this section radio signal propagation is characterized in indoors environments. Indoor path loss can be modelled as having three components: free space loss, shadow fading due to attenuating objects, and multipath fading due to interference between multiple signal paths. Each of these components is reviewed in turn.

3.2.1 Free space loss

In free space, the relationship between transmitted power P_t and received power P_r is given by

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \tag{3.1}$$

where G_t and G_r are the transmitter and receiver antenna gains, respectively, d is the distance between the transmitter and the receiver, $\lambda = c/f$ is the wavelength of the transmitted signal, and c is the velocity of radio wave propagation in free space, which is equal to the speed of light [80].

Defining $P_0 = P_t G_r G_t (\lambda/4\pi)^2$ (that is, d = 1) as the normalized received power at a distance of 1m, the above equation reduces to

$$P_r = \frac{P_0}{d^2} \tag{3.2}$$

In logarithmic form (decibel scale):

$$10 \log_{10} P_r = 10 \log_{10} P_0 - 20 \log_{10} d \tag{3.3}$$

Environment	m
Office Building [112]	4.2
Corridor [112]	1.2
Laboratory [65]	3.4

Figure 3.1: *Values of m for a 2.45GHz signal empirically determined in different environments.*

3.2.2 Shadow Fading

Indoor signal propagation, however, does not follow a free space model as there are usually objects blocking the direct signal path. The simplest method of modelling this is to parameterize the distance-power relationship given in equation 3.2 as

$$P_r = \frac{P_0}{d^m} \tag{3.4}$$

m is adjusted to fit the environment and the frequency of the signal. In corridors, m is usually less than 2 as the enclosing walls act as a waveguide; across entire buildings m is usually greater. Table 3.1 shows some empirically determined values of m from previous work.

Equation 3.4 expressed in decibels is

$$10 \log_{10} P_r = 10 \log_{10} P_0 - 10m \log_{10} d \tag{3.5}$$

If we define the path loss in decibels at a distance of 1m as $L_0 = 10 \log_{10} P_t - 10 \log_{10} P_0$ then the total path loss L_p in decibels is

$$L_p = L_0 + 10m \log_{10} d \tag{3.6}$$

Using this equation, the value of m can be used to summarize the entire environment by modelling it as 'thick space', including the effects of distance, multipath fading, attenuation through solid objects, and scattering. However, indoor environments tend to be complex and therefore this model is an insufficient approximation for the purposes of a positioning system [66]. One approach to dealing with this problem is to add a random variable to represent random path loss and consider the scenario probabilistically:

$$L_p = L_0 + 10m \log 10d + l \tag{3.7}$$

l is a lognormal-distributed random variable representing *shadow fading*; the average loss caused by a large number of objects over a large area. The variance l is adjusted according to the environment.

Alternatively, the effects of attenuation can be deterministically modelled by considering the effects of individual objects, such as the wall attenuation factor approach used by Bahl and Padmanabhan [4], Li et al. [65]:

$$L_p = L_0 + 10m \log 10d + \begin{cases} nw & n < C \\ Cw & n \leq C \end{cases} \tag{3.8}$$

n is the number of walls between the transmitter and receiver, w is the wall attenuation factor (in decibels, determined empirically), and C is the maximum number of walls up to which the attenuation factor makes a difference. More fine-grined approaches allow for each attenuating obstacle to be modelled individually according its particular construction [65].

3.2.3 Multipath Fading

In most radio channels the transmitted signal arrives at the receiver from various directions over several paths. This is particularly the case indoors, where more reflecting surfaces are located along the signal path than outdoors. The composite received signal is the sum of the signals arriving along different paths; reflected signals will be attenuated and delayed compared to the direct path signal. The amount of attenuation depends on how many times the signal was reflected before reaching the receiver and what materials constitute the reflecting objects. The amount of delay depends on the extra distance travelled compared to the direct path.

Because the received signal is the sum of multiple paths, the received power is the result of constructive and deconstructive interference across the paths. The interference pattern varies according to location and the frequency of the signal. At some locations the phases of the reflected signals will be aligned, causing constructive interference, at others the signals will nearly cancel each other, causing large reductions in received power. Fluctuations occur as the receiver or other objects in the environment moves on the order of a wavelength of the signal.

Multipath interference at any particular location is very difficult to predict as indoors environments are typically complex and modelling all the signal paths within them is not computationally practical. The multipath component can be modelled probabilistically as a Rayleigh distribution unless there is a strong LOS path, in which case it can be modelled as a Rician distribution [80]. However, as far as I know this model has not been applied to signal-strength based positioning; Bahl and Padmanabhan [4] explicitly avoided it in their work as the parameters of the model are too difficult to determine because this requires isolating the direct wave from the scattered components.

3.3 Inputs and Outputs

In this section signal strength based positioning algorithms are characterised by their inputs and outputs in order to place them in context.

3.3.1 Input

802.11 positioning systems infer position from data incidental to the networking operations of 802.11 wireless LAN. As discussed in section 2.3, the majority of these systems rely on the *signal strength* of 802.11 signals to characterize and identify locations. Other parameters, such as angle of arrival and time of flight, have been used successfully in other RF positioning systems, such as GPS and aviation radar, but with less success in 802.11; angle of arrival cannot be calculated without rotating antennas and cannot compute position quickly enough to tracking a moving target [77], while time of flight calculations are inaccurate because accurate timing information is not made available by most consumer 802.11 hardware [41]. Our research uses signal strength information only, as it is the mostly widely available and practical parameter.

Each 802.11 packet received by an 802.11 device has an associated signal strength reading that expresses the received power of the signal at the time the packet was received. Signal strength readings are used internally to dynamically adjust speed according to conditions, and to inform the end user so they can select the access point with the best reception. Signal strength readings also constitute the input to signal strength based positioning algorithms.

Signal strength is reported as the Received Signal Strength Indicator (RSSI) in units of a quantity defined by the hardware manufacturer. Usually the units are expressed as dBm, a logarithmic unit conversion of milliwatts that allows the expected range of values to be more effectively described. If not directly reported in dBm, RSSI can usually be directly converted using a lookup table [5]. The range of observable signal strengths is typically between approximately -10dBm and -100dBm, reported in integers, depending on the chipset [5, 38].

It is often difficult to programmatically access signal strength information from user applications. The method varies depending upon the operating system and hardware used. Sometimes the information is not available at all; sometimes only the names and MAC (media access control) addresses of access points are available without signal strength data; sometimes the signal strength data is misreported. Examples of such problems, and our methods for extracting signal strength for the experimental work in this thesis, are described in Appendix A.

Signal-strength based positioning algorithms take as input a *vector* containing the most recently detected signal strength readings from each access point. This vector can be obtained by either requesting a scan of the access points from the 802.11 hardware, or

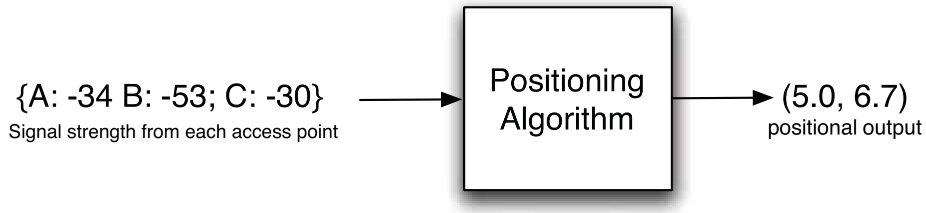


Figure 3.2: *Diagram showing an example of inputs and outputs to the 802.11 positioning algorithm. In this example, the units of RSSI are dBm and the output is given in two-dimensional co-ordinates. The input is a vector of signal strengths from three access points. A, B and C represent the access points used for positioning.*

by directly observing packets emitted by the access points over a period of time using a packet sniffer such as Kismet [54]. In this work, the latter approach is used (Section A). Figure 3.2 shows an example input vector. Separate signal strength values from three access points A, B, and C are observed and the resultant vector is used as input to the positioning algorithm. Note that in reality, the access point identifiers used are unique 802.11 Media Access Control (MAC) addresses rather than letters; letters are only used here for the sake of clarity.

3.3.2 Output

The final output format of the algorithm depends upon the client application. For most indoor applications, a tuple containing a two-dimensional x,y co-ordinate (usually in metres) and a floor number should be sufficient. This is the output generated by Bahl and Padmanabhan [4], Roos et al. [90] and Li et al. [66]. It is also commonly acceptable to report position in terms of labelled locations (e.g. ”Physics Lecture Theatre”) rather than co-ordinates. This is the output generated by Castro et al. [14] and Haeberlen et al. [43]. The example shown in Figure 3.2 has an output in two-dimensional co-ordinates.

Height estimation within a single floor is usually inaccurate: differing heights at the same x,y position typically share a similar path of obstructions and distance to nearby devices and hence have similar signal strengths. On the other hand, positions on different floors of a building should be easy to distinguish due to the large attenuation from the separating ceiling and floor and the fact that most 802.11 devices focus transmission power on a horizontal plane, causing signal strength to decline abruptly at angles not acute to the horizontal.

Rather than just the conventional output of a single positional estimate, sophisticated applications might benefit from output in the form of a probability map. A probability map associates each of a discrete set of locations with the probability that the mobile device is at that particular location. This form of output could allow an application to make probabilistic decisions based on uncertainty or take into account the possibility of the device being at multiple different locations.

3.4 Centroid method

The simplest 802.11 positioning algorithm is the centroid method [16]. The centroid method is not tested in this research, as the focus is signal strength map based strategies; however discussing it here illustrates the fundamental difficulties faced in a signal strength based RF positioning system.

Assuming free air between transmitter and receiver, radio frequency (RF) signals decay inversely according to the square of distance [105]. Using this formula, it is possible to convert signal strength into a distance (in metres), and therefore convert a set of signal strengths from neighbouring devices into a set of distances from each access point. The set of distances can be used to trilaterate a position, assuming the positions of the access points is known. Unfortunately, for indoors environments the path between transmitter and receiver is typically blocked by obstacles (such as walls), which cause the signal to be attenuated. Obstacles also partially reflect the signal, which results in the receiver observing multiple reflected signals in addition to the direct path; these echoes can sum to form significant constructive or destructive patterns. These factors cause signal strength to be heavily characterized by the environment in addition to distance, and not obey the inverse square law. Therefore, the centroid method is inaccurate indoors, as it does not take the position specific effects of the obstacles in the environment into account.

Although it is inaccurate, the centroid algorithm is simple to implement, fast to calculate, and the only prior knowledge required is the location of the access points, the transmission powers of respective base stations, and the gain of the receiving antenna.

3.5 Location Fingerprinting

This section discusses the technique of *location fingerprinting*, the method this research focuses on. The motivation for the technique is explained, and it is placed in the context of known models of indoor RF signal propagation. Two different types of fingerprinting are described: point and sector surveying, and finally some of the practicality trade-offs involved are discussed.

3.5.1 Signal Strength Maps

The problems with the centroid method lead to researchers generating *signal strength maps* that characterised the environment by associating signal strength with position [4]. These maps can be generated either empirically, by performing a site survey of the area, or predictively, by calculating signal strength using ray-tracing based on a model of the environment. Empirical methods have been shown to be more accurate [4].

The attraction of predictive modelling is that it removes the need for arduous and time consuming site surveying. However, accurate signal strength modelling requires inputting detailed information about the locations and materials of obstacles (such as walls and furniture). This process also takes some time, depending on the availability and organisation of building plans. This research exclusively focuses on empirical methods, although the algorithms can also be used with a predictively generated map.

Each entry in a signal strength map is known as a *fingerprint*, the idea being that the signal strength (from each base station) that each fingerprint contains identifies the associated position as uniquely as possible. The exact format of these fingerprints varies according to the algorithm, but generally involves some representation of the signal strength to be expected at that position. For example, fingerprints used by the nearest Neighbour Algorithm [4] (Section 3.6.1) contain the *mean signal strength* observed at that position during the surveying phase.

The signal strength maps are used to calculate position by comparing the input with each fingerprint and calculating which fingerprints are most similar to the input. The locations of the most similar fingerprints are used to generate the output.

3.5.2 Indoor RF Signal Propagation

The fingerprinting approach is based on the assumption that provided there is enough space between each fingerprint location, the signal strength fingerprints are sufficiently unique to be distinguishable by a positioning algorithm. This is an effective approach due to the natural attenuation of signal strength with increasing distance, as well as the attenuation of the signal as it passes through solid objects.

From a positioning accuracy perspective, a rate of signal strength decay yields poorer accuracy, as the distance between two points with distinguishable signal strength becomes smaller. For example, it is easy to distinguish between two positions of identical distance to the transmitter where the signal path to one is obstructed by a thick wall and the other is not; the attenuation through the wall is enough to be measurable. By comparison, it is harder to distinguish between two positions in the same hallway (assuming the hallway is free of obstacles) when the transmitter is at one end of the hallway, due to the low rate of signal decay caused by the enclosed reflecting walls.

Multipath fading presents problems for any signal strength based positioning system as it varies rapidly according to location. The spatial variance makes it difficult to build a sufficiently comprehensive signal strength map by surveying the area. It tends to create many small zones of interference which have a different signal strength spread to the adjacent areas. The small size of these areas can mean they are missed during the surveying process. Even worse, sometimes a particular fingerprint is recorded in one of these zones that is not representative of the local area.

The fundamental accuracy of any signal strength based system is largely defined by these two factors: the rate of signal strength decay and the degree of multipath fading.

Transient interference from other RF devices, and side lobe interference from other 802.11 devices can also cause problems with positioning as they unpredictably alter the signal strength. Ideally, the channels used for positioning should be sufficiently distant from other active channels. Gast [38] recommends using the separated channels 1, 6, and 11 for 802.11b and 802.11g in data transmission to reduce side lobe interference; this should also be sufficient for positioning purposes.

3.5.3 Point and Sector Surveying

This section compares two strategies for recording the signal strength map: point and sector surveying.

Surveying is complicated by the fact that the human body affects signal strength readings as it both attenuates and reflects the RF signal [4]. Hence, different directions faced at a position leads to different readings as the signal path is blocked, or not, by the person holding the device. The way the device is held also affects readings: a device in the user's pocket is reached by different signal path than a device held in their hand. Furthermore, although signal strength varies continuously over the environment, it is only practical to sample it at a limited number of positions.

Bahl and Padmanabhan [4] survey by recording signal strength at several discrete locations over the environment facing north, south, east and west. The locations were spaced approximately 1.5m from each other. This distance was chosen based on observations of signal strength propagation in their environment. This approach is simple to implement but captures a limited image of the environment as the map only represents the set of positions each fingerprint was captured at. I will refer to this type of surveying as *point surveying*.

Youssef et al. [122], Castro et al. [14] record data associated with sectors (small areas of the environment) rather than specific points but are not specific in whether they faced different directions during surveying, nor whether they stood still or moved the recording device around. Haeberlen et al. [43] records signal strength for room sized cells by walking about the sector during recording to capture the complete signal strength image of the sector; they claim this is more accurate than point sampling but do not provide an explicit comparison. I will refer to this type of sampling as *sector surveying*.

Point surveying misses a lot of space because it records data at a small set of positions within a relatively larger area. Sector sampling records much more as the surveyor sweeps the device through different positions constantly, so the image of signal strength is more complete. Furthermore, point-based signal strength maps can contain fingerprints distorted by pockets of multipath interference that are specific to the point locations, but not the immediately surrounding area. Conversely, pockets of multipath interference located at positions other than those sampled are not captured. Sector sampling captures these pockets, and assuming the surveyor records the sector reasonably evenly, they will be represented in the fingerprint data.

However, point sampling can be viewed as a more accurate description of the data. Point fingerprints have an associated direction, whereas sector samples contain data recorded facing in all directions. Also, because the surveyor is moving around the area during recording, it is difficult to record data exactly evenly about the space. This can lead to the recording being biased towards sub-areas where the device recorded more samples than others.

In Chapter 5, experimental results are presented comparing the accuracy of point and sector sampling.

3.5.4 Density of Fingerprints

Another way of increasing map coverage is to simply increase the density of fingerprints, using point sampling. However, this approach increases the amount of time taken during the surveying phase substantially. It will also not increase accuracy unless m is sufficiently high. The number of fingerprints n_f in a grid of width w and height h, for a spacing between points of s is

$$n_f = (\frac{w}{s} + 1)(\frac{h}{s} + 1) \propto \frac{1}{s^2} \tag{3.9}$$

As an example, Table 3.1 shows the minimum time required to survey a 50x50 metre area (the size of a large office floor) at several densities, assuming a sampling time of five seconds per direction faced. The time required becomes rapidly impractical for spacings below 2m.

Spacing (m)	Points	Time (hours)
2	676	3.8
1	2601	14.5
0.5	10201	56.7

Table 3.1: *Surveying time required for a 50x50 metre area with different fingerprint spacings.*

As well as the increased time required, increasing the density of discrete sampling locations still suffers from the problem that areas in between the fingerprints remain uncaptured. Many small pockets of fluctuation due to multi-path interference would not be recorded. One advantage that increasing density has over our suggested approach is that the extra captured data is locationally tagged with a finer granularity.

A spacing of approximately 2m is used in the experimental section of this work. This value has been selected as a compromise between accuracy and whether data could be gathered within a reasonable time in a real-world scenario. This spacing is similar to values used in the literature (see Table 2.2).

The surveying procedure is discussed in detail in Chapter 4.

Surveying Time

Factors to consider when evaluating surveying time are the battery life of the scanning device, ergonomic factors, the times in which the area is vacant of people, and how often the area is likely to have to be re-sampled.

The device used for scanning (a Sony VAIO U8G, see section A.1) had a battery life of approximately 2 hours, and a recharge time of approximately 1 hour. I also experienced some pain in the wrists after holding the device for these extended periods of time, which necessitated a break regardless. In the larger CSE Level 3 area, I gathered data in 2 hour stints, once per day, over a period of several days. The area was usually sufficiently vacant after 9pm.

For ergonomic and efficiency reasons it may be advantageous to employ multiple surveyors in real-world scenarios. The surveyors should be careful not to survey too close each other as they may interfere with the signal strength readings, particularly if they block the direct signal path.

Prior work relating to fingerprint density

Several authors have assessed the impact of fingerprint density versus accuracy as practicality versus accuracy is an important trade-off in 802.11 positioning systems. There is a definite trend that increasing fingerprint density above a certain threshold results in only small gains in accuracy.

Bahl and Padmanabhan [4] found that a data set of 40 fingerprints performed less than 10% worse than a set of 70 fingerprints. In other words, they almost halved the data set with only a minimal effect on accuracy - roughly equivalent to doubling their fingerprint spacing of 1.5m. Figure 3.3 shows the 25th and 50th percentile (median) distance errors according to the number of points used on a logarithmic scale.

They conclude that

> The diminishing returns as n becomes large is due to the inherent variability in the measured signal strength. This translates into inaccuracy in the estimation of physical location. So there is little benefit in obtaining empirical data at physical points spaced closer than a threshold.

Roos et al. [90] found that the average distance error increased from approximately 1.6m using all points available (approximately a 5m^2 area covered per point) to only 2.0m

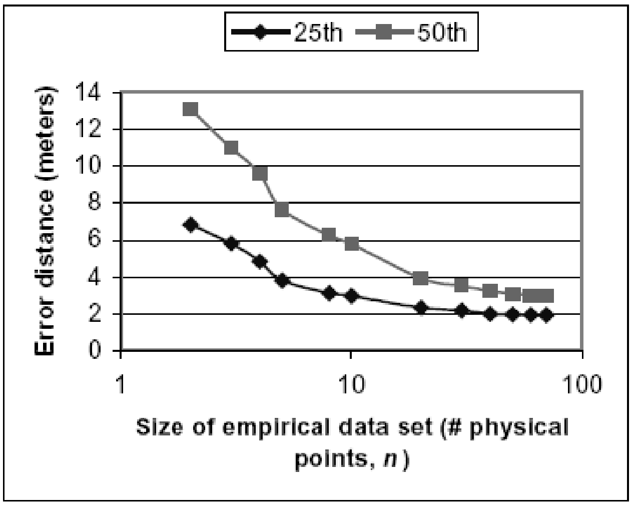


Figure 3.3: *Number of fingerprints versus 25th percentile and 50th percentile (median) distance error using a logarithmic scale from Bahl and Padmanabhan [4].*

when using points covering a 25m^2 area (see figure 3.4). Although not explicitly stated in their paper, by our calculations, this involved reducing the number of fingerprints from 155 to about 25 fingerprints - so the data set was reduced to 16% of the original with only a 25% performance penalty. In comparison, merely halving the number of points (approximately equivalent to increasing the area per fingerprint from 5m to 10m) resulted in a negligible difference in average error (see figure 3.4).

Li et al. [66] produced a similar result (see figure 3.5) demonstrating only a very small accuracy penalty after halving their data set.

These results all suggest that a substantial performance improvement from increasing fingerprint density is unlikely as there are only small increases in accuracy above a spacing threshold of approximately 3-4m.

3.6 Positioning Algorithms

In this section, our implementations of three classes of positioning algorithm are described. The first two are the Nearest Neighbour [4] and Bayesian methods [14, 90]. The third is a novel method called the Distribution Distance. These three classes of algorithm are used in the experimental comparison of algorithm accuracy in Chapter 5.

3.6.1 Nearest Neighbour method

The Nearest Neighbour 802.11 positioning algorithm, proposed by Bahl and Padmanabhan [4], treats signal strength data as vectors, where each dimension corresponds to data from a single base station.

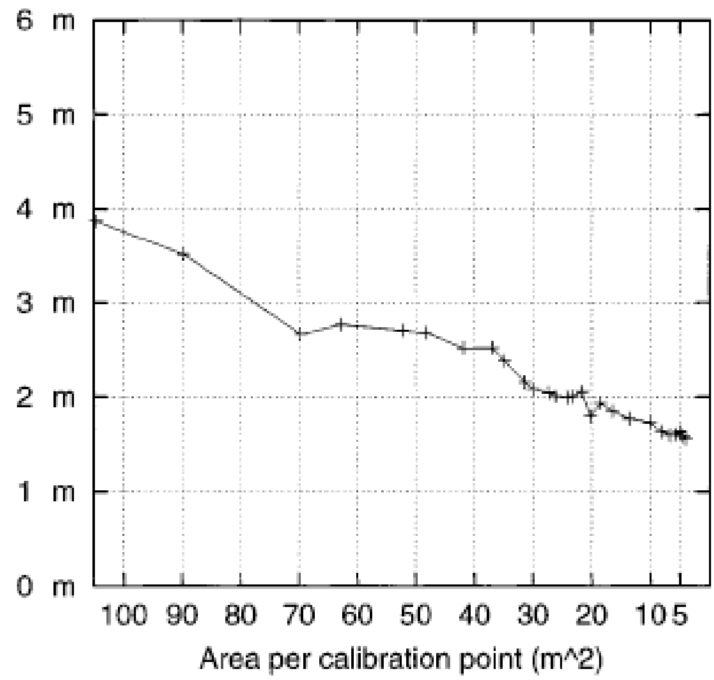


Figure 3.4: *Average error of the histogram method (y-axis) versus fingerprint density (x-axis, expressed as the area covered by each fingerprint, in metres) from Roos et al. [90].*

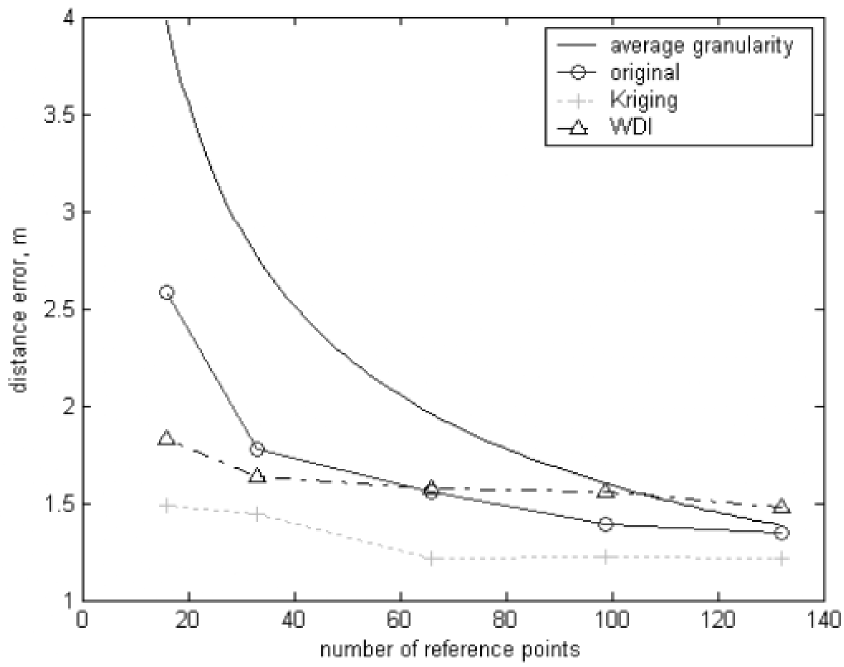


Figure 3.5: *Number of reference points (fingerprints) versus distance error from Li et al. [66] using 3 different algorithms. Regardless of the method used, halving the number of fingerprints only has a very slight impact on the distance error.*

The problem is modelled using a state space of fingerprint locations $L = \{l_1, \ldots, l_n\}$, and an observation space $O = \{o_1, \ldots, o_m\}$. Each state l_i corresponds to the location at which fingerprint i was recorded. Each observation is a signal strength vector $o = \{\lambda_1, \ldots, \lambda_J\}$, where λ_j is a signal strength reading from access point j and J is the number of access points in range. This notation will be used to describe this and subsequent algorithms.

Fingerprints are generated by standing at a location in the coverage area and recording signal strength data for some period of time. Using this data, the mean signal strength for each access point is calculated; each mean forms an element in a signal strength vector associated with that location. Hence, a fingerprint can be expressed as $f_i^{NN} = (l_i, \{\mu_{i,1}, \ldots, \mu_{i,n}\})$, where l_i is the location of the fingerprint, and $\mu_{i,j}$ the mean signal strength for access points b_j at location l_i. Here the location l can refer to either a Cartesian coordinate, with or without a direction (in the case of point sampling), or refer to a less well defined region such as a room or part of a hallway, as is common with sector sampling.

Once this map has been generated, online positioning is performed by calculating the distance between the currently observed signal strength o and each fingerprint. The fingerprints are then ranked according to this distance, with the top-ranked fingerprint containing the most likely position and the bottom-ranked fingerprint the least likely.

The p-norm distance is used to calculate the distance between the fingerprint and observation vectors.

$$D(f_i, o) = \left(\sum_j |\mu_{i,j} - \lambda_j|^p \right)^{1/p} \tag{3.10}$$

Bahl and Padmanabhan [4] use the Euclidean distance, $p = 2$ to calculate the distance between vectors, although any $p \geq 1$ can be used.

Increasing p might unfairly penalise momentarily noisy readings, but emphasising the largest differences while ignoring smaller discrepancies could be useful in this scenario. Li et al. [66] compare results with a range of p values and find that the results did not vary greatly.

When ranking distances, it is not actually necessary to apply the outer $1/p$: for any positive x and y and $p >= 1$, if $x >= y$, then $x^{1/p} >= y^{1/p}$. The same p is used in all distance calculations and therefore applying the outer $1/p$ does not actually affect the final ranking. Omitting this step can be useful for increasing performance. However, it may affect the accuracy of weightings, if the weighted mean of the results needs to be calculated as described below.

Chapter 5, presents empirical results comparing accuracy of the Nearest Neighbour algorithm with values of p from 1 to 10.

Dimensional Mismatches

If the positioning area is large enough, not all access points will be in range for the entire area. Hence, it becomes necessary to calculate a distance between two vectors whose access points do not match. For example, a fingerprint might not contain any data for a particular access point as it was recorded in an area where it was out of range, but the input data might contain such a value as the mobile device is currently in range.

The simplest method of dealing with this is to simply ignore the extra dimension, but this discrepancy can also be used to further disambiguate fingerprints. All fingerprints and input vectors are padded with a value slightly lower than the minimum receivable RSSI (-95dBm, and the minimum receivable is -90dBm on our hardware) for access points that are part of the positioning system but not in range. In other words, each access point that is not present in the input but present in the map, or vice versa, causes the distance sum to be increased by a value proportional to strength of the signal. The proportionality to signal strength is appropriate because lower signal strength values are logically closer to no signal being received at all. This is similar to the method described by Roos et al. [90].

3.6.2 Output averaging

Taking the output ranking of fingerprints and their associated Nearest Neighbour distances, we can spatially average the locations of the k closest fingerprints. This technique is directly applicable to point sampling, where l_i is a Cartesian co-ordinate; for sector sampling the geometric centre of the sector is used. Either the arithmetic or weighted mean can be used. The arithmetic mean position is:

$$l_a = \frac{1}{k} \sum_{n=i}^{k} l_i \tag{3.11}$$

The positional estimate l_a is a Cartesian co-ordinate. The weighted mean position [66] is calculated with weights equal to the inverse of the distance of the particular fingerprint, i.e. the weight of the ith fingerprint is

$$w_i = \frac{1}{D_i} \tag{3.12}$$

where D_i is the signal strength distance between the ith nearest fingerprint and the incoming signal strength.

The weighted mean position l_w is

$$l_w = \frac{\sum_{n=i}^{k} w_i . l_i}{\sum_{n=i}^{k} w_i} \tag{3.13}$$

where l_i is the position of the ith nearest fingerprint, and k is the number of fingerprints used from the head of the output list.

As an example, figure 3.6 shows the results produced by taking the arithmetic and weighted mean of the nearest 3 fingerprints. The arithmetic mean weights each fingerprint equally, so it lies in the centre of the fingerprints, but the weighted mean uses the inverse of the signal strength distance which causes the result to be closer to the fingerprint with shortest distance at (2,0).

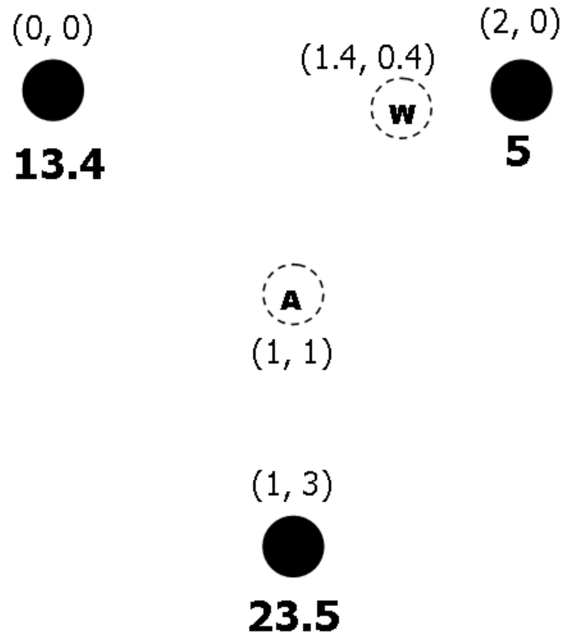


Figure 3.6: *Birds-eye view showing position averaging using arithmetic and weighted means. The black filled circles are the locations of the nearest 3 fingerprints, with co-ordinates labelled above and signal strength distance labelled below each circle. Position A is the arithmetic mean, position W the weighted mean.*

Averaging can help reduce distance errors by considering several nearest fingerprints rather than just the first, which might be noise affected or otherwise anomalous. Note that the nearest fingerprints frequently have very similar signal strength distances and these differences are often only due to small fluctuations in signal strength.

The weighted mean can achieve better accuracy than the arithmetic mean only if the algorithm used generates appropriate distances for each fingerprint (in the case of the Nearest Neighbour, this is the p-norm distance). If the distances to the considered k fingerprints are all similar, the result will be similar to the arithmetic mean. If the distance to fingerprints after the first are much lower than the first, the result will be similar to $k = 1$.

Chapter 5 presents empirical results comparing the accuracy between arithmetic weighted mean output averaging.

3.6.3 Bayesian methods

Bayesian algorithms for 802.11 positioning [14, 90, 43, 122] use Bayes' rule to calculate the mobile agent's location. The final output is a discrete probability distribution, $\overrightarrow{P}$, over L, representing the probable location of the agent. $\overrightarrow{P}(l_i)$ expresses the probability that the mobile agent is at the location of fingerprint i. In other words, the output is a one to one mapping between each fingerprint location and the probability of the agent being at that location.

$\overrightarrow{P}$ is calculated upon receiving each new signal strength observation vector o using Bayes' Rule as follows:

$$\overrightarrow{P}(l_i) = p(l_i|o) = \frac{p(o|l_i)p(l_i)}{p(o)} \tag{3.14}$$

$p(l_i|o)$ is a conditional probability representing the likelihood that the mobile agent is at location l_i, given observation o. This is known as the *posterior* probability. $p(o|l_i)$ is the *likelihood function*: the probability of making observation o at location l_i. $p(l_i)$ is the *prior probability* and represents the probability that the device can be at location l_i. $p(o)$ is a normalizer that can be calculated within this context as $\sum_{i=1}^{n} p(o|l_i)p(l_i)$. It can also be used as a measure of the *confidence* of the distribution.

The likelihood function can be calculated using the signal strength map. Prior to positioning, the training data is recorded similarly to the nearest neighbour method. However, instead of associating each location l_i with a vector of average signal strengths for each access point, it is associated with a vector of empirical probability density functions. Fingerprints can be expressed as $f_i^{Bayesian} = (l_i, \{\Lambda_{i,1}, \ldots, \Lambda_{i,n}\})$. $\Lambda_{i,j}$ is the empirical probability at location l_i for signal strength from access point b_j. Each empirical probability density function is generated by taking the recording of signal strength values for an access point, calculating the frequency of each signal strength value, and normalizing the frequencies (the maximum likelihood approach). Hence, $\Lambda_{i,j}(\lambda) = p(\lambda_j|l_i)$: the probability of observing signal strength λ from access point b_j, at the location l_i.

The likelihood function is calculated by multiplying the conditional probability of observing each reading in the observation vector $o = \{\lambda_1, \ldots, \lambda_n\}$:

$$p(o|l_i) = \prod_j p(\lambda_j|l_i) = \prod_j \Lambda_{i,j}(\lambda_j) \tag{3.15}$$

This calculation assumes independence between the readings from different access points for a fixed location. This is not strictly true: proximate access points broadcasting on overlapping channels can interfere with each other, but in a properly configured network such interference is minimal. The alternative to this method is to maintain multi-dimensional distributions, but this greatly increases the complexity of the calculations and the size of the signal strength map, and is unlikely to deliver better results assuming the dependency violation is insignificant.

The prior can be used to bias the result towards one location or another; this can be useful for considering historical information or the presence of impassable objects such as walls. A uniform prior distribution was used which introduces no bias towards any particular location, for simplicity.

To use the output in other applications, the single most likely location can be used, or either the weighted or arithmetic means for the most likely k results can be used, as with the Nearest Neighbour method. If the weighted average is used, the weights are the probabilities. Note that the type of summarization used can affect the fidelity of the weights and can change the threshold of useful k; this is discussed further in Chapter 5.

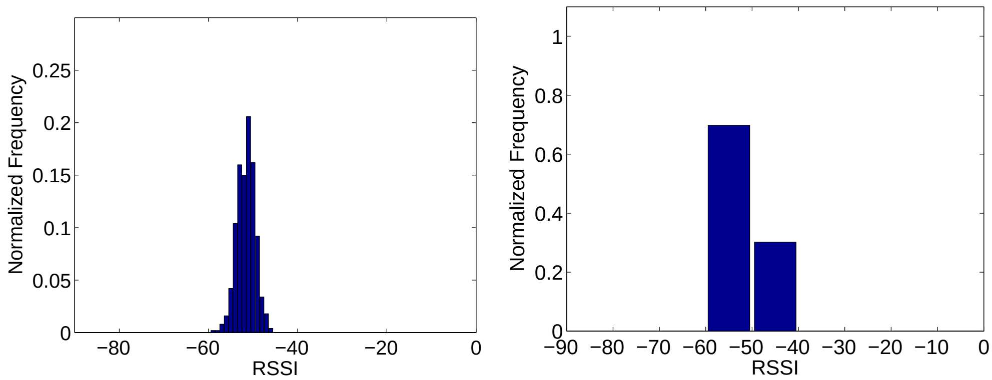
The strength of the Bayesian method over the Nearest Neighbour method is that in theory it uses more of the information in the training data. While the Nearest Neighbour's signal strength map uses only signal strength means, the probability density functions maintained in a Bayesian map provide a more detailed image of the signal strength at any particular location. For example, the Bayesian method should perform better in situations where $\Lambda_{i,j}$ is multi-modal (that is, when the probability density function has two distinct peaks). In that case, the mean lies between the peaks and is not a common signal strength value at that location - so the characterization as the mean is inaccurate.

More generally, the Bayesian method should beat the Nearest Neighbour in any situation in which a detailed characterization of the spread of the values - rather than just the mean - could provide greater distinction between fingerprints. On the other hand, the increased detail can be counter-productive when the sample size is relatively small; transient minor modes can have a greater effect than with the Nearest Neighbour.

The Bayesian method faces the same problem as the Nearest Neighbour algorithm when dealing with out of range access points (section 3.6.1). A heuristic is required to deal with the case where an access point distribution is missing from the fingerprint but not the input. The probability is multiplied by a fixed small value that indicates the probability that this is an artifact observation. A more sophisticated solution might be to scale this value according to magnitude of the signal strength.

Summarization methods

Within this framework, implementations adopt different derivations of $\Lambda_{i,j}$. It is possible to use the empirical density functions directly in the Bayesian framework, but the density functions are only a rough estimation of the underlying probability based on a limited amount of data. Further grooming of the distribution can lead to better results. Two methods are used in this work, empirically compared in chapter 5: a histogram method similar to Roos et al. [90], and summarization of the signal strength as a normal distribution similarly to Haeberlen et al. [43].



(a) $b = 1$ **(b)** $b = 10$

Figure 3.7: *Two signal strength histograms representing the same data with bin widths of 1 and 10. Each histogram contains the normalized frequencies of received signal strength values for a particular access point.*

Histogram

The *histogram method* [90, 43] generates a probability histogram $H_{i,j}$ in place of the density function $\Lambda_{i,j}$. Rather than separating the frequency of each and every signal strength value λ, the frequencies are grouped into q contiguous bins $\{h_1, \ldots, h_q\}$ of fixed and equal width b. Each bin covers a continuous, non-overlapping range of signal strength values. The histogram constitutes a more discretized version of the original density function $\Lambda_{i,j}$, and is used directly in its place when calculating the likelihood function.

The histogram summarizes the information with granularity proportional to the size of the bins. Note that a histogram with $b = 1$ is identical to the original probability density function. This summarization process reduces the noisiness of the distribution by spreading the bin frequencies between their neighbours; this helps balance an incomplete distribution when there are gaps. It also tends to widen the signal strength ranges, which tends to make $H_{i,j}$ more representative of the area surrounding l_i and less specific to the exact point the data was gathered at. This is helpful up until a threshold of bin width, but bins that are too large result in too much similarity between distributions.

A small delta value is added to each bin to avoid problematic probabilities of zero occurring outside the range of the training data. A value of $1/n$ is used (similarly to Roos et al. [90]), where n is the number of samples used to generate the histogram.

Figure 3.7 shows two examples of signal strength histograms. Figure 3.7a is the normalized frequency histogram for $b = 1$; i.e. the original data. 3.7b shows the same data using a bin width of 10.

The drawback of the histogram method is that it returns similarly low probabilities for input signal strengths outside the range of the training data. For example, consider the histogram in Figure 3.7a. The range of training data lies mostly between approximately -56 and -46dBm. Outside this range, the normalized frequency is a small constant value generated by the small delta value added to all bins (invisible in the figure due to the scale). Therefore, the probability assigned to -20dBm is the same as that assigned to -40dBm. Hence, accuracy can be poor if the input value falls outside the training data, as the algorithm cannot accurately rank fingerprint similarity in that case.

This situation can occur if the input is offset by interference or objects unexpectedly blocking the direct line of sight to an access point. It can also occur if the current location is sufficiently spatially distant from the nearest fingerprint that the signal strength falls outside its range. This is less likely to happen with sector based fingerprint maps, which tend to have wider ranges and a higher degree of spatial coverage. Increasing the bin widths can somewhat compensate for this problem, by artificially widening the ranges, but at the expense of reducing the resolution of the data.

Another problem with the histogram method is that wider bin widths tend to lead to increasing distortion of the probability distribution due to *bin alignment*. This can be illustrated by the example in Figure 3.7b, which shows a histogram with a bin width of 10. The histogram indicates that probability of receiving an input value between -51 and -60dBm (inclusively) is 0.7, and the probability of receiving an input between -61 and 70dBm is close to zero. Therefore, the probability of inputs -60 and -61dBm are 0.7 and close to zero respectively - despite only being separated by 1dBm. This represents a fairly unrealistic model, and is due to the wide bin widths causing sharp changes in probability. The exact cutoffs vary according to bin width, because it defines the alignment of the bins. For example, if the bin width is 11dBm, instead of 10, the bins represent the probability of observing inputs between -90 and -80dBm, -79 and -69dBm, -68 and -58dBm, and so on. In that case, the probabilities at -60dBm and 61dBm are the the same as they are both fall within the -68 to -58dBm bin.

This effect leads to results where the outcome is highly sensitive to bin width, and the lowest distance errors are not necessarily found in a single range of bin widths. This effect can be seen in our experimental results using the histogram method in Section 5.2.2.

Gaussian

In the Gaussian method, $\Lambda_{i,j}$ is summarized as a Gaussian probability distribution by storing only its mean $\mu_{i,j}$ and standard deviation $\sigma_{i,j}$. The probability of observing signal strength λ_j at location l_i is given by density function of the Gaussian distribution:

$$p(\lambda_j|l_i) = \frac{1}{\sigma_{i,j}\sqrt{2\pi}}\exp\left(-\frac{(\lambda_j - \mu_{i,j})^2}{2\sigma_{i,j}^2}\right) \tag{3.16}$$

Although the underlying signal strength data is not necessarily Gaussian, or even uni-modal, fitting the data to a Gaussian distribution reduces the influence of outliers and smoothes any gaps in the distribution. The Gaussian method can also result in a more compact fingerprint map than the Histogram method: only the mean and standard devia-

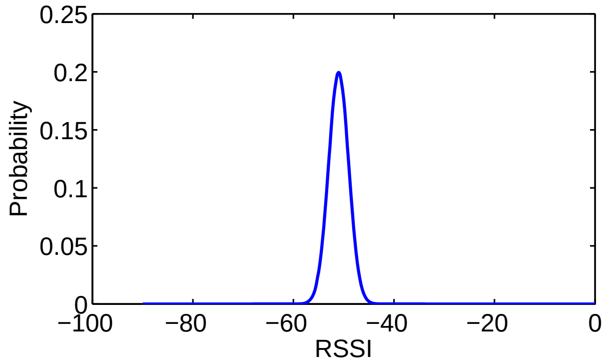


Figure 3.8: *Example of a Gaussian summarized fingerprint with mean -51dBm and standard deviation 2.*

tion need be stored for each access point rather than an entire distribution. Fingerprints generated using this method can be expressed as $f_i^{Gaussian} = (l_i, \{(\mu_{i,1}, \sigma_{i,1}), \ldots, (\mu_{i,n}, \sigma_{i,n})\})$.

Figure 3.8 shows an example of a Gaussian summarized fingerprint with mean -51dBm and standard deviation of 2; the Gaussian summarization of the fingerprint in Figure 3.7a. The probabilities drop sharply to close to zero for RSSI values outside the immediate range of the training data, similarly to the histogram summarization method. However, unlike the histogram method, the probabilities decrease further away from the mean, which means that the algorithm can make some distinction between input values close to and further away from the mean.

3.6.4 Distribution Difference method

The algorithms described above take as input *snapshots* of the most recent signal strength readings. Readings prior to this are not taken into account when calculating the new position; the algorithms use only an instantaneous image of the signal strength. In contrast, the *Distribution Distance method* uses a frequency histogram of recent input readings rather than just a single instantaneous reading. Fingerprints are ranked using the distance between the input and signal strength map histograms. Our hypothesis is that this method's use of a greater amount of information can provide accuracy greater than that of snapshot type algorithms.

Youssef and Agrawala [121] also describes a method for taking into account multiple input samples, but uses time-series techniques to perform averaging of distributions rather than the approach described below.

The signal strength map is calculated identically to the Bayesian Histogram method (Section 3.6.3), so each fingerprint contains a set of normalized frequency histograms $f_i^{DD} = (l_i, \{\Lambda_{i,1}, \ldots, \Lambda_{i,n}\})$. During online positioning, the sequence of input readings is used to generate another set of normalized frequency histograms for each access point $I = \{I_1, \ldots I_n\}$; I_j is the histogram of readings from base station b_j. Both histograms have an equal fixed bin size b, and a small delta value is added to each bin equal to $1/n$ as for the Bayesian histogram method.

When tracking a moving target, only relatively recent readings can be considered, otherwise the output can appear to lag behind the current position of the mobile agent. Therefore, the input histogram is constructed over a limited window of previous data. The window length can be adjusted according to how quickly the mobile agent is likely to move; this would usually be fixed before use according to the expected positioning scenario (e.g. walking, running, in-vehicle) but the window length can also be adjusted dynamically if necessary.

Similarly to the Nearest Neighbour, the output is a ranked list of fingerprints with associated distances; hence it can be output averaged using weighted or arithmetic techniques (Section 3.6.2).

The distance between a fingerprint f_i and the input I is calculated as the sum of the distances between the corresponding histograms:

$$D_i = \sum_{j=1}^{n} d(I_j, \Lambda_{i,j}) \tag{3.17}$$

Here D_i is the Distribution Distance between the input I_j and the ith fingerprint $\Lambda_{i,j}$. $d(X, Y)$ is the *distance function*. Two methods were trialled for calculating this distance: the *p-norm distance* and the *match distance* [91], based on their simplicity. Other possible methods for calculating the distance between histograms include the Earth Mover's distance [91] and the Kullback-Leibler divergence [59], but are not considered in this work.

As with previous methods, a method is required for comparing inputs and fingerprints that do not necessarily contain the same sets of access points (Section 3.6.1). In this case, where data from some access points is missing due to them being out of range at the time of recording, both fingerprints and inputs are padded with *null histograms* containing a single value at -95dBm (slightly below the minimum receivable RSSI of -90dBm). A small delta value is added to all bins to avoid zero probabilities.

Chapter 5 contains empirical results showing the accuracy of the Distribution Distance method using both the p-norm and Match Distance methods.

p-norm Distance

The p-norm distance between two histograms [92] treats the two histograms as one dimensional vectors. The distance is calculated as

$$d_{pnorm}(I_j, \Lambda_{i,j}) = \left(\sum_{\lambda} \left| I_j(\lambda) - \Lambda_{i,j}(\lambda) \right|^p \right)^{1/p} \tag{3.18}$$

Here $\Lambda_{i,j}$ and I_j are fingerprint histograms and input histograms, respectively, for the signal strength readings from base station j. $I_j(\lambda)$ is the value of I_j at λ; i.e. the normalized frequency of observations of signal strength λ from base station b_j during the input window. Similarly $\Lambda_{i,j}(\lambda)$ is the normalized frequency of observations of signal strength λ from base station b_j, at location l_i.

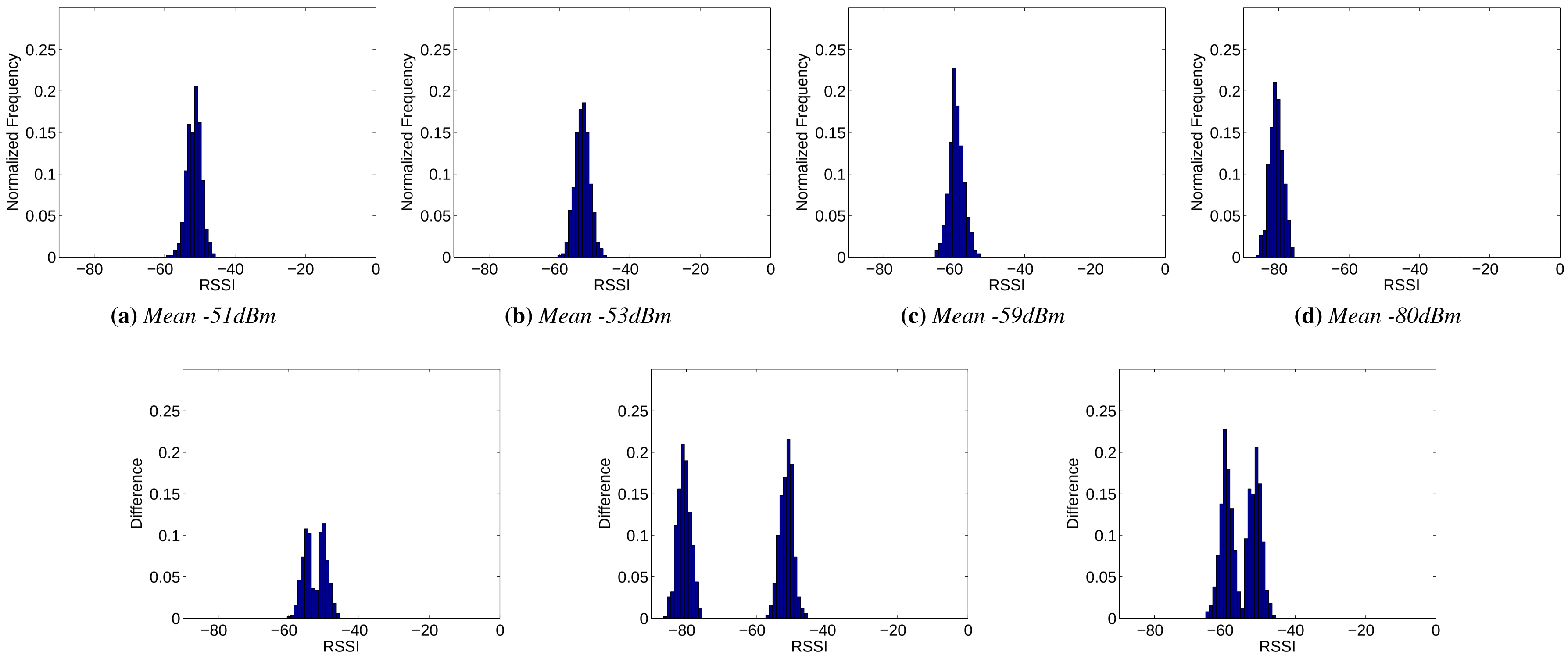
Note that the p-norm distance formula is similar to that used in the nearest neighbour calculation, except here it is used to calculate the difference between two histograms rather than the difference between a vector of averages and an input snapshot. In this work, $p = 2$ is used (the Euclidean distance).

Figure 3.9 illustrates how the p-norm distance method works using some example histograms with a bin width of 1 and $p = 2$. Figures 3.9a to 3.9d show the sample histograms, and Figures 3.9e to 3.9g show the bin-wise differences between selected combinations of these histograms. The bin-wise difference is the absolute value of the difference between two corresponding bins; $|I_j(\lambda) - \Lambda_{i,j}(\lambda)|$ in equation 3.18.

The example histograms are similar to the data recorded in our experiments, although they are actually simulated for the sake of this example rather than being drawn from experimental data (random values normally distributed, with a standard deviation of 2). Note that the values outside the modes of the plots are very small, non-zero values due to the addition of the delta value to each bin. They are not visible on these plots due to the scale.

Figure 3.9e shows the bin-wise differences between the histograms in Figure 3.9a and Figure 3.9b, which have means of -51 and -53dBm respectively. The ranges of the histograms overlap due to their similar means, and the 2-norm distance (the p-norm distance with $p = 2$) is lower (0.87) than for the two following examples, which compare non-overlapping histograms. Figure 3.9f shows the bin-wise differences between the histograms in Figure 3.9a and Figure 3.9d. They do not overlap, and their means are separated by a substantial amount: 27dBm, which usually corresponds to a spatial distance of at least 10m, according to the signal strength maps in Section C. Accordingly, the 2-norm distance is higher (1.41) than for the previous example. The final example (Figure 3.9g) compares the histograms in Figure 3.9b (mean of -53dBm), and Figure 3.9c (-59dBm). Despite their means being separated by only 6dBm, the 2-norm distance is 1.38, almost the same as the previous example, where the difference in means was much greater.

This occurs because the p-norm distance method does not measure the distance between means or ranges, only the differences of corresponding bins. Therefore, the p-norm distance between histograms that do not overlap will be similar regardless of their content. This is a drawback for fingerprint positioning because it means that accuracy is poor if the input does not closely resemble any of the fingerprints, which can occur if the location of the mobile agent's reception is affected by multipath fading, if there is channel interference, or if an object is unexpectedly blocking the direct path to one or more access



(a) *Mean -51dBm*

(b) *Mean -53dBm*

(c) *Mean -59dBm*

(d) *Mean -80dBm*

(e) *Bin-wise differences between (a) and (b). The 2-norm distance is 0.91.*

(f) *Bin-wise differences between (a) and (d). The 2-norm distance is 1.41.*

(g) *Bin-wise differences between (a) and (c). The 2-norm distance is 1.38*

Figure 3.9: *Several normalized histograms and examples of p-norm differences between them.*

points. Similarly to the Bayesian histogram, this problem can be somewhat remedied by increasing the bin width, which widens the ranges of the histograms at the expense of training data resolution and increasing bin alignment effects.

Figure 3.10 repeats the examples using a bin width of 10. The wider bin width substantially reduces the information in the histograms, but widens their ranges such that there is a higher probability of overlap. The previously problematic example using a bin width of 1 was the comparison between the -51dBm mean histogram (Figure 3.10a) and the -59dBm mean histogram (Figure 3.10c). The increased bin width causes the two histograms to overlap, such that the bin-wise differences are reduced. Accordingly, the 2-norm distance is reduced to 0.79 (Figure 3.10g), compared to 1.38 using bin widths of 1 (Figure 3.9g).

Figure 3.11 shows an example of distortion caused by bin alignment effects. A distribution of mean -45dBm (Figure 3.10b) is compared to the distribution of mean -53dBm (Figure 3.10b), using a bin width of 10. Because the bins are aligned to values divisible by 10, the normalized frequencies for each histogram are found in non-corresponding bins, between -40 and -50dBm for the mean -45dBm histogram and -50 and -60dBm for the -53dBm histogram. As a result, the histograms do not overlap and high bin-wise differences occur for both the populated bins (Figure 3.11b). The end result is a high 2-norm distance (1.36), despite the difference in means being only 8dBm. In comparison, the 2-norm distance between the -51dBm mean histogram and the -59dBm mean histogram (also separated by 8dBm) is 0.79 (Figure 3.10g).

Match Distance

The match distance [92] is the difference between the *cumulative histograms* of I_j and $\Lambda_{i,j}$:

$$d_{match}(I_j, \Lambda_{i,j}) = \sum_{\lambda} \left| \hat{I}_j(\lambda) - \hat{\Lambda}_{i,j}(\lambda) \right| \tag{3.19}$$

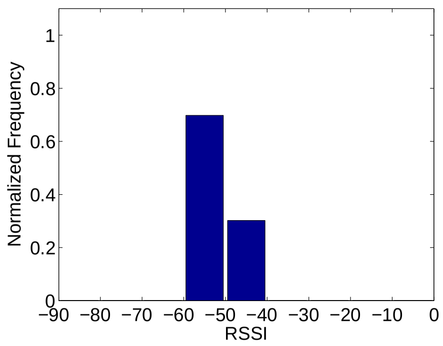
$\hat{I}_j$ is the cumulative histogram of I_j, such that

$$\hat{I}_j(\lambda) = \sum_{\lambda_{min} \leq \lambda_i \leq \lambda} I_j(\lambda_i) \tag{3.20}$$

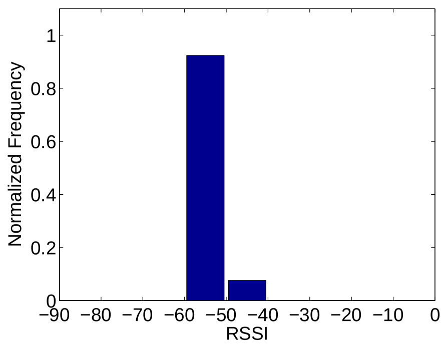
and $\hat{\Lambda}_{i,j}$ is the cumulative histogram of $\Lambda_{i,j}$, such that

$$\hat{\Lambda}_{i,j}(\lambda) = \sum_{\lambda_{min} \leq \lambda_i \leq \lambda} \Lambda_{i,j}(\lambda_i) \tag{3.21}$$

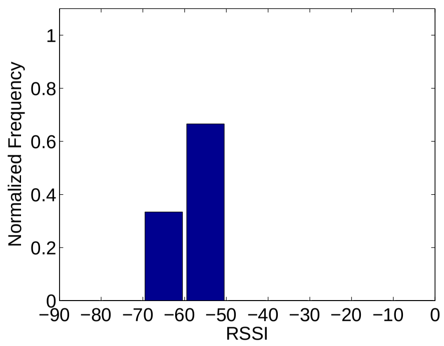
λ_{min} is the minimum signal strength value that can be observed, which was -90dBm using our hardware. The ith bin of the cumulative histogram is equal to the sum of the frequency histogram bin values in the range λ_{min} to i inclusive. Comparing the cumulative



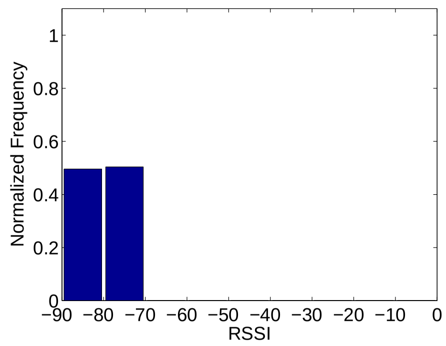
(a) *Mean -51dBm*



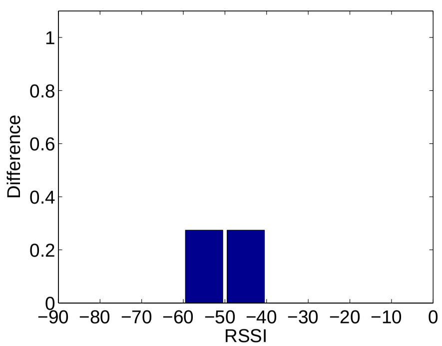
(b) *Mean -53dBm*



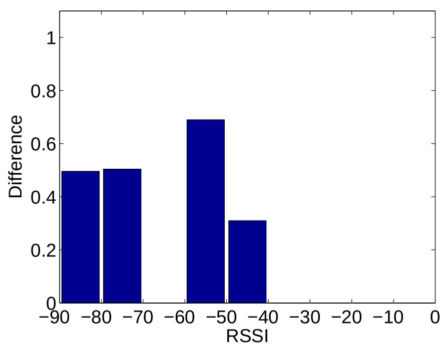
(c) *Mean -59dBm*



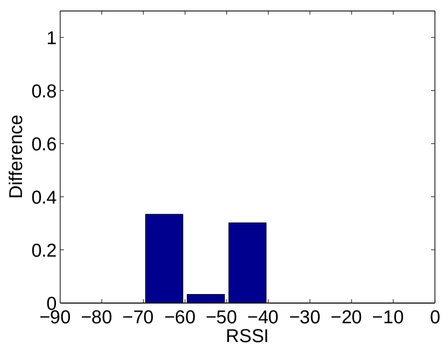
(d) *Mean -80dBm*



(e) *Bin-wise differences between (a) and (b). The 2-norm distance is 0.67.*

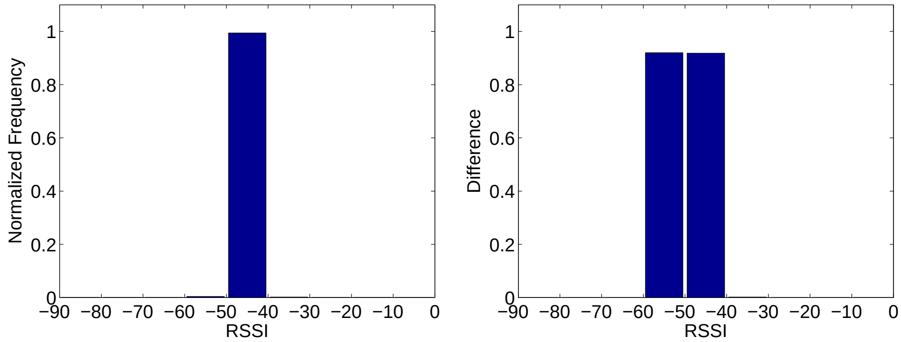


(f) *Bin-wise differences between (a) and (d). The 2-norm distance is 1.41.*



(g) *Bin-wise differences between (a) and (c). The 2-norm distance is 0.79*

Figure 3.10: *Several normalized histograms and examples of p-norm differences between them.*



(a) *Mean -45dBm*

(b) *Bin-wise differences between (a) and 3.10b. The 2-norm distance is 1.36.*

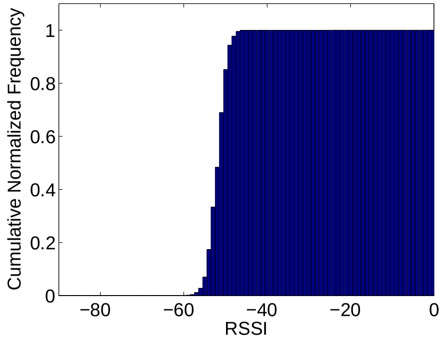
Figure 3.11: *Histogram with mean -45dBm, and the bin-wise differences between it and the -53dBm mean histogram in 3.10b, using a bin width of 10.*

histograms rather than the normalized frequency histograms allows differences between non-corresponding bins to be considered.

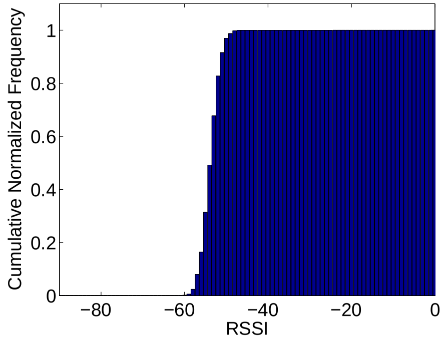
To illustrate how this method works, we re-use the example histograms from the previous section. Figure 3.12 shows the cumulative distributions of the example histograms (Figures 3.12a to 3.12d) and their bin-wise differences (Figures 3.12e to 3.12g). The bin-wise differences are calculated by taking the absolute value of the differences between the cumulative histograms; $|\hat{I}_j(\lambda) - \hat{\Lambda}_{i,j}(\lambda)|$ from equation 3.19. The match distance is the sum of these bin-wise differences.

Because the frequency histograms (Figures 3.9a to 3.9d) are unimodal and have low variances, the corresponding cumulative histograms are characterized by values close to zero to the left of the range of training data, a sharp increase in cumulative normalized frequency in the data range, and a plateau close to 1.0 to the right of the data range. Frequency histograms with greater variance would result in a more gradual gradient in the cumulative histogram.

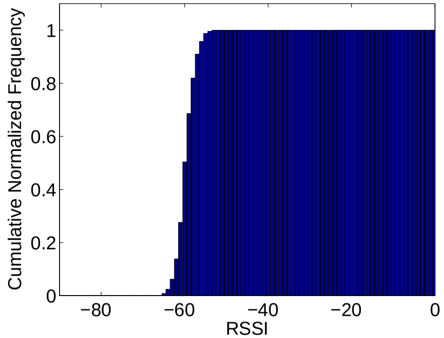
Figure 3.12e shows the bin-wise differences between the -51dBm mean cumulative histogram and the -53dBm mean cumulative histogram. Because the corresponding frequency histograms have similar means and standard deviations, the cumulative histograms have sharp increases in y-value at similar RSSI values; this generates small bin-wise differences. The match distance (2.07) is similar to the difference between their means (2dBm), and is smaller than the following two examples, which compare less similar histograms. Figure 3.12f shows the bin-wise difference between the -51dBm mean cumulative histogram and the -80dBm mean cumulative histogram. In this case, the difference between means is large and there are a corresponding number of large bin-wise differences. The match distance is high (29.09) and similar to the difference in means (29dBm). In the final example (Figure 3.12a), the cumulative histograms with mean -51dBm and -59dBm are compared. The difference (7.80) is again similar to the difference in means



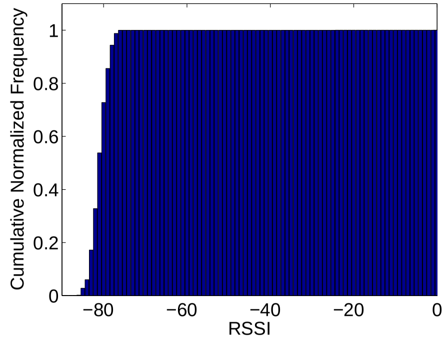
(a) *Mean -51dBm*



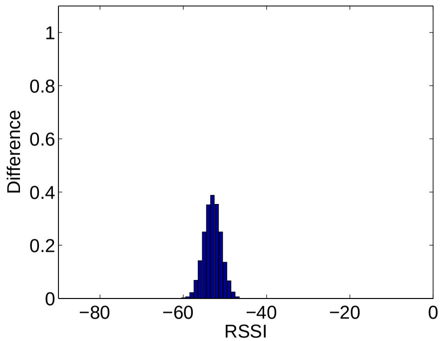
(b) *Mean -53dBm*



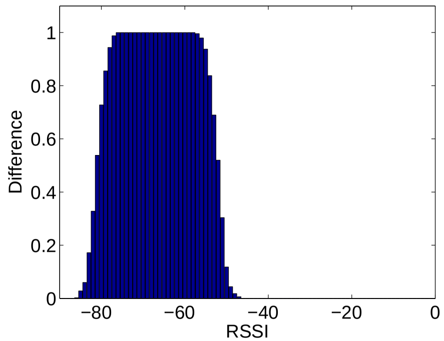
(c) *Mean -59dBm*



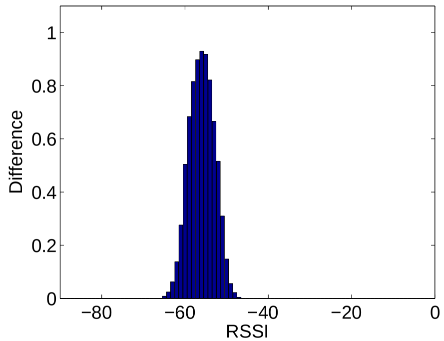
(d) *Mean -80dBm*



(e) *Bin-wise differences between (a) and (b). The match distance is 2.07.*



(f) *Bin-wise differences between (a) and (d). The match distance is 29.09.*



(g) *Bin-wise differences between (a) and (c). The match distance is 7.80*

Figure 3.12: *Bin-wise differences between cumulative histograms corresponding to the frequency histograms in Figure 3.10*

(8dBm).

The tendency for the match distance to be similar to the difference in means is due to the nature of the histograms used as examples and the size of the bins. Because they have low variances, the bin-wise differences between their modes tend to be close to 1, and with a bin size of 1 (i.e, 1dBm), the number of bins separating the means is the same as the difference in dBm. The sum of bin-wise differences is therefore close to the difference in means. For bin widths greater than 1, the result will be proportional to the difference in means rather than similar.

The match distance method operates well with small bin sizes. Unlike the p-norm distance, it does not require large bin sizes to widen the range of the data to reduce the score assigned to similar but non-overlapping histograms. Hence, it operates accurately using higher resolution data than the p-norm distance. If necessary, bin size can be increased to reduce the amount of memory occupied by the signal strength map; this does not lead to a substantial degradation in accuracy, as shown in our experimental results (Section 5.2.4).

3.7 Conclusions

This chapter comprehensively discussed the operation of 802.11 fingerprint-based positioning systems. Two methods of signal strength surveying have been described: the point and sector methods. Five positioning algorithms have been described: the Nearest Neighbour method, Bayesian Histogram and Bayesian Gaussian methods, and the p-norm and Match Distance methods. The output of each of these algorithms can also be spatially averaged using a process called output averaging; using either arithmetic or weighted output averaging.

The Bayesian and the Distribution Distance methods are, in theory, more promising than the simpler Nearest Neighbour algorithm due to the increased amount of information they take into account when comparing input data to signal strength fingerprints. The two Distribution Distance methods take into account the most information of all the algorithm classes, as they consider the distribution of signal strength values over a sliding window of input data, rather than a single snapshot, and use a detailed representation of the fingerprint data.

I predicted that this greater amount of information would allow the Bayesian and Distribution Distance methods to distinguish more accurately between similar fingerprints, and therefore produce better accuracy than the Nearest Neighbour method. Disappointingly, this prediction was not confirmed by the results of empirical testing (Chapter 5), which show similar results for all algorithm classes when using the most accurate parameter values. This suggests that considering more information is not necessarily beneficial to accuracy; reasons for this are discussed in detail in Section 5.7.

The next chapter describes the test environments and procedures used to gather empirical data to evaluate the effectiveness of the techniques discussed in this chapter. Chapter 5 compares the results of using each surveying method, positioning algorithm, and spatial output averaging method. The results are generated using a range of parameters for each positioning algorithms and output averaging method, to determine which parameter values produce the best accuracy.

Chapter 4

Test Environments and Procedures

This chapter describes the two test environments and procedures used to record experimental test data. In each environment, point and sector signal strength maps (section 3.5.3) and separate test points were recorded. This constituted the experimental data that was used to derive results in Chapters 5, 6 and 7.

Each test environment is described in terms of construction material and the location of access points, with maps of fingerprints and test points. This is followed by the details of the data recording procedures, and a discussion of the effects of varying signal strength sampling rates on our experimental data.

4.1 Test Environments

The two data sets were deliberately recorded in different environments and conditions so that the positioning algorithms would be exposed to a wide range of test cases, and to avoid the possibility of results specific to one environment. Data was recorded in two quite different test locations: a small residential house, and the third floor of the UNSW Computer Science and Engineering (CSE) building. The house is a small area of relatively simple construction with high access point density, while the CSE environment is a larger, more complex environment constructed of many different materials and with less dense access points.

4.1.1 House Environment

The house consists of several small enclosed rooms, with no open floor spaces or large rooms. The walls of the house vary in thicknesses between 10 and 20cm, constructed of brick and plaster. The floors are mostly carpeted, with wooden floorboards approximately 2cm thick. Beneath the floorboards a low basement runs the length of the house.

In the residential area the house is located in, signals from several neighbouring 802.11 base stations could been detected at the time of the experiment. However, they

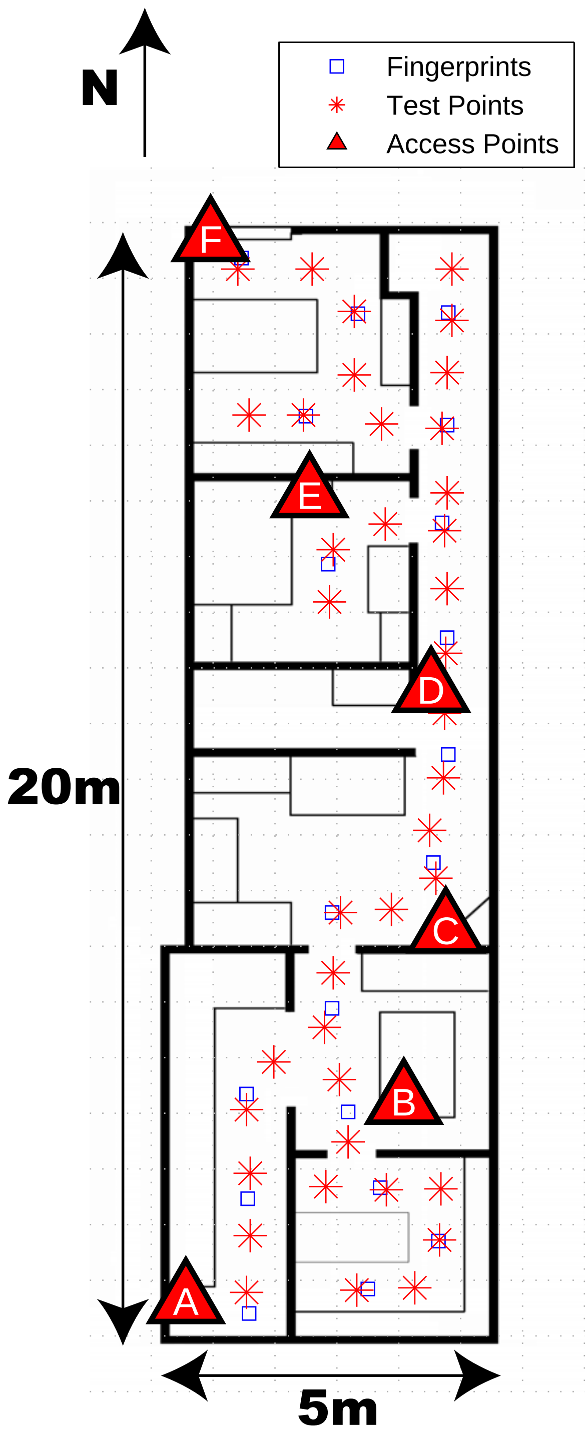


Figure 4.1: *House test bed, showing 19 fingerprints, 39 test points, and 6 access points, with an overlaid 1 metre grid.*

Letter	Model
A	Netgear WGR614
B	Proxim AP2000
C	Proxim AP2000
D	Proxim AP2000
E	Linksys WRT54GL
F	SparkLAN WX1590

Table 4.1: *Access points used in the house environment.*

were broadcasting at low power levels on channels distant from our experimental base stations and it is unlikely that they affected the results.

Figure 4.1 shows a map of the house environment showing the locations of access points, fingerprints, and test points. It contains 6 802.11b base stations of various models (shown in table 4.1), 19 fingerprint locations, and 39 test point locations. The rationale behind the placement of fingerprints and test locations is described below in section 4.2.

The six access point arrangement is much denser than most 802.11 networks used solely for data transport. In fact, one base station in the centre of the house is sufficient for coverage of the entire area, as can be seen in the signal strength maps in Appendix C. More access points than necessary were used so that it was possible to simulate positioning with lower numbers of access points by considering only a subset of access points (Chapter 6).

The access points were all set to broadcast on the same frequency (channel 11), such that channel hopping (switching between channels) was unnecessary. This greatly increased the sampling rate as all packets broadcast were able to be received. Channel hopping is discussed in more detail later in this chapter in section 4.3.

4.1.2 CSE Level 3 Environment

Our second test environment was the third floor of the UNSW CSE building (Figure 4.2). The floor consists of a laboratory area (I) populated by mostly cubicles, with an open area on the lower right used for robot testing (II). The robot testing area contained a miniature soccer field used for robot soccer and was off limits to us. A central foyer area (III) provides access to elevators and connects several hallways which are lined with small offices.

The area contains a wide variety of materials. The internal walls are mostly concrete and plasterboard, and the external walls are mostly reinforced concrete. There are many metallic objects, and objects of a wide range of shape and size; these properties are known to cause multipath fading [105, 106, 102]. For example, in the laboratory, foyer and hallway areas there is complex metallic air conditioning and cable trays exposed in the ceiling, and the area is full of desktop computers with metal casings and robotics and electrical equipment with metal parts. There are also several large metal lockers in the

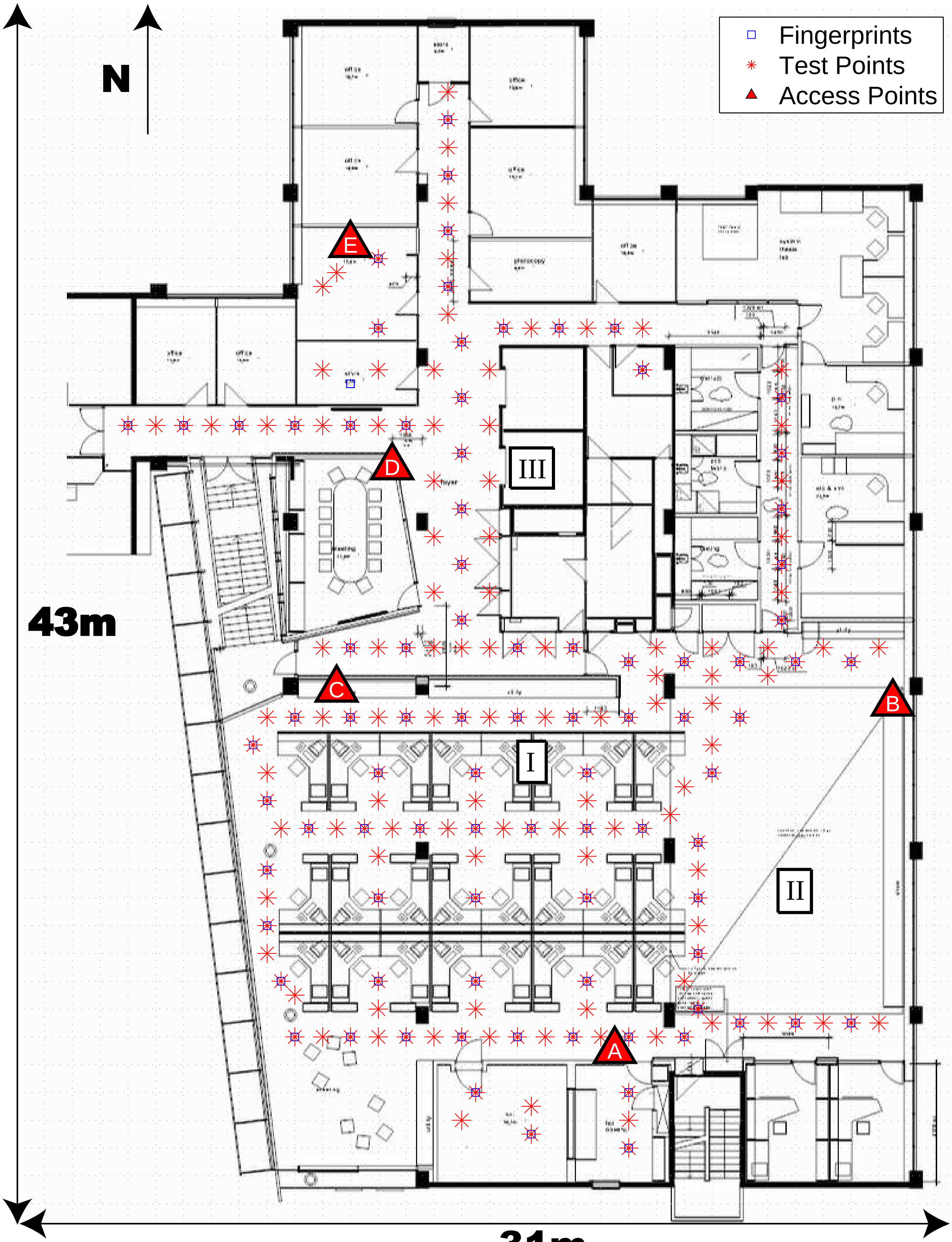


Figure 4.2: *CSE Level 3 test bed, showing 87 fingerprints, 183 test points, and 5 access points, with an overlaid 1 metre grid.*

Letter	Model
A	Netgear WRT54GL
B	Proxim AP2000
C	Netgear WRT54GL
D	Proxim AP2000
E	Netgear WRT54GL

Table 4.2: *Access points used in the CSE environment.*

laboratory area. The left wall of the laboratory is made entirely of glass and constitutes the side of the building, looking out onto an open courtyard, entirely protected by a metal grating. The signal fading caused by these objects is detrimental to accuracy as described in section 3.5.2.

Data was recorded in areas I had security access to and where it was practical to do so. It may have been possible to record more complete coverage had I acquired a master key of the area, but I deemed this unnecessary on the basis that the data set would be suitably large and varied regardless.

There were already two pre-existing 802.11g access points on the floor setup for use by staff. Several other access points were visible when I recorded the data, of unknown location and administration. Packets from these access points were ignored in our testing. Three additional access points (for a total of five) were added to the environment; It is shown that these extra access points substantially boosted accuracy in chapter 6.

Figure 4.2 shows the fingerprint map and test points: 87 fingerprints and 183 test points. Access points B and D belong to the pre-existing staff network, A, C and E are the units added for this study. The models of each access point are shown in Table 4.2. Photographs and signal strength maps of the area are in Appendix C.

Because the pre-existing access points were set to different channels, it was necessary for the device to channel hop while recording data. This limited the amount of data it was possible to record compared to the house, and the fingerprints contained much less data as a result. The effects of channel hopping are discussed below in section 4.3.

Signal strength data was recorded on weekends and on weeknights to avoid electrical interference and transient effects from people moving around the laboratory. It was difficult to find times when the floor was completely free of people, as the laboratory is open 24 hours. Some of the data was recorded when there were one or two people on the floor, but no data was recorded in situations where they would have blocked the direct signal path to any of the access points.

A possible criticism of our approach is that no testing was performed during hours when the environment was populated with many people, which could be detrimental to accuracy. This step was omitted because the environment is fairly sparsely populated even during the daytime and the results would probably have been inconclusive. Prior research already exists on this aspect; Haeberlen et al. [43] performed an extensive analysis of the performance of indoors 802.11 positioning over a larger and heavily populated area, and suggest algorithmic methods for adapting to these conditions. They experienced a degradation in accuracy of approximately 20% during peak hours, but were able to recover most of this deficit by scaling the input signal strength, calibrating the scaling factor by recording signal strength at known locations.

4.2 Test Procedure

Data was recorded in two phases: fingerprint maps, followed by separate test points. Both are recorded in the same format; effectively the test points are a second fingerprint map. In this section, the procedures for recording both are described.

Our procedure is most similar to that used by Bahl and Padmanabhan [4], Roos et al. [90] and Li et al. [66]. Although the procedure is similar, comparing results between research directly is still problematic as differences in environment, equipment, and access point placement can cause major variations in results. This means that the relative performances of difference techniques can only be reliably analysed using the same sets of test and fingerprint data.

Fingerprints

Both point and sector fingerprint maps (section 3.5.3) were recorded in each environment[1]. The fingerprints were recorded at approximately 2m intervals. Point fingerprints were recorded facing north, south, east and west for 5s per direction. Corresponding sector fingerprints were recorded for 20s each; each sector covered a $2m^2$ area centred on a corresponding point fingerprint location. The boundaries of each sector fingerprint were adjoining, providing contiguous blanket coverage. Each sector was assigned a two-dimensional coordinate identical to the point fingerprint in its centre; this value was used for output averaging and for calculating the distance error.

Our surveying software prompts the surveyor to move to each position in turn and start recording data at a position indicated by a graphical map. In this and other fingerprint research [4, 14, 90, 122, 43, 66, 113] it is implied that the surveyor stations herself at that position by using nearby objects as reference points. There is a limit to the precision with which a human can perform this task; from my experience I estimate around 50cm depending on the care taken. Hence small errors will inevitably be present in the locations of fingerprints and test points. Nonetheless, this is the most practical and efficient method of surveying, and such errors are relatively small compared to the accuracy of the positioning system. It might be possible to eliminate these human errors if a highly accurate

[1]For further reference, Appendix C contains illustrations of the fingerprints' signal strength in each environment using colour hue to represent signal strength magnitude.

alternate positioning system was used in the surveying phase.

Test Points

In both environments, test points were recorded at a separate set of locations, several days apart from the fingerprint recording. Test points were spaced approximately 1m apart while facing north, south, east and west, 5s per direction. The tests were located both in between and close to fingerprint locations to test both the harder and easier cases. Points between the fingerprints are the most difficult to localize because their signal strength usually differs the most from the surrounding fingerprints, while points close to or coinciding with fingerprint locations are relatively easier because they share greater similarity to the nearby fingerprint.

For the house experiments separate test points were selected without considering the relative locations of fingerprints; as a result the test locations are scattered among the fingerprint locations at varying distances (Figure 4.1). For the CSE environment test points were evenly placed between and coinciding with location of each fingerprint (Figure 4.2). The uniformity of the latter approach produces a slightly more reliable result because it provides a more uniformly distributed set of tests locations relative to the fingerprint locations.

Limitations

There are limitations to this test procedure in the sense that accuracy is only examined at a finite set of locations. However, it is the most practical and quantitative method for testing the positioning system that I am aware of, and the results are a suitable metric to roughly compare performance with other systems, especially those using similar procedures. A better method might be to use a third party, highly accurate positioning system and calculate the difference between its output and the 802.11 system; this would produce a more reliable result and raise the possibility of testing the system in a larger number of locations in a smaller amount of time.

4.3 Sampling Rates and Channel Hopping

The signal strength *sampling rate* is the rate at which signal strength values are captured, and determines the rate at which positions can be output. This rate is critical for mobile tracking applications; if it is low the position will appear to lag behind motion. Higher sampling rates can also increase the amount of information in each fingerprint as a larger number of samples can be recorded in a shorter amount of time.

In this section, the sampling rates and number of signal strength samples in the fingerprint maps are described. This discussion is specific to the passive scanning approach used in our signal strength extraction software, described in Appendix A. Using the passive scanning approach, the positioning device listens on the channel for packets emitted by the access points and recording their signal strength. No packets are emitted by the device during passive scanning.

The rate at which signal strength data could be gathered differed considerably between the two environments because *channel hopping* was used in the CSE environment. Channel hopping is a process where the wireless interface rapidly switches its receiving channel (i.e. receiving frequency) to scan its surroundings for possible client access points. In the house environment all access points were running at the same frequency, but in the CSE building the access points were set to different channels and hopping was necessary.

If channel hopping is not activated, signal strength values from each access point can be recorded at the rate packets are received on the single channel. At a minimum, *beacon packets* are broadcast by access points at a constant rate (usually approximately 10Hz) on their set channel to indicate their area of coverage [48]. If there are data packets on the channel the sampling rate will be higher.

As an aside, although no empirical testing was performed on heavily trafficked networks, there is no reason that the amount of traffic should have an effect on positioning accuracy. 802.11 media access control ensures that data transmissions from devices sharing a channel should not collide [48], and therefore the signal strength should be unaffected.

If channel hopping is activated, the sampling rate will be much lower. Each channel must be scanned for at least 100ms to ensure the receipt of any beacon packets, which are transmitted once every 100ms. Kismet [54], the packet sniffer used in our implementation (described in Appendix A), has a scan interval of 200ms by default. Although this variable is ostensibly user configurable, it was found that (at least with the specific hardware used) changing its value had no effect.

The number of channels varies according to the country, but in Australia (where our study took place) and the U.S. there are 11 defined channels for 802.11b/g [48]. In an 11 channel context, Kismet listens to each channel once every 2.2s, and the theoretical minimum sampling rate is 0.45 Hz.

Table 4.3 shows the sampling rates in each environment, based on data from the fingerprint maps. Only beacon packets were considered in these statistics. The house fingerprints achieved close to the predicted 10Hz; the discrepancy is probably due to packet loss. The CSE fingerprints had a sampling rate (1.20Hz and 1.25Hz for point and sector maps respectively) more than twice the potential minimum rate of 0.45Hz; this is probably due to the fact that packets on neighbouring channels are often (although not always) observed when channel hopping. Note that the sampling rate for both point and sector

Environment	Mean Packets per AP		Interval (s)		Mean Sampling Rate (Hz)	
	Point	Sector	Point	Sector	Point	Sector
House	46.53	181.75	5	20	9.31	9.09
CSE	6.02	25.06	5	20	1.20	1.25

Table 4.3: *Fingerprint beacon packet statistics for both environments. The mean packets per access point (AP) is the mean number of packets recorded from each access point, per fingerprint. The interval is the recording time for each fingerprint. The sampling rate, for each access point, is calculated as the mean packets divided by the interval.*

maps is over 7 times higher for the house environment than for the CSE environment.

4.4 Input Averaging

The data produced by Kismet is received as a series of individual signal strengths recorded at the time of each packets arrival. Converting a dump of this packet data is straightforward for building fingerprint maps, but using it as input in the positioning phase is more problematic. When constructing a fingerprint map, all packets are considered when building the mean signal strength vectors for, say, the Nearest Neighbour algorithm, or the frequency histograms for the Bayesian Histograms. When using the data as input, it is necessary to restrict the scope to only recently observed packets, to provide a snapshot that reflects the current position of the mobile agent.

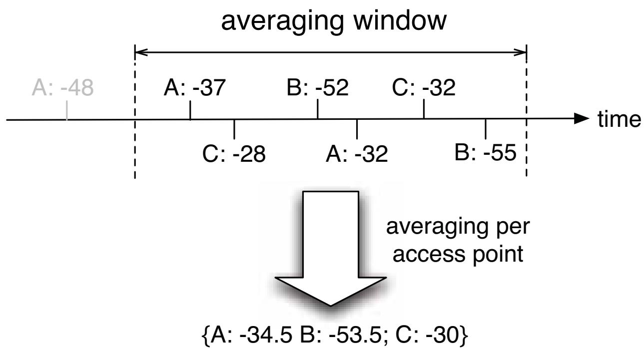


Figure 4.3: *Diagram showing input averaging of incoming signal strength data from 3 access points A, B and C. Each letter:number pair represents a single packet from a particular access point.*

For the Nearest Neighbour and Bayesian algorithms the stream of packets are converted into a series of snapshots by a process called *input averaging*. This process is illustrated in Figure 4.3. Packets are received from Kismet in the sequence they are received by the hardware; effectively unordered with respect to their originating access point. The input averaging process accumulates the signal strengths for each access point over a window of time and calculates the average signal strengths within this window.

The resultant vector of mean signal strength values can be used as input to the positioning algorithm.

The averaging process stabilises the noisy input readings, and reduces the number of positional calculations necessary, compared to calculating a new position upon receiving each new packet. This comes at the expense of responsiveness; if the window is too long the positional output will appear to lag behind the current position of the mobile agent.

In the house environment, a window of 1s is used to generate the experimental results. Each 5s test point generates a sequence of 5 positional outputs. This short interval is possible due to the high sampling rate of the test data. A positioning rate of 1Hz is similar to that of most consumer GPS devices, and should be sufficient for tracking a moving target.

In the CSE environment, a window of 5s is used; i.e. the position is updated once every 5s and each test point generates one positional output. A value of at least 2.2s is necessary to ensure that each test contained data from all channels (the time taken to scan all channels). This lower sampling rate means the system would be less responsive if it were tracking a moving target.

4.5 Conclusions

This chapter described the test environments used for our experiments. Each environment was described in terms of fingerprint, test point, and access point locations, construction materials, and the signal strength sampling rates. The data gathered constitutes test data for the results described in Chapter 5, in which algorithms and surveying methods compared, and Chapter 6, in which the effects of varying access point density and layout are compared.

Chapter 5

Comparison of Algorithms and Sampling Methods

In this chapter, the accuracy of the techniques described in Chapter 3 is evaluated, using test data recorded as described in Chapter 4. The point and sector surveying methods, Nearest Neighbour, Bayesian, and Distribution Distance positioning algorithms, and arithmetic and weighted output averaging methods, are compared using data from the house and CSE environments. The aim of this chapter is to determine the best parameter values for each algorithm and the most accurate algorithms and surveying methods spanning both test environments.

Firstly, results are summarised by tabulating the most accurate combination of parameters for each algorithm and the resulting accuracy, in terms of distance error (in metres). These results were generated by exhaustively testing all possible combinations of parameters for each algorithm. The remainder of the chapter examines the results in more detail. Detailed descriptions of the results for each algorithm are provided, showing how accuracy varies across a range of algorithm-specific parameter values. The effects of output averaging are then analysed, and the effectiveness of the two surveying methods. Finally, results are compared to the literature and overall conclusions are presented.

5.1 Summary of Results

The results shown in this chapter were generated by simulation, using the test data and fingerprint maps gathered as described in Chapter 4. Accuracy results were exhaustively generated for all possible combinations of environment, surveying method, output averaging method, and algorithm specific parameters. Due to the large number of combinations, this resulted in an extremely large set of results. For clarity, this chapter focuses on the combinations which produced the best accuracy. Appendix B contains the results in their entirety.

Algorithm	Map	Averaging	Best Parameters	Distance Errors (m)			
				Mean	50th	90th	Max
Nearest Neighbour	Sector	Weighted	p=1 k=2	1.35	1.15	2.65	4.52
Bayesian Histogram	Sector	Weighted	b=4 k=3	**1.25**	1.12	2.36	5.12
Bayesian Gaussian	Sector	Arithmetic	k=2	1.37	1.13	2.77	5.97
p-Norm Distibution Distance	Sector	Weighted	b=6 k=2	1.42	1.32	2.67	5.34
Match Distribution Distance	Sector	Weighted	b=2 k=2	1.30	1.13	2.47	5.10

Table 5.1: *Parameter and map type combinations that achieved the lowest distance error, displayed by algorithm, for the house environment. The Bayesian Histogram algorithm has the lowest mean distance error of 1.25m (in bold).*

Algorithm	Map	Averaging	Best Parameters	Distance Errors (m)			
				Mean	50th	90th	Max
Nearest Neighbour	Point	Weighted	p=2 k=8	**2.86**	2.45	5.14	17.45
Bayesian Histogram	Point	Weighted	b=8 k=11	3.18	2.61	6.39	14.00
Bayesian Gaussian	Point	Arithmetic	k=3	3.76	3.10	6.93	21.55
p-Norm Distribution Distance	Point	Weighted	b=12 k=6	3.14	2.75	5.72	13.22
Match Distribution Distance	Point	Weighted	b=6 k=7	2.87	2.36	5.29	14.93

Table 5.2: *Parameter and map type combinations that achieved the lowest mean distance errors, displayed by algorithm, for the CSE environment. The Nearest Neighbour has the lowest mean distance error of 2.86m (in bold).*

Tables 5.1 and 5.2 show the best results arranged by algorithm. The tables represent the best results for the house and CSE environments, respectively. Each row shows the combination of parameters and surveying method that produced the lowest mean distance errors for each algorithm. For example, the first row of Table 5.1 (which represents the results for the house environment) shows that the Nearest Neighbour algorithm produced the best results using the sector map, weighted output averaging, and the algorithm parameters $p = 1$ and $k = 2$. The actual distance errors are tabulated in the final four columns.

Several measures of the *distance error* are shown - the distance between the output of the positioning algorithm and the actual location the test data was recorded at. The measures shown are the mean, 50th percentile, 90th percentile and maximum distance errors. The mean distance error provides a good indication of overall accuracy, as it factors in all results. This is the most common measure of accuracy, and is found in most of the comparable studies reviewed in Chapter 2 [4, 90, 43]. The 50th percentile distance error is otherwise known as the median distance error, that is, half the distance errors fall below this value. The 90th percentile error for which 90% of the errors are less than, and gives some indication of the accuracy bound of the algorithm without considering the most pathological cases. The maximum distance error is the largest error result, and gives some indication of the worst case scenario; however because it represents an outlying value it is unreliable in terms of ranking the results.

The following results can be observed from Tables 5.1 and 5.2:

- **The mean distance errors produced by each algorithm are relatively similar.**

In the house environment, the best algorithm was the Bayesian Histogram, with a mean distance error of 1.25m; the worst algorithm was the p-norm distance, at 1.42m, only 14% higher. In the CSE environment, the best algorithm was the Nearest Neighbour with a mean distance error of 2.86m; the worst algorithm was the Bayesian Gaussian algorithm at 3.76m, 32% higher. Furthermore, the Nearest Neighbour algorithm is competitive with the more sophisticated Bayesian and Distribution Distance based algorithms, which suggests that taking more information into account does not necessarily improve accuracy. The individual results of each algorithm are discussed in Sections 5.2.1– 5.2.4.

- **Output averaging improves accuracy.** The best results, regardless of the algorithm used, occurred for $k > 1$ (k is the number of fingerprints to use when applying output averaging, see Section 3.6.2). The effects of output averaging are discussed in Section 5.3.

- **The best surveying method differs between the two environments.** In the house environment the sector map has the best accuracy, while in CSE environment the point map has the best accuracy (column 2 of Tables 5.1 and 5.2). I speculate that this is due to point maps being relatively more robust to any interference that occurs during the recording of the fingerprint map, as multiple fingerprints are recorded at each point, providing some level of redundancy. By comparison, sector maps only have one fingerprint per location. This is discussed in Section 5.4.

- **The house results are more accurate than the CSE environments in all cases**. The two best case mean distance errors are 1.25m in the house environment and 2.86m in CSE environment (more than double the house result). This is a consistent trend for all algorithms and is due to a combination of differing conditions in each environment, including access point density, construction materials, room sizes, and the presence of interference. The difference in accuracy between the two environments is discussed in Section 5.5.

Note that mean distance errors for both environments are always greater than the 50th percentile error. This indicates that the errors above the mean, although fewer in number, have a relatively higher magnitude (as they include the most pathological errors).

5.2 Algorithm Specific Parameters

This section investigates how accuracy varies according to algorithm-specific parameter values: either p (the p-norm distance parameter) for the Nearest Neighbour algorithm, or b (bin width) for the other algorithms. For each algorithm the mean distance error is plotted using a range of parameter values for each combination of surveying method and output averaging method.

Note that this section does not include results for the Bayesian Gaussian method, as it has no parameters to vary.

5.2.1 Nearest Neighbour

Figure 5.1 shows the mean distance errors using the Nearest Neighbour algorithm (Section 3.6.1) for $1 \leq p \leq 10$ (p is the parameter used in the p-norm distance formula to calculate the distance between the input and the fingerprints). These results were generated using values of k (the output averaging threshold, Section 3.6.2) that produced the lowest mean distance errors for each combination of surveying method and output averaging technique. For example, in the house environment, using arithmetic output averaging and point surveying, the best combination of parameters is $p = 1$, $k = 2$ (row 1 of Table 5.1). Therefore, the corresponding blue plot in Figure 5.1a contains the distance errors generated by varying p with fixed $k = 2$. This approach is also taken to plotting the results of the other algorithms in the remainder of this section.

Note that this does not necessarily imply that $k = 2$ is the best k for *all* p; it only indicates that it is the best k value when $p = 1$, and that the combination of these two parameter values produces the best overall results, when using point sampling and arithmetic output averaging. It is possible that for other values of p, different values of k produce better results. However, the best k is usually similar for each p (see Appendix B), and fixing k allows us to demonstrate, using a 2-dimensional plot, how the result varies with p. I experimented with 3-dimensional plots to represent the results more exhaustively, but they provided insufficient clarity and accuracy. If more detail is required, Appendix B contains the mean distance errors for all parameter combinations.

Using point surveying (Figures 5.1a and 5.1b), the distance errors are very similar for all values of p. In the sector results (Figures 5.1c and 5.1d), distance errors gradually increase with p, but the difference is still only marginal. This suggests that varying p produces only a marginal, and inconsistent, difference in accuracy.

5.2.2 Bayesian Histogram

Figure 5.2 shows the mean distance errors using the Bayesian Histogram algorithm (Section 3.6.3) for $1 \leq b \leq 30$. The best values of b vary depending on the environment and the surveying method, and to a lesser extent on the output averaging technique.

Using point surveying, the house results (the blue plots in Figures 5.2a and 5.2b) are poor; the distance errors are quite variant according to bin width and relatively inaccurate compared to using sector sampling in the same environment (the blue plots in Figures 5.2c and 5.2d). Using point surveying, the lowest mean distance error is 1.54m

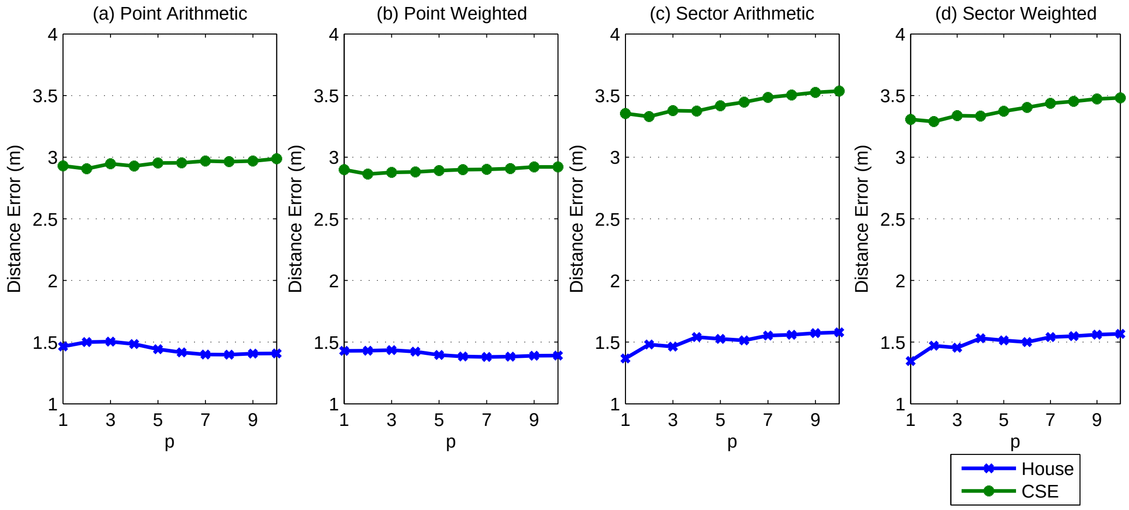


Figure 5.1: *Nearest Neighbour mean distance errors for $1 \leq p \leq 10$; p is the p-norm distance parameter for the Nearest Neighbour algorithm. From left to right: point surveying and arithmetic output averaging, point surveying and weighted output averaging, sector surveying and arithmetic output averaging, and sector surveying and weighted output averaging. Results from the house environment are blue, results from the CSE environment are green.*

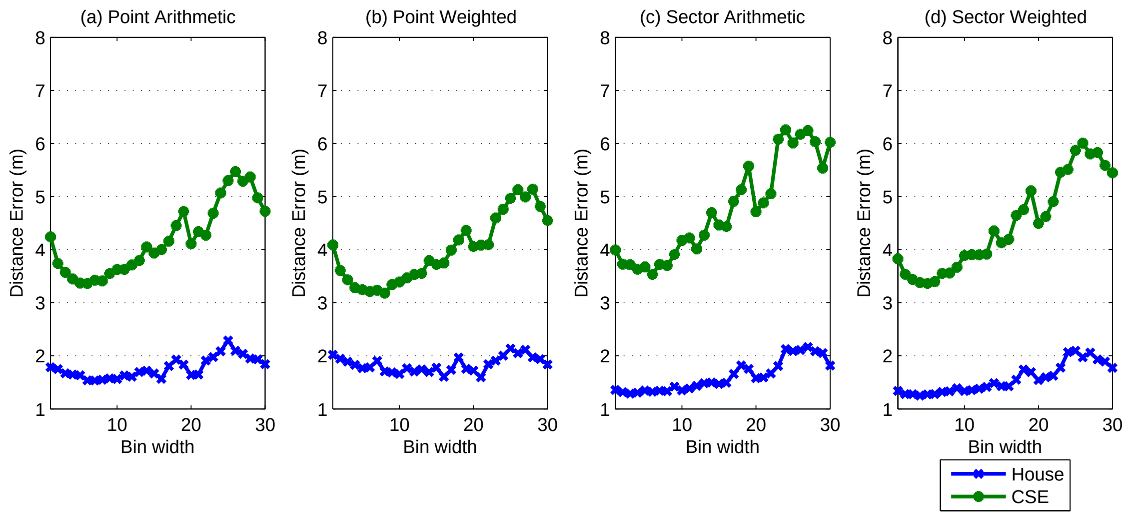


Figure 5.2: *Bayesian Histogram mean distance errors for $1 \leq b \leq 30$.*

using arithmetic output averaging; in contrast, the sector surveying results have a lowest mean distance error of 1.25m. The underlying reasons for this are discussed in detail in Section 5.4.

In the CSE environment (the green plots in Figure 5.2), irrespective of surveying method, the distance errors are relatively large for small b values. This effect is more pronounced than in the house results, probably because of the lower sampling rate and the noisier, more multipath affected environment, which necessitates wider bin widths to smooth the data. Increasing the bin width always improves the results, with the lowest mean distance error (3.18m) occurring at $b = 8$ using point surveying and weighted output averaging (green plot in Figure 5.2b).

For all sets of results, for approximately $b \geq 15$ the distance errors tend to increase and show large, bin width specific variations in mean distance error. This is most apparent in the CSE environment, which had more interference than the house environment. This variation is probably due to bin alignment causing inconsistent distortions in the final result that are specific to particular values of b (Section 3.6.3).

5.2.3 p-norm Distance

Figure 5.3 shows how the mean distance errors vary for different bin widths b using the p-norm Distance algorithm (Section 3.6.4). Similarly to the histogram method, the best bin width ranges vary according to the environment, the surveying method and to a lesser extent, the output averaging method.

Most of the best results occur for approximately $6 \leq b \leq 12$. The wide bin widths smooth the histograms, which in raw form (that is, $b = 1$) are noisy and tend to incompletely sample the underlying signal strength distribution. There is a greater degree of bin specific variation for $b > 12$, although some of the lowest distance errors are in this range. This is likely to be due to varying degrees of distortion caused by bin alignment (Section 3.6.4), specific to particular values of b.

In the house environment, using sector surveying (the blue plots in Figures 5.3c and 5.3d), the distance errors are comparatively lower for $b < 6$. I speculate that this is because the sector fingerprints contain a larger amount of data and are therefore relatively less noisy than the point fingerprints, requiring less smoothing. This effect is not apparent in the CSE sector results (the green plots in Figures 5.3c and 5.3d), probably because the CSE tests have a much lower sampling rate and therefore have relatively less data.

5.2.4 Match Distance

Figure 5.4 shows how distance errors vary for different values of b using the Match Distance algorithm (Section 3.6.4). Low distance errors occur at relatively small bin widths

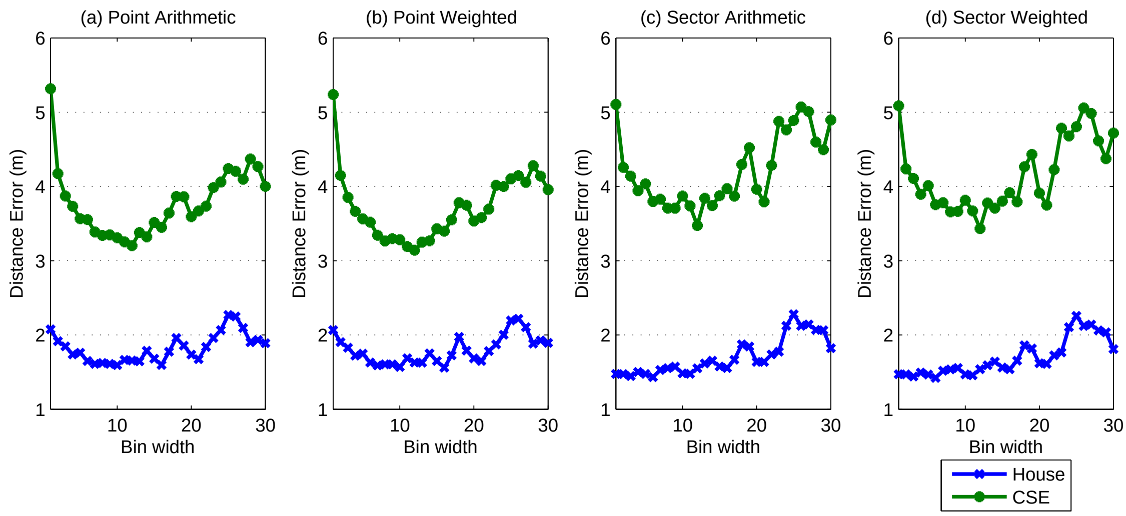


Figure 5.3: *p-norm Distance mean distance errors for* $1 \leq b \leq 30$.

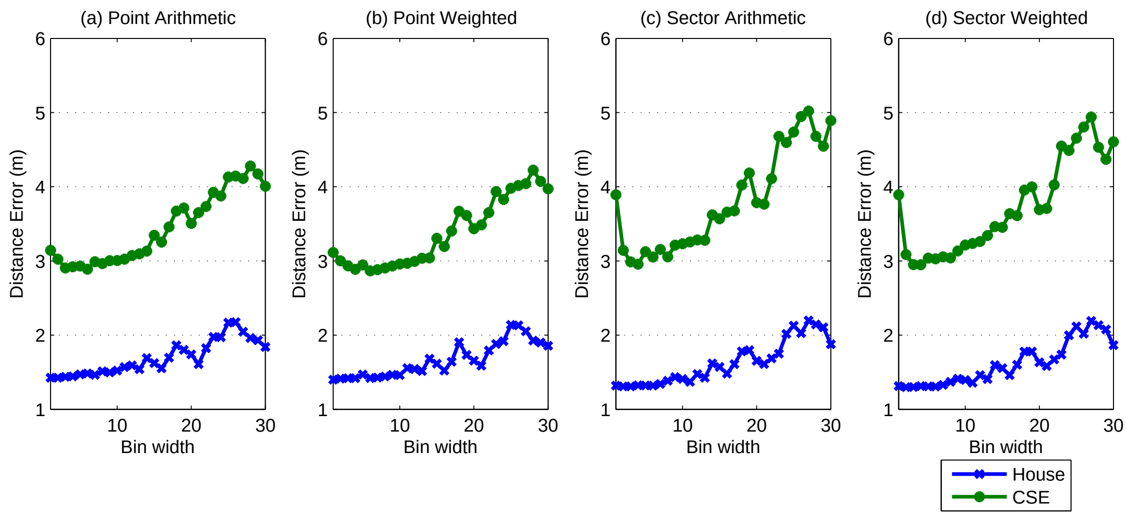


Figure 5.4: *Match Distance mean distance errors for* $1 \leq b \leq 30$.

compared to the other algorithms. Even using the raw data ($b = 1$) directly produces a good result in most cases, except in the CSE environment using sector surveying (the green plots in Figures 5.4c and 5.4d) where the results are poor for $b = 1$ specifically, but accurate for $b \geq 2$.

Similarly to the Bayesian Histogram and p-norm distance methods, higher values of b (approximately $b > 12$) tend to lead to greater variations in distance error, due to greater amounts of bin alignment related distortion.

5.2.5 Best Parameter Ranges

Algorithm	Best Parameter Range
Nearest Neighbour	$1 \leq p \leq 10$
Bayesian Histogram	$4 \leq b \leq 10$
p-Norm Distribution Distance	$6 \leq b \leq 12$
Match Distribution Distance	$2 \leq b \leq 10$

Table 5.3: *Summary of the best parameter ranges for each algorithm.*

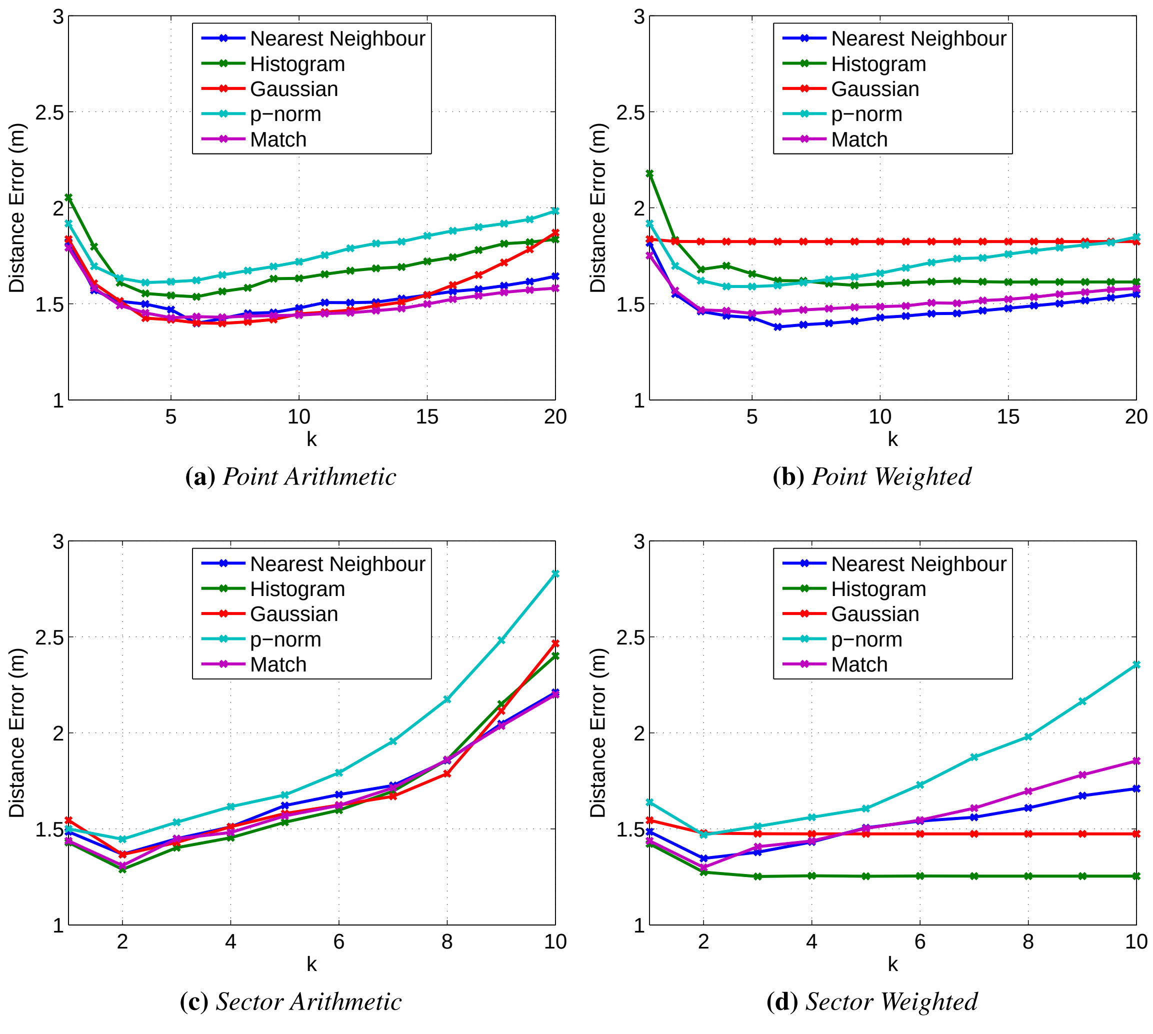
Table 5.3 summarises the best parameter ranges for each algorithm based on the previously presented results in this section. The results are an approximation, taking into account the results for both environments, each surveying method, and each output averaging method.

In comparison to the Bayesian Histogram and p-norm Distance algorithms, the Match Distance algorithm produces its best results at lower values of b (and in a wider range). This reflects the fact that the Match Distance algorithm is less vulnerable to incomplete and noisy fingerprint data, and therefore does not require the data to be summarised as much as the other methods; in other words, it can take more information into account. I speculate that this contributes to its accuracy being slightly superior to the Bayesian Histogram and p-norm methods despite the fact that all three algorithms use histograms to represent signal strength data.

5.3 Spatial Output Averaging

This section shows the effects of *spatial output averaging* (Section 3.6.2); the process of taking the spatial average of the first k ranked fingerprints to produce the final result.

Figures 5.5 and 5.6 show the how the mean distance errors of each algorithm vary according to k (the first k ranked fingerprints are used in the average, the others are excluded). These figures were plotted using the algorithmic parameters (that is, b or p) that achieved the lowest mean distance errors for each algorithm overall, for each environment (Tables 5.1 and 5.2). Note that k is not independent of the other algorithm parameters;



(a) *Point Arithmetic* **(b)** *Point Weighted*

(c) *Sector Arithmetic* **(d)** *Sector Weighted*

Figure 5.5: *Mean distance errors from the house environment for each algorithm using* $1 \leq k \leq 20$ *with the point map and* $1 \leq k \leq 10$ *with the sector map.*

generally the most accurate parameters produce a wider range of accurate k values, because the output list usually contains a more accurate ranking of fingerprints. However, this approach to visualizing the results allows the effects of varying k to be clearly shown, and the observations are generally relevant under the assumption that the values of the algorithm-specific parameters are optimal.

House Environment

In the house environment (Figure 5.5), the mean distance error with respect to k usually monotonically decreases between $k = 1$ and the k value which produces the minimum mean distance error, then monotonically increases for values of k greater than that. For example, Figure 5.5a shows results from the house environment using point surveying and arithmetic output averaging. Using the p-norm Distance method as an example, at $k = 1$ the mean distance error is 1.9m; the mean distance error declines monotonically until $k = 4$, where the distance error is 1.6m; the mean distance error then monotonically increases for greater k values. This is because the best value of k is a trade-off between smoothing the output such that multiple estimates are taken into account, and the output estimate being diluted by fingerprints too distant from the correct fingerprint.

The results (in terms of the relative decreases in distance error, and the values of k that produce the best results) are similar for all algorithms, suggesting they all respond fairly similarly to arithmetic output averaging. The k values that produce the minimum mean distance error range between 4 (using the p-norm Distance) and 6 (the Bayesian Gaussian, Histogram, and Nearest Neighbour methods). The absolute distance errors vary, but the degree to which the mean distance error is reduced is similar for each algorithm. The exact magnitude of this improvement is described in Section 5.3.2.

In contrast, the results differ between algorithms when using *weighted* output averaging (Figure 5.5b), due to the differences in methods used to generate output weights between the algorithms. The Bayesian Histogram and Gaussian algorithms in particular produced flat k curves after a certain threshold, suggesting that the weights are too small above a certain k to influence the output. This is discussed further in Section 5.3.1. The other algorithms have results similar to those using arithmetic output averaging. The main difference is that when using weighted output averaging, the curves have a lower gradient for k greater than the value that produces the minimum distance error. This is because the gradually decreasing weights from the output fingerprint list reduce the degree to which fingerprints contribute to the weight average. Despite this difference, the minimum mean distance errors for each output averaging method are similar.

Figure 5.5c shows the mean distance errors for the house using sector surveying and arithmetic output averaging. The distance errors for all algorithms are reduced for $k = 2$, but for $k = 3$ are similar to the results using *no* output averaging (that is, $k = 1$) and monotonically increasing in distance error for $k > 2$. The lowest k ranges differ between the point and sector maps because of the differing spatial coverage of the two map types. In a point map, four fingerprints represent a single point, one recording for each direction. In the sector map, there is only one fingerprint for each sector. For the point map, this means that the first k fingerprints are likely to share locations; for the sector map the fingerprints represent unique locations. Therefore, the first k point fingerprints tends to represent a smaller dilution of the estimate than the first k sector fingerprints.

Note that output averaging also reduces the distance errors in the sector results proportionally less than the point results, compared to when not using output averaging (k=1). The improvement is approximately 20% using point surveying, and 10% using sector surveying (Section 5.3.2). However, in the house environment, the $k = 1$ estimate using the sector map is relatively more accurate than using the point map, so the lowest mean distance errors are overall slightly lower than the point results.

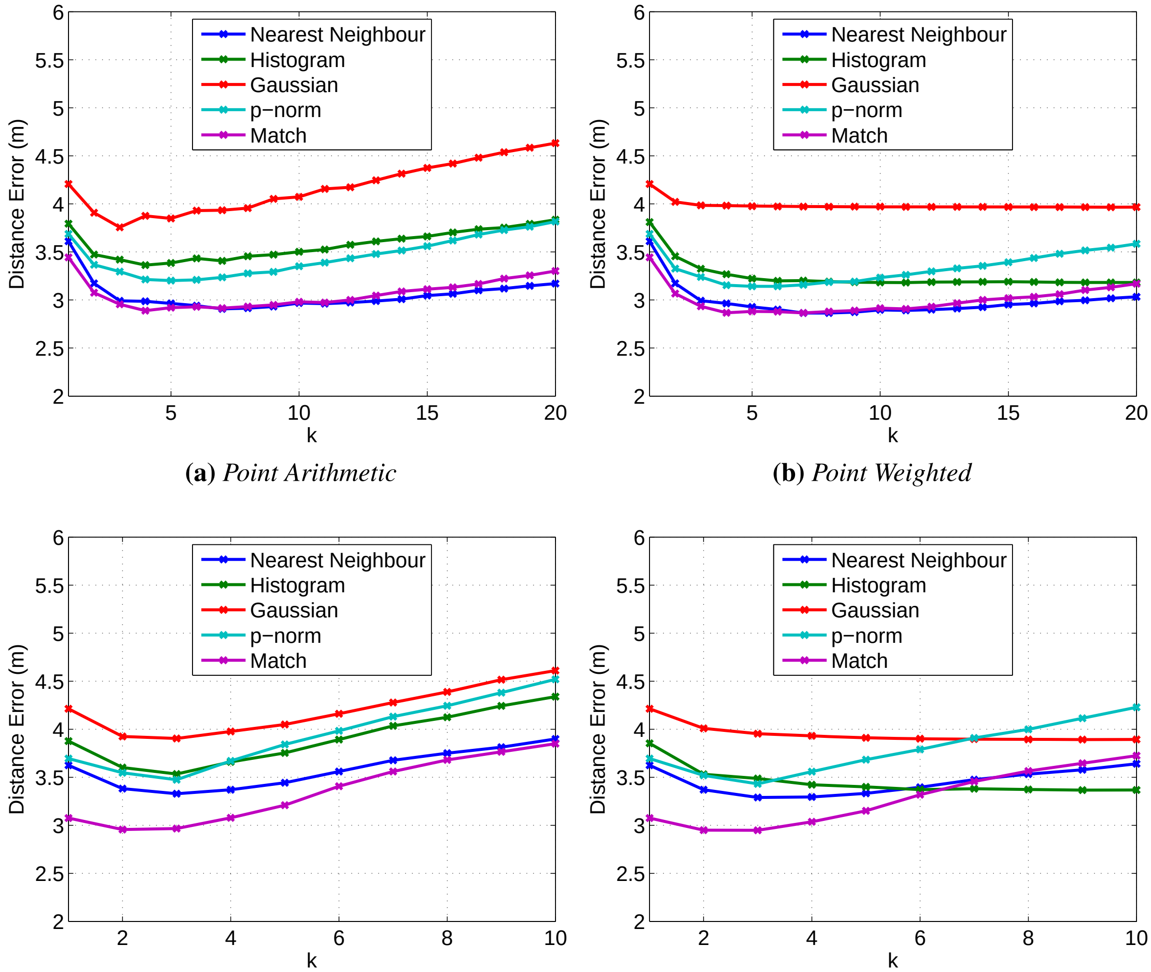
Figure 5.5d shows the mean distance errors using sector surveying and weighted output averaging. Again, the Bayesian algorithms show very little change in distance error after a threshold k, while the other algorithms show similar mean distance errors to when using arithmetic output averaging.

CSE Environment

Figure 5.6 shows the mean distance errors for varying k from the CSE environment. In terms of the most accurate values of k, the point results are quite similar to the house environment, but the sector results are slightly different. Figure 5.6c shows the mean distance errors using sector surveying and arithmetic output averaging. Output averaging improves the result (compared to $k = 1$) for a wider range of k, depending on the algorithm. Except when using the Match Distance algorithm, the results are improved for $2 \leq k \leq 4$. By contrast, in the house environment only $k = 2$ produced a lower distance error. I speculate that this is because the initial estimates (i.e. the highest ranked fingerprints) without output averaging are less accurate to begin with (between 3.1 and 4.2m, compared to the house range between 1.8 and 2.3m), and therefore the CSE environment benefits relatively more from averaging a greater number of fingerprints as this increases the probability that the correct fingerprint will be part of the average.

5.3.1 Bayesian methods and Weighted Output Averaging

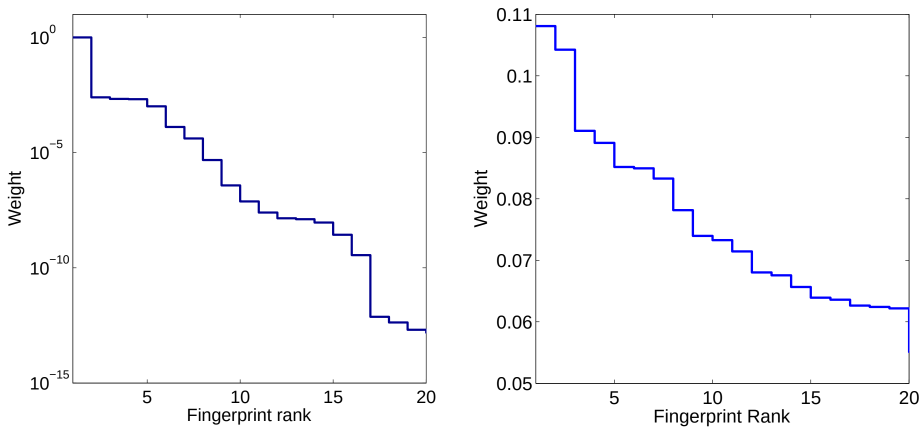
The Bayesian methods produce a probabilistic output; each fingerprint is associated with the probability that the mobile agent is nearest each fingerprint (Section 3.6.3). These probabilities are used directly as weights for weighted output averaging. When using either the Bayesian Histogram or Gaussian methods, these probabilities tend to sharply decline after the first few fingerprints; hence the final result remains fairly static after a low k threshold as seen in Figures 5.5b, 5.5d, 5.6b and 5.6d.



(a) *Point Arithmetic* **(b)** *Point Weighted*

(c) *Sector Arithmetic* **(d)** *Sector Weighted*

Figure 5.6: *Mean distance errors from the CSE environment for each algorithm using* $1 \leq k \leq 20$ *with the point map and* $1 \leq k \leq 10$ *with the sector map.*



(a) *Histogram Weights* **(b)** *Nearest Neighbour Weighted*

Figure 5.7: *Comparison of output weights using the Histogram and Nearest Neighbour methods. Note that 5.7a uses a logarithmic scale over a large range so that the decrease in weights can be visualised, while 5.7b uses a linear scale over a much smaller range.*

An example comparing two lists of fingerprint weights is shown in Figure 5.7. These weights are the results from a test case from the house point data using the Bayesian Histogram method. Both plots show the weights for the 20 highest ranked fingerprints from the output of the algorithm. The first Bayesian Histogram weight is 0.9921, followed by 0.0025, 0.0021, and degrading rapidly by many orders of magnitude over the first 20 ranked fingerprints; so rapidly that it is necessary to plot the values on a logarithmic scale to visualise them. By comparison, the Nearest Neighbour weights are within a range of only 0.05.

The relatively sharp decline in Histogram method weights is related to the low probabilities the method generates for input signal strengths outside the ranges of the training data (Section 3.6.3). Generally, there tends to be only a few fingerprints with training data that contain the input signal strength entirely; that is, each element of the input vector falls within the range of previously recorded fingerprint data (summarised as a histogram) for the corresponding access point. These fingerprints are ranked highest, and have the largest weights. The next ranked fingerprints tend to be those for which the input vector has one element outside the recorded range, followed those for which the input vector has two elements outside the recorded range, and so on. When the input data falls outside the recorded range of a fingerprint, the final probability associated with that fingerprint is sharply reduced because the histograms in each fingerprint have low probabilities associated with data outside the recorded range. Effectively, this means a penalty is attracted for each element in the input vector that does not fall within the range of recorded data for the fingerprint.

The end result is that large probabilities are assigned to a small number of fingerprints, with very small probabilities assigned to the remainder. Therefore, when weighted output averaging is used, the weights (which are equal to the probabilities) are too small to make a difference to the final result above a certain k.

Weights generated by the Bayesian Gaussian algorithm decline even more sharply after the first few ranked fingerprints, because the probabilities generated by the Gaussian function drop extremely sharply outside the standard deviation. The probabilities are usually lower than those outside the recorded data range for the Bayesian Histogram. This results in weighted output averaging performing generally worse than arithmetic output averaging for the Bayesian Gaussian method, as the number of fingerprints significantly contributing to the output averaging process is too few.

In contrast, weighted output averaging with the Bayesian Histogram method performs similarly to arithmetic output averaging, because the weights are more accurate. Using weighted output averaging has the advantage that a fixed k (which may be specific to the surveying method or environment) is unnecessary; instead, k can be be set to a high value (say, 20) and the positioning algorithm will determine, for each input, how much each fingerprint should contribute to the final average.

Algorithm	Surveying Method	Averaging Method	Reduction in Error (%)		
			House	Lab	Mean
Nearest Neighbour	Point	Arithmetic	23.2	19.5	21.4
		Weighted	24.1	20.7	22.4
	Sector	Arithmetic	7.9	8.2	8.1
		Weighted	9.3	9.3	9.3
Bayesian Histogram	Point	Arithmetic	25.2	11.4	18.3
		Weighted	26.7	16.5	21.6
	Sector	Arithmetic	9.8	8.8	8.9
		Weighted	12.0	12.6	12.3
Bayesian Gaussian	Point	Arithmetic	23.8	10.7	17.3
		Weighted	0.7	5.7	3.2
	Sector	Arithmetic	11.5	7.3	9.4
		Weighted	4.6	7.7	6.2
p-norm Distance	Point	Arithmetic	16.0	13.2	14.6
		Weighted	17.1	14.8	16.0
	Sector	Arithmetic	3.6	6.0	4.8
		Weighted	10.3	7.1	8.7
Match Distance	Point	Arithmetic	20.4	16.1	18.3
		Weighted	17.2	16.8	17.0
	Sector	Arithmetic	8.9	3.9	6.4
		Weighted	9.6	4.2	6.9

Table 5.4: *Reductions in distance error as a result of output averaging.*

5.3.2 Overall Effect of Output Averaging

Table 5.4 shows the effects of output averaging in terms of the reduction in distance error, as a percentage of the distance error without using input averaging. For the results in this table, each algorithm used the algorithm-specific parameters that produced the lowest mean distance errors in each case, as shown in Section 5.1.

The results are mostly similar when using either arithmetic or weighted output averaging, except in the case of the Bayesian Gaussian using point surveying, where weighted output averaging produced a considerably smaller reduction distance error. This is due to the poor weights output by the Bayesian Gaussian method in that instance; this is examined in detail the previous section.

Output averaging using point surveying consistently produces a higher reduction in distance error than when using sector surveying; the mean reduction in error ranges between 14.6% and 21.6% (ignoring the Gaussian point weighted result). Using sector surveying, the mean reduction in error ranges between 4.8% and 10.4%. This is because higher values of k can be used with point surveying due to the differences in fingerprint organisation, which produces a correspondingly higher reduction in accuracy.

		Difference in mean error using sector surveying	
Algorithm	Averaging	House	CSE
Nearest Neighbour	Arithmetic	-2.1	+14.4
	Weighted	-2.2	+15.0
Histogram	Arithmetic	-16.2	+5.0
	Weighted	-21.9	+6.0
Gaussian	Arithmetic	-2.1	+3.7
	Weighted	-19.2	-2.0
p-norm	Arithmetic	-4.0	+8.4
	Weighted	-4.7	+9.2
Match	Arithmetic	-10.2	+2.4
	Weighted	-9.0	-8.9

Table 5.5: *Differences in mean error between the point and sector results, showing the differences between the two as a percentage of the point surveying mean distance error. Negative values indicate a decrease in mean distance error using sector surveying, compared to using point surveying; positive values indicate an increase.*

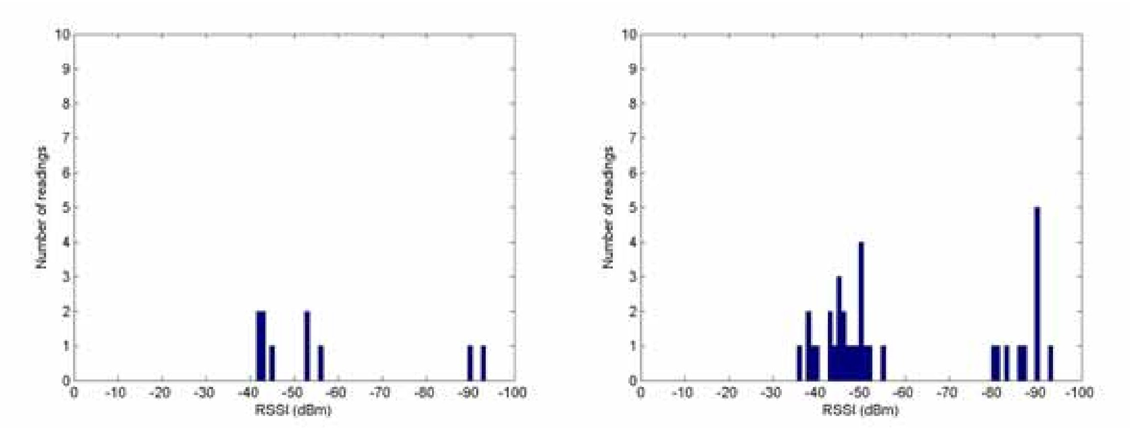
5.4 Sector versus Point Surveying

This section analyses the relative accuracy of using point and sector surveying in each environment. Table 5.5 shows the difference in mean distance errors between the point and sector maps for each algorithm. The best performing algorithm parameter values (that is, values of p for the Nearest Neighbour and b otherwise) were used for this comparison. Differences are represented as a percentage of the point surveying result; + indicates an increase in mean distance error (that is, less accuracy) and - indicates a decrease (more accuracy).

The results are mostly inconsistent across the two environments. The sector map shows similar or better accuracy in the house, but in most cases accuracy was similar or slightly worse in the CSE environment. I spectulate that the disparity in comparative accuracy across the two environments is due to interference in the CSE environment caused by the large number of 2.4GHz RF devices in and around the laboratory area, and the differences in the robustness of each algorithm to such interference.

Figure 5.8 shows two examples of interference-affected fingerprints from the point and sector maps. In the point fingerprint (Figure 5.8a), the data is mostly clustered in between -42dBm and -56dBm, with two readings of -90dBm and -93dBm. The data in the sector fingerprint (Figure 5.8b) is clustered in two distinct areas of -36dBm to -56dBm and -80dBm to -93dBm. A small number of fingerprints in both the CSE point and sector maps show similar effects, all in data recorded from one particular access point (denoted access point C in Figure 4.2). This suggests that the interference is specific to that channel (11). Note that this kind of interference-affected data is not found in either house fingerprint map.

Interference has a subtly different effect depending on the sampling method and algo-



(a) *Point* **(b)** *Sector*

Figure 5.8: *Example of two multi-modal fingerprints from the CSE point and sector fingerprint maps. The point fingerprint represents 5s worth of data, while the sector map represents 20s of data.*

rithm being used. Using *point* sampling, each location is covered by four recordings of signal strength recorded for 5s each, so any transient interference (assuming it lasts for only a few seconds, as in the examples shown) affects only one or two of the fingerprints, and the remaining recordings at that location for different directions are unaffected. Often, the data recorded while facing different directions is quite similar, and so the remaining recordings effectively act as a backup. By contrast, using *sector* sampling each location is covered by only one fingerprint recorded for 20s; if the positioning algorithm is unable to make effective use of the interference-affected fingerprint, then the map effectively contains no accurate representation of the associated location, resulting in poor accuracy at that location.

Therefore, the results using the sector map in the CSE environment somewhat reflect the effectiveness of each algorithm in dealing with interference-affected fingerprints. The Nearest Neighbour and p-norm Distance algorithms are the most affected (15.0% and 9.2% worse using the sector map), while the other algorithms have only marginally worse sector accuracy compared to the point map, which suggests they are relatively more robust to interference.

The remainder of this section discusses the effects of using point and sector maps for each algorithm and suggests which surveying method should be generally better in each case, based on the empirical results and the underlying mechanics of each algorithm.

5.4.1 Nearest Neighbour

In the house environment, the Nearest Neighbour algorithm produces similar results using either surveying method. In the CSE environment, the results are worse using the sector map by 14.4% and 15.0% for the two averaging methods. As described above, the

CSE environment contains fingerprints affected by interference, which has a greater effect when using a sector map; this is reflected by the differing results for each environment.

The Nearest Neighbour method summarizes the data as the mean, which can produce inaccurate results if the data occurs as several separate clusters. This is because the mean may not fall within the domain of the observed data, at a signal strength value which is not actually observed very often at that location. In other words, the mean value can be unrepresentative of the location. Therefore, interference-affected fingerprint data can degrade the accuracy of the Nearest Neighbour relatively more than other algorithms, such as the Bayesian and Distribution Distance algorithms, which use more information than just the mean.

For example, consider the example point and sector fingerprints in Figure 5.8. The mean of the point data (Figure 5.8a) is -56dBm, which falls on the lower bound of the cluster between -41dBm and -56dBm. The mean of the sector data (Figure 5.8b) is -59dBm, which falls in between the two clusters. Because of the interference-affected readings, neither mean falls in the middle of the cluster of readings not affected by interference (which is what will be observed at that location the majority of the time), and therefore they are not representative of the fingerprint location. This results in poor accuracy using the Nearest Neighbour algorithm, particularly using the sector map.

This suggests that in environments that might contain interference, it may be better to use the Nearest Neighbour algorithm with a point map rather than a sector map, as this can somewhat reduce the impact of interference. Sector surveying does not provide any advantage to the Nearest Neighbour algorithm in any case, as the extra information it provides is discarded when the information is summarised as the mean.

5.4.2 Bayesian Histogram

In the house environment, the Histogram results using sector sampling are better than those using point sampling by 16.2% and 21.9%. These superior sector results are due to the differences in coverage that the two maps provide, and the Histogram method's inefficiency when processing inputs that lie outside the training data (Section 3.6.3). Table 5.6 shows the percentage of test cases in each environment for which the test input vector was within all corresponding training data ranges for at least one fingerprint (in other words, when the input fell at least partially inside the training data). In the house environment, the test input is within the training data range only 11.4% of the time using the point map, increasing to 92.4% of the time using the sector map.

The table reflects the difference in coverage the sector and point maps provide. The point maps contain signal strength data that is relatively sparse within the signal strength domain, because the fingerprints are recorded only at specific points. By contrast, the sector maps contain data that covers the entire signal strength domain, because the finger-

	Point Surveying	Sector Surveying
House	11.4%	92.4%
CSE	68.7%	99.1%

Table 5.6: *Percentage of test cases in which the test vector was within the range of at least one fingerprint's training data.*

prints are recorded over the entire area. Therefore, using sector maps the input falls within the training data more often. This factor affects the Bayesian Histogram in particular as it can be very inefficient at processing inputs that do not fall directly within the training data (Section 3.6.3).

In contrast to the house results, the CSE sector results were very similar to the point results; in fact the mean distance errors are marginally worse by 5% and 6%. This is due to the relatively higher proportion of inputs that fall within the training data. Table 5.6 shows that in the CSE environment, the test input is within the training data range 68.7% of the time using the point map, increasing to 99.1% of the time using the sector map; a 30.4% improvement, compared to the 81.0% improvement in the house. Because the sector map provides relatively less benefit to the Histogram method, there is no accuracy benefit. The reason a relatively high proportion of inputs fall within the training data in the CSE environment is probably because a relatively larger number of the test points were recorded at point fingerprint locations.

This result suggests that the locations of test points (relative to the locations of the fingerprints) may affect the outcome of algorithm comparisons. In particular, the Histogram method may be more sensitive to test point locations than others, because the test point locations determine whether the input falls within the training data range. More experiments are needed to confirm this hypothesis; for example, a direct comparison of accuracy when using different sets of test point locations. However, such an investigation is outside the scope of this work.

Generally, sector surveying should perform similarly or better than point surveying when using the Bayesian Histogram algorithm; our empirical results support this view. This is because the blanket coverage provided by sector surveying reduces the number of inputs that fall outside the range of the training data, which can substantially increase accuracy.

5.4.3 Bayesian Gaussian

The Gaussian method is relatively indifferent to whether point or sector surveying is used, with only small variations in relative accuracy for the house arithmetic and CSE arithmetic and weighted results. The exception is the house weighted result, where sector surveying produces an improvement of 19.2%; this is because weighted output averaging performed particularly poorly in the house environment as described in Section 5.3.1.

5.4.4 p-norm Distance

In the house environment, there is a very small improvement (4.0% and 4.7% for the arithmetic and weighted results, respectively) in the p-norm distance results. In the CSE environment, the sector map produced worse results (by 8.4% and 9.2%). We believe this is due to the low sampling rate and the inherently wide signal strength ranges of sector maps (due to the device being moved around during recording), which result in histograms with many gaps in the data. These gaps are problematic for the p-norm distance method, which compares the histograms bin by bin; the gaps mean that larger bin widths are required to avoid bins being compared, which reduces the resolution of information. To avoid such gaps, point surveying is probably a better choice if the p-norm distance must be used, although the match distance is a better choice of Distribution Distance algorithm.

5.4.5 Match Distance

The Match Distance algorithm is more accurate by between 8.9% and 10.2% when using the sector map for all test sets except the in the CSE arithmetic output averaging results, where the mean distance error is marginally greater (2.4%). The algorithm is relatively robust to a small amount of interference because the interference affected readings contribute to the difference measure in proportion to their number; if there are only a small handful they will not greatly affect the final estimate. In our tests, sector surveying always produces similar or slightly better results than point surveying when using the Match Distance, regardless of environment. In general, either surveying method should perform reasonably well using this algorithm; sector surveying may provide a slight advantage because it provides a more detailed image of the environment.

5.5 House Versus CSE Environment

The difference in accuracy between the two test sets is due to a combination of several factors:

- The **constitution of the environments** differ in many respects: the CSE environment is larger, has a wider variety of rooms and open areas, and has more metallic objects and objects of varying size (leading to more multipath fading).

  Crucially, the CSE environment contains two large open areas: a large laboratory filled with cubicles, and a foyer area. By contrast, the house area is composed of relatively small rooms. Open areas are detrimental to accuracy because they result in a low *signal strength gradient*; i.e. the rate of decay of signal strength over space. Enclosing walls (that reach from floor to ceiling) are greatly beneficial to accuracy

Environment	Access Point	Range (dBm)	Mean Signal Strength Gradient (dBm / m)
House	A	34.43	2.09
	B	26.40	1.55
	C	22.55	1.91
	D	28.23	2.55
	E	30.67	2.50
	F	39.28	2.26
CSE	A	39.08	1.22
	B	40.28	1.21
	C	39.31	1.28
	D	37.24	1.65
	E	39.38	1.31

Table 5.7: *Signal strength ranges and mean signal strength gradients for each access point for both environments.*

as they sharply attenuate signal strength and allow any fingerprints that they divide to be more easily distinguished.

Table 5.7 shows the overall ranges of signal strengths and the mean signal strength gradients by access point, for each environment. The range is the difference between the lowest and highest mean signal strengths from the fingerprints, calculated using the sector maps. The signal strength gradient of each fingerprint is calculated as the difference between its signal strength and that of the fingerprint with the highest relative signal strength, divided by the distance between the fingerprint and the access point. This gives a rough measure of how sharply the signal strength decays at increasing distance from the access point.

The gradients in the house range between 1.55 and 2.55dBm/m, while the gradients in the CSE environment range between 1.21 and 1.65dBm/m. The lower gradient has the effect of degrading accuracy in the CSE environment due to greater signal strength similarity between neighbouring fingerprints.

- The **sampling rate** in the house environment was much higher than in the CSE environment (Section 4.3), resulting in a higher resolution signal strength map. Further experiments are required to isolate the effects of this factor on accuracy. A lower sampling rate may reduce the effectiveness of sector sampling as it reduces the amount of data that can be captured as the device is moved about the fingerprint area; because less readings are recorded, smaller multipath affected pockets may be skipped. Point sampling should be less affected as the device is static during fingerprint recording.

- The **interference in the CSE environment**, and its absence in the house environment, was demonstrated in the previous section. Interference is detrimental to accuracy: if it occurs during the recording of the fingerprint map, the training data is inaccurate; if it occurs during online positioning, the input will not resemble the training data for the current location.

5.6 Comparisons with other studies

The difference in overall results between the two environments emphasises the difficulty of comparing results between different studies, which not only differ in the environments used, but also have different algorithm implementations and use different equipment. Nonetheless, our results roughly agree with the distance errors found in similar quantitative studies using similar testing methods [4, 90, 66]. mean distance errors of 1.25 and 2.86m and median errors of 1.06 and 2.36m were achieved in the house and CSE environments respectively. In contrast, Bahl and Padmanabhan [4] achieved a best median distance error of 2.13m, Roos et al. [90] had a best mean distance error of 1.56m using 10 access points, 2.60m with 3 access points, and Li et al. [66] had a best case mean distance error of 1.19m. Roughly speaking, mean distance errors of less than 3m are the norm. These results from the literature are summarized in Table 2.2.

Similarly to our work, Roos et al. [90] compare Bayesian methods with the Nearest Neighbour algorithm. They use point surveying in an open laboratory environment relatively similar to our CSE environment, with no output averaging. Their Nearest Neighbour method used the Euclidean distance ($p = 2$); the bin widths used to achieve the results are not given.

Environment	Algorithm	Parameters	Mean Distance Error (m)
House	Nearest Neighbour	$p = 2$	1.76
	Bayesian Histogram	$b = 8$	1.78
CSE	Nearest Neighbour	$p = 2$	3.61
	Bayesian Histogram	$b = 6$	3.79
Roos et al. [90]	Nearest Neighbour	$p = 2$	1.67
	Bayesian Histogram	?	1.56

Table 5.8: *Comparison between Nearest Neighbour and Bayesian Algorithms in the house and CSE environments, and the results in Roos et al. [90]*

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Table 5.8 shows a comparison of our results with Roos et al. [90]. To approximate their methods, the results were generated using point surveying, no output averaging, $p = 2$ for the Nearest Neighbour algorithm, and the best performing value of b (bin width) in each environment. Our results show the performance of the Bayesian Histogram to be almost identical to the Nearest Neighbour in the house environment, and 5.0% worse than the Nearest Neighbour in the CSE environment. By comparison, their results show a 6.6% reduction in mean distance error using the Bayesian method.

In terms of the comparison between the two algorithms, the results are quite similar: almost no difference in accuracy was found, while Roos et al. [90] found a very marginal improvement. The slight discrepancy can be accounted for by differences in environment, implementation, device hardware, and test procedures.

In contrast to our work and Roos et al. [90], Youssef et al. [122] found that the Nearest Neighbour performed much worse than their algorithm, based on the Bayesian Histogram method. No mean distance errors are presented, but the 50th percentile error is approximately 4m for the Nearest Neighbour algorithm and approximately 1.5m for their Bayesian method. However, they do not provide any details of their Nearest Neighbour implementation, and in most other research [4, 66] the Nearest Neighbour produces much lower mean distance errors (usually below 3m). This makes the comparison suspect. It is possible that their implementation lacks the improved heuristic for dealing with mismatched vectors in the Nearest Neighbour algorithm suggested by Roos et al. [90] and also used in our work (Section 3.6.1); this can substantially improve accuracy.

5.7 Conclusions

This chapter presented a comparison of surveying methods, algorithms, and output averaging methods, based on empirical results generated using test data from two separate environments, gathered as described in Chapter 4. The results show that overall, the best accuracy achieved with each algorithm type is relatively similar. They have also shown that output averaging improves accuracy, and that the best surveying method can differ depending upon the environment. The house results were shown to be much more accurate than the CSE environment; this can be attributed to a combination of different conditions for each environment, such as the number of access points, construction materials, room sizes, and the presence of interference. It is also possible that the differences in sampling rate used to record data in each environment, and the differences in the test locations relative to the fingerprint locations, had an effect on the result.

The fact that there is little difference in performance between the algorithm types suggests that there may be little advantage to taking a greater amount of information into account (as the Bayesian, and to an even greater extent, the Distribution Distance algorithms do, compared to the Nearest Neighbour). This can be explained by examining the grounds for the assumption that extra information *does* improve the result. The idea is that the extra information helps distinguish fingerprints; that if only the means are considered, two fingerprints may be similar, but if the entire distribution of signal strength is taken into account, they may be able to be distinguished based on the extra information, such as different variances, characteristic peaks in the data, and so on. I speculate that in reality the the underlying signal strength data is too stochastic, and the time intervals used to record fingerprints and input data is too small, so the distributions are generally incomplete, and fail to sufficiently characterise the underlying data.

It is possible that with substantially longer recording intervals for both fingerprints and inputs, there would be sufficient information to provide a benefit to the Bayesian and Distribution Distance algorithms, but this might be impractical. Increasing the fingerprint recording time increases the amount of time required to survey an area, which can be prohibitive if the area is large. Tracking a moving target requires that only the last few seconds of input are considered, otherwise the estimate will lag behind the movement of the target.

Chapter 6

Access Point Layout

In this chapter, it is shown that accuracy is dependent on the *layout* of access points. The layout is defined as the *number* of access points and their *placement* within the environment. Increasing the number of access points helps improve robustness to interference and tends to increase the uniqueness of fingerprints due to the higher dimensionality of the data. Good placement can sometimes improve accuracy for a given number of access points because the underlying signal strength data can be more favourable to positioning.

To investigate the effects of layout on accuracy, the test data from both the house and CSE environments (Chapter 4) is used to simulate various layouts with less access points than the original data set, as described in Section 6.1. The results of accuracy testing on the simulated data are described in Section 6.2. It is shown that accuracy varies according to both placement and the number of access points. This is followed by investigations of two examples (one in each environment) of instances where unfavourable placement caused a variation in accuracy (Section 6.3). It is demonstrated how differences in the respective signal strength maps cause the variations in distance error through visual representations of signal strength maps side by side with maps showing the distance error for each test case.

6.1 Simulation of Layouts

To demonstrate variations in accuracy according to access point layout, existing test data (Chapter 4) was modified to simulate many different layouts in addition to those in the original data sets. Each layout was simulated by removing data associated with certain access points from both the test and fingerprint data to produce a data set containing less access points than the original. In the house environment, layouts were simulated containing 1 and 2 access points; in the CSE environment, layouts were simulated containing 2 and 3 access points. Table 6.1 shows the combinations used. Each letter refers to a single access point; for example, CD is a layout with only access points C and D.

Environment	Access Points
House	A, B, C, D, E, F, AB, AE, BE, AF, BF, EF
CSE	CD, BD, BC, AE, CDE, BCD, ACE, ABE

Table 6.1: *Access points used in the simulated layouts for each enviroment. Each letter represents an access point as shown in Figures 4.1 and 6.2.*

Figures 6.1 and 6.2 show maps of the two environments with the locations of each access point. These are reprinted from figures 4.1 and 4.2 here for convenience.

Results using only one access point have been omitted for the CSE environment, as a single access point was insufficient to provide reasonable coverage of the area. Information was not analysed exhaustively on all possible subsets, but rather focused on a small number of possible arrangements for each environment. Both lopsided (i.e. with access points all on one side of the area, such as A, B, E and F from the house environment, and CD and CDE from the CSE environment) and evenly distributed arrangements (the remaining combinations) were used.

The analysis in this chapter was performed using the Nearest Neighbour algorithm (Section 3.6.1), using the Euclidean Distance formula (i.e. $p = 2$), arithmetic output averaging with $k = 6$, and the sector surveyed maps, in each environment. These parameters were selected based on the consistent experimental results they produced and the fact that the Nearest Neighbour is something of a benchmark due to its prominence in the literature, for example [4, 90, 122, 66, 65, 67].

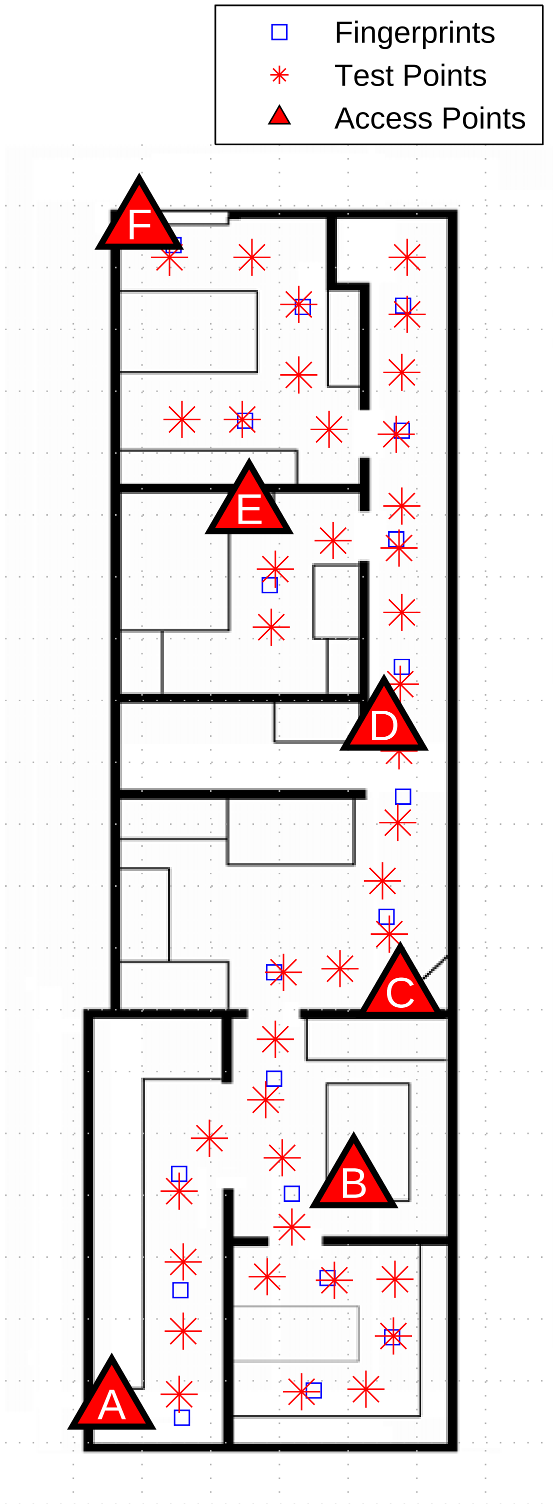


Figure 6.1: *House test bed, showing fingerprints, test points, and access points locations.*

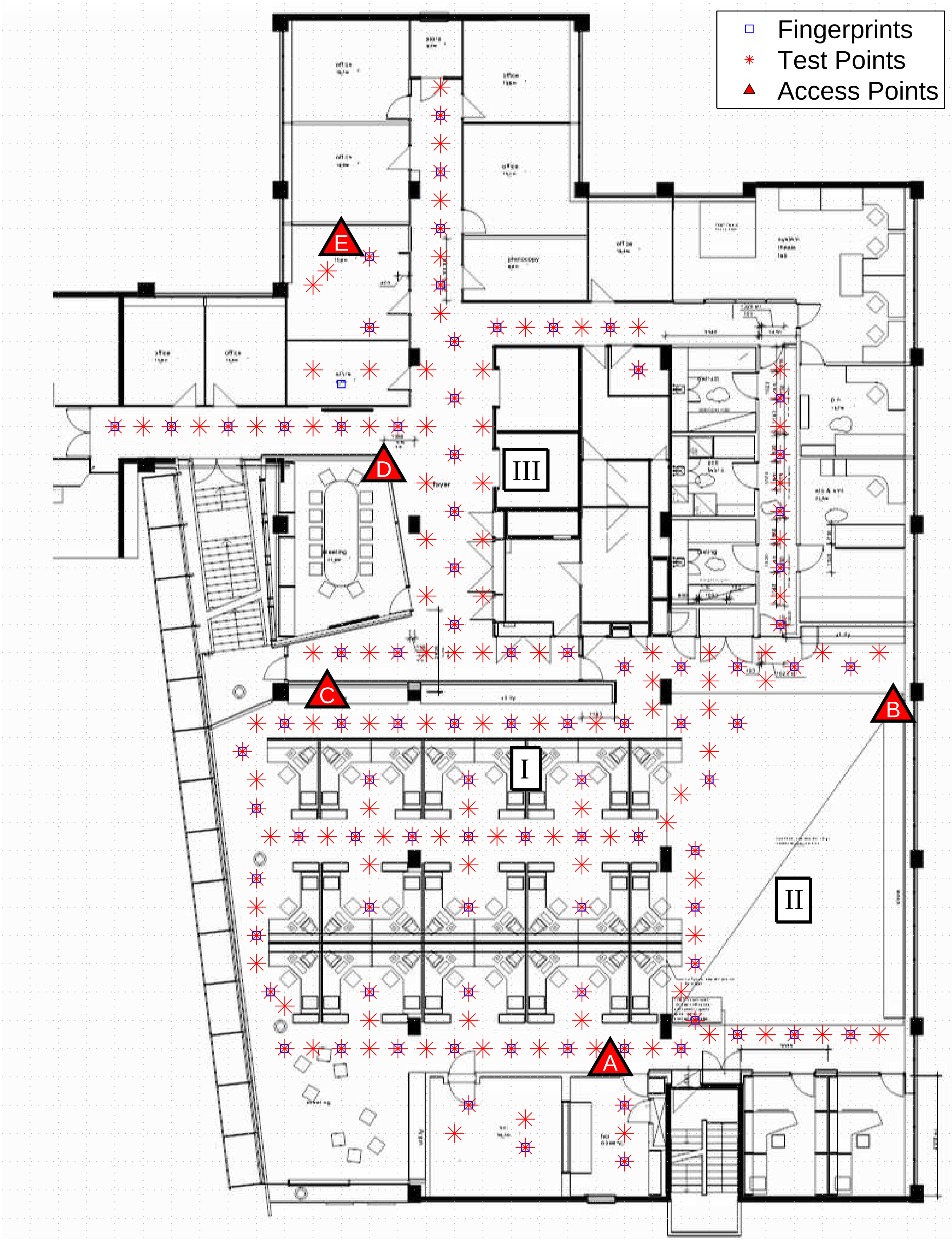


Figure 6.2: *Level 3 floor plan, showing fingerprints, test points and access point locations.*

6.2 Results

This section shows the results of accuracy testing on each simulated layout.

6.2.1 House

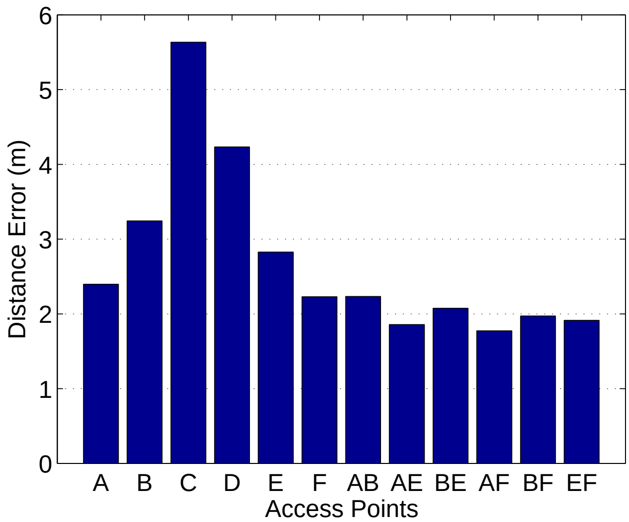


Figure 6.3: *Mean distance errors using 1 and 2 access point layouts in the house environment. Each letter represents a single access point; for example, BD is the combination of access points B and D.*

Figure 6.3 shows the mean distance errors for each simulated layout in the house environment. Interestingly, accuracy is not that much worse than the results generated using the full set of 6 access points (Section 5.1). The mean distance error using the original 6 access points and the same parameters was 1.4m; for the dual access point setups simulated here it varies between 1.8 and 2.3m, and is as low as 2.2m using only one access point (F). This suggests that 2 or 1 access points may be enough to generate a set of sufficiently unique fingerprints for viable positioning in the house environment.

However, accuracy varies substantially according to layout when using only one access point. When only a single access point is placed in the centre of the house (layouts C and D), the mean distance error is much higher than when a single access point is placed towards either vertical end (layouts A and F). The worst case is when only access point C is used, resulting in a mean distance error of 5.6m. By comparison, the accuracy is relatively similar for all layouts using 2 access points.

6.2.2 CSE

Figure 6.4 shows results of accuracy testing in the CSE environment. The 2 access point layouts (CD, BD, BC and AE) produce similar high mean distance errors, ranging between 4.9 and 4.8m. Note that BD is the layout using only the pre-existing CSE access

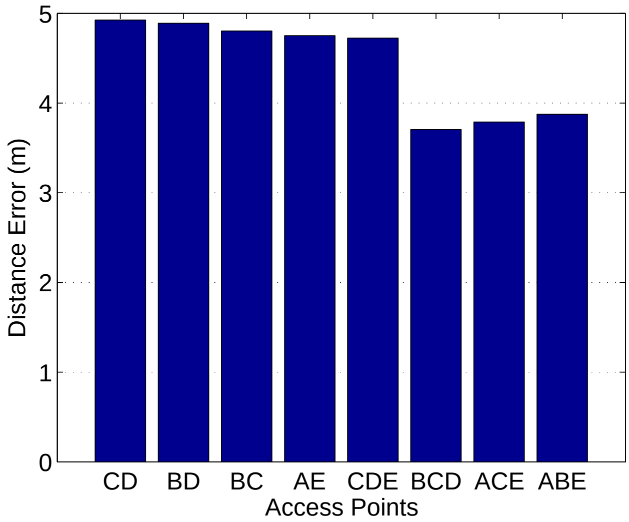


Figure 6.4: *Mean distance errors using 2 and 3 access point layouts in the CSE environment. Each letter represents a single access point; for example, BCD is the combination of access points B, C and D.*

points. Of the layouts using 3 access points (CDE, BCD, ACE and ABE), BCD, ACE and ABE have similar mean distance errors ranging between 3.7 and 3.9m, but CDE has a relatively high mean distance error of 4.8m (30% greater then BCD, the most accurate 3 access point layout), despite using the same number of access points. Similarly to the results for the house environment layouts, this confirms that the placement of access points (in addition to their number) can have an effect on accuracy.

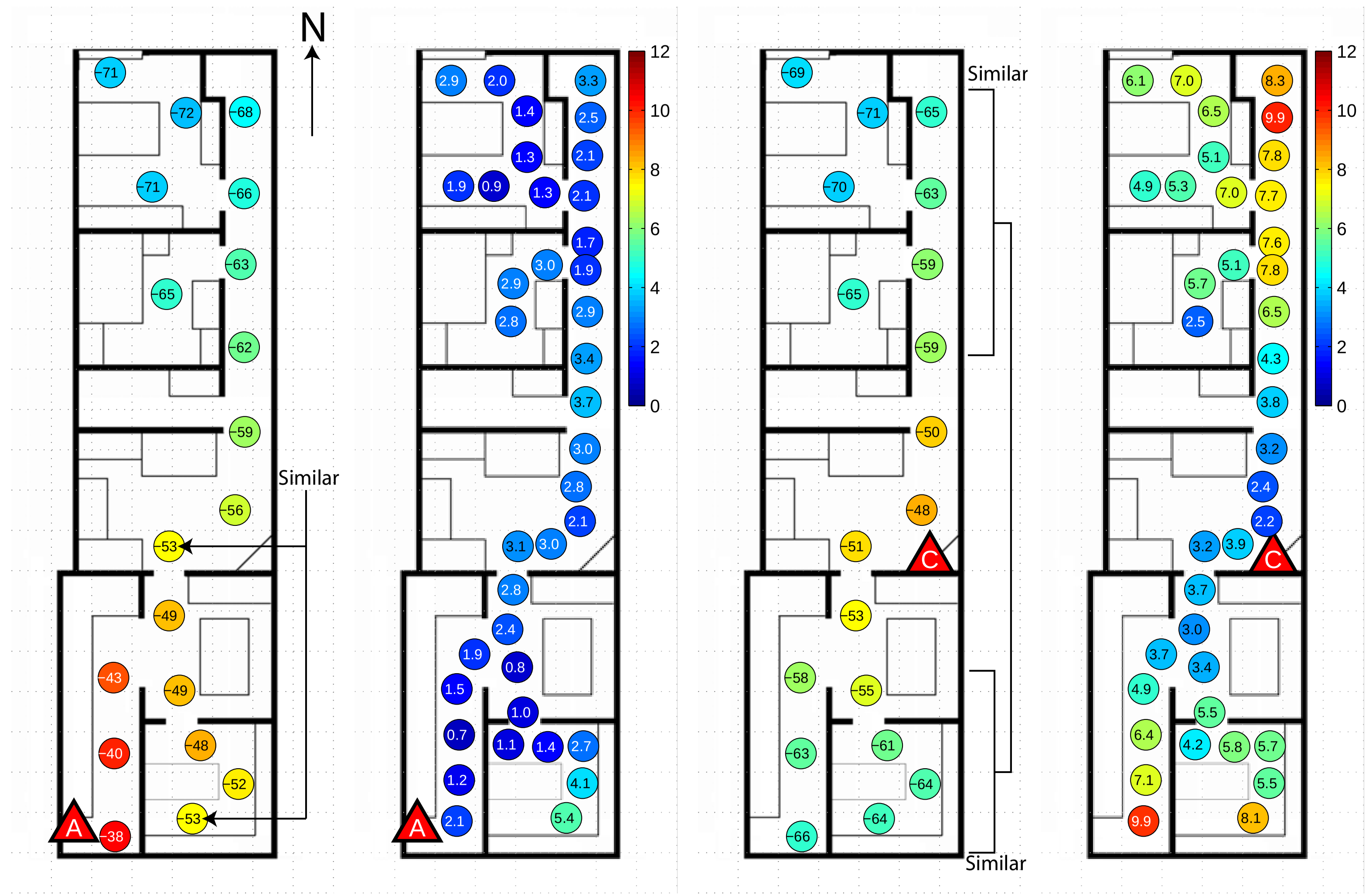
6.3 Investigation of variations in accuracy

This section investigates two cases from each environment where the accuracy substantially differed for two different access point layouts using the same number of access points (layouts A and C from the house environment, and CDE and BCD in the CSE environment). The investigation demonstrates why the different access point layouts caused a variation in accuracy using illustrations of the signal strength maps for each layout and maps of distance error for each test case.

House

The test cases used for comparison are the signal strength maps using only access points A and C, which have relatively low and high distance errors respectively, as previously shown in Figure 6.3.

Figure 6.5 shows the signal strength maps and distance errors for each test position, when using only access points A and C. These maps were generated using the sector map of the house environment. Each point shows the mean signal strength value at that



(a) *Signal strength map for AP A (dBm)* **(b)** *Distance Errors using AP A(m)* **(c)** *Signal strength map for AP C (dBm)* **(d)** *Distance Errors using AP C(m)*

Figure 6.5: *Signal strength maps and distance errors when using only access points A and C.*

location. Consider the signal strength map for access point A (Figure 6.5a). The signal strength is constantly decreasing from south to north such that most of the fingerprints have values unique to the local area. The only exceptions are the similar values of -53 and -52 in the southernmost room and -53 and -56 in the 3rd room from the bottom.

Because the means are sufficiently unique, it is reasonably easy for the positioning algorithm to distinguish between locations. As shown in figure 6.5b, this results in low distance errors for most of the house, with higher distance errors in the southernmost room where the signal strength is ambiguous.

Contrast this with the signal strength and error maps for the results using access point C (Figures 6.5c and 6.5d). There are many distant fingerprints with similar means, and the result is mostly relatively high distance errors

In this example, the distance errors are smallest for test locations near the access point. This is because the highest signal strength values (-48 to -51dBm) are only found in the room containing the access points; therefore they are unique to that room only. The distance errors are largest at the ends of the house, which have relatively similar distance errors. For example, the fingerprint with mean -66dBm at the south west corner of the house is similar to the fingerprint with mean -65dBm in the north east corner.

CSE

Here, CDE and BCD are used as contrasting arrangements to demonstrate the difference that access point placement can make. Figures 6.6 and 6.8 show the signal strength maps and figures 6.7 and 6.9 show the test point accuracy. A different format for the signal strength maps diagrams is used in order to visualize the signal strengths from three access points simultaneously. Each access point is represented by a specific shape, coloured according to signal strength. As in the house environment, the values shown are the mean signal strength values from the sector map.

The problems here are more complex than in the house example due to the increased number of access points and relative complexity of the environment. Examine the distance errors map for CDE (Figure 6.7). The greatest distance errors (between 5.9 and up to 14.4m) are concentrated slightly above the robot testing area in the middle right of the map (I), and in the adjoining north south hallway (II). There are also large errors in the two rooms at the bottom of the map (III, IV), and less acute but generally poor performance in the bottom aisle running east-west (V).

Each of these problem areas will now be examined. Firstly, consider area I in Figure 6.6. The signal strengths here are very similar to the area on the opposite side of the robotics testing area (VI), causing ambiguity between the two and resultant large distance errors. Area II suffers poor accuracy because it contains similar signal strength to room IV to to the south, and because there is not a great deal of difference between the fingerprints inside the hallway itself. Rooms III and IV have large errors because they are similar to

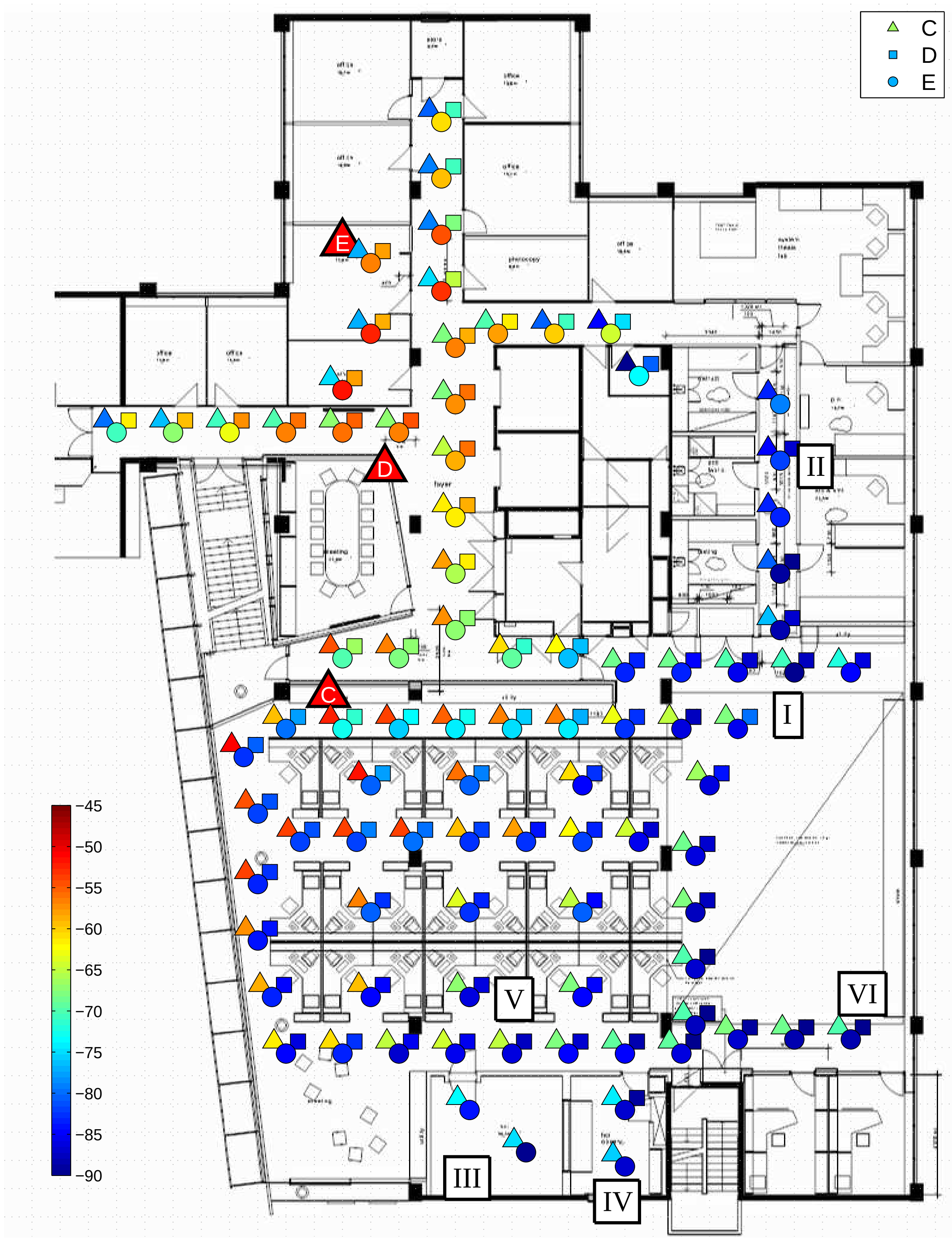


Figure 6.6: *Signal strength map for access points C, D and E (dBm).*

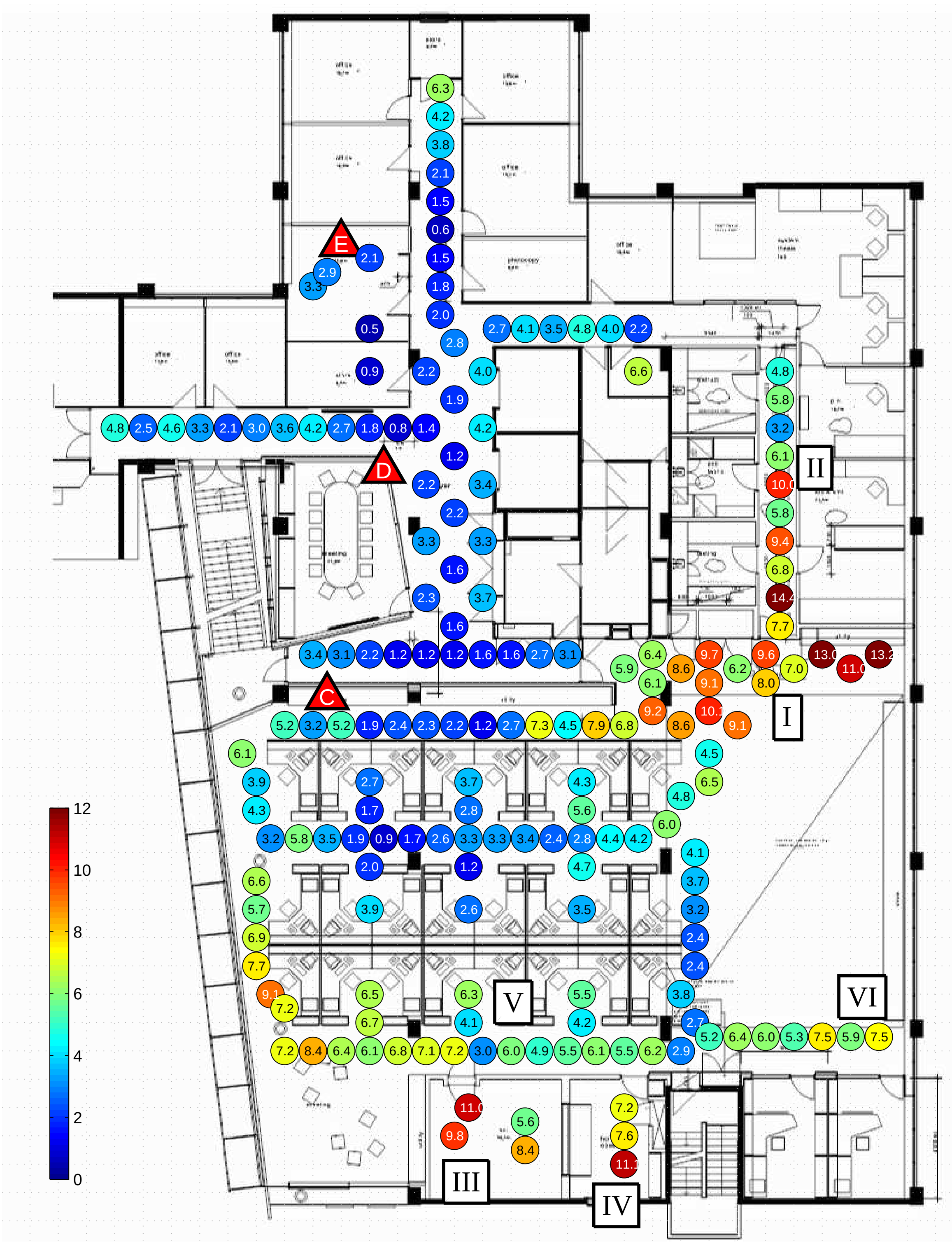


Figure 6.7: *Distance Errors (m) for the test points using access points C, D and E.*

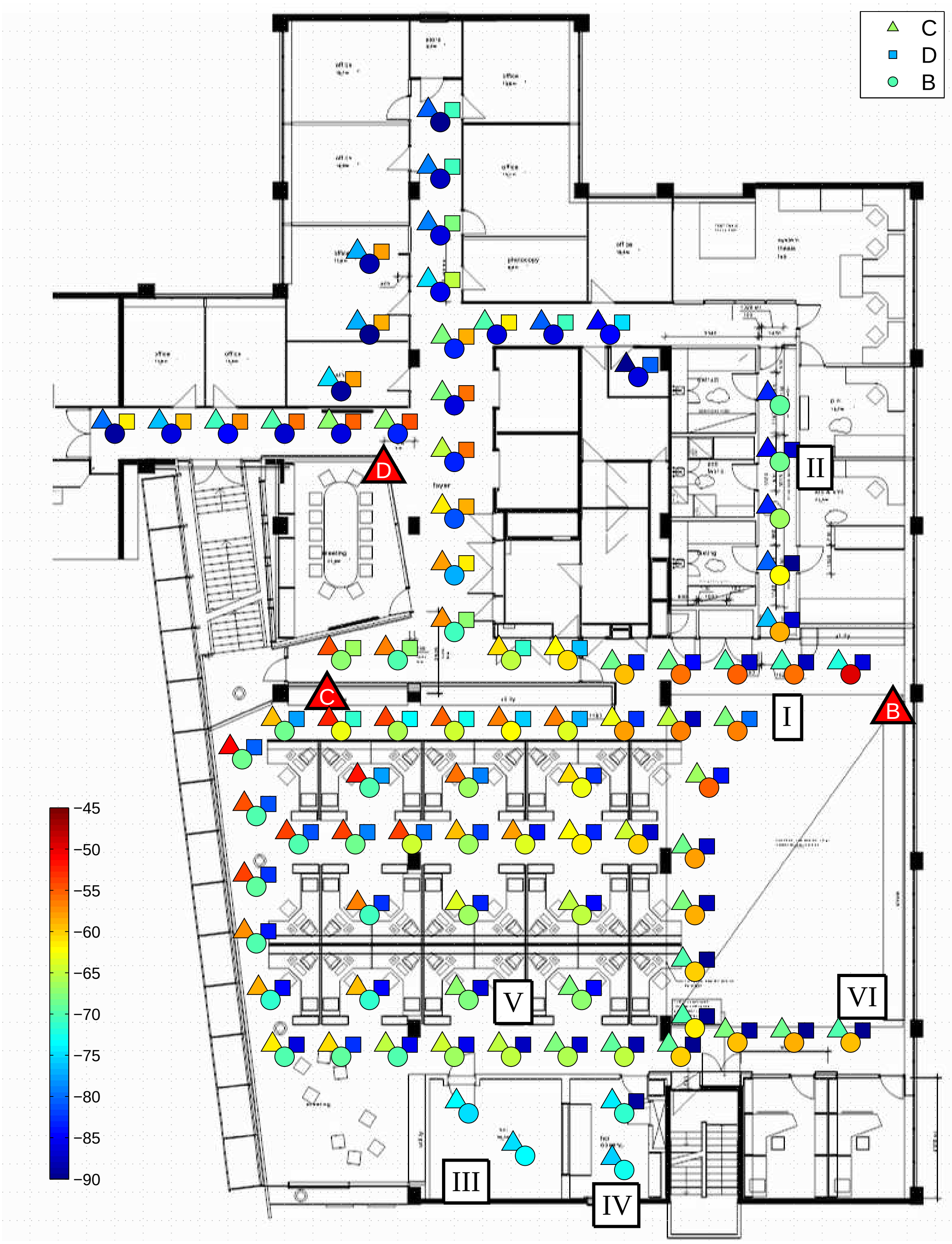


Figure 6.8: *Signal strength map and distance errors map when using only access points B, C, and D*

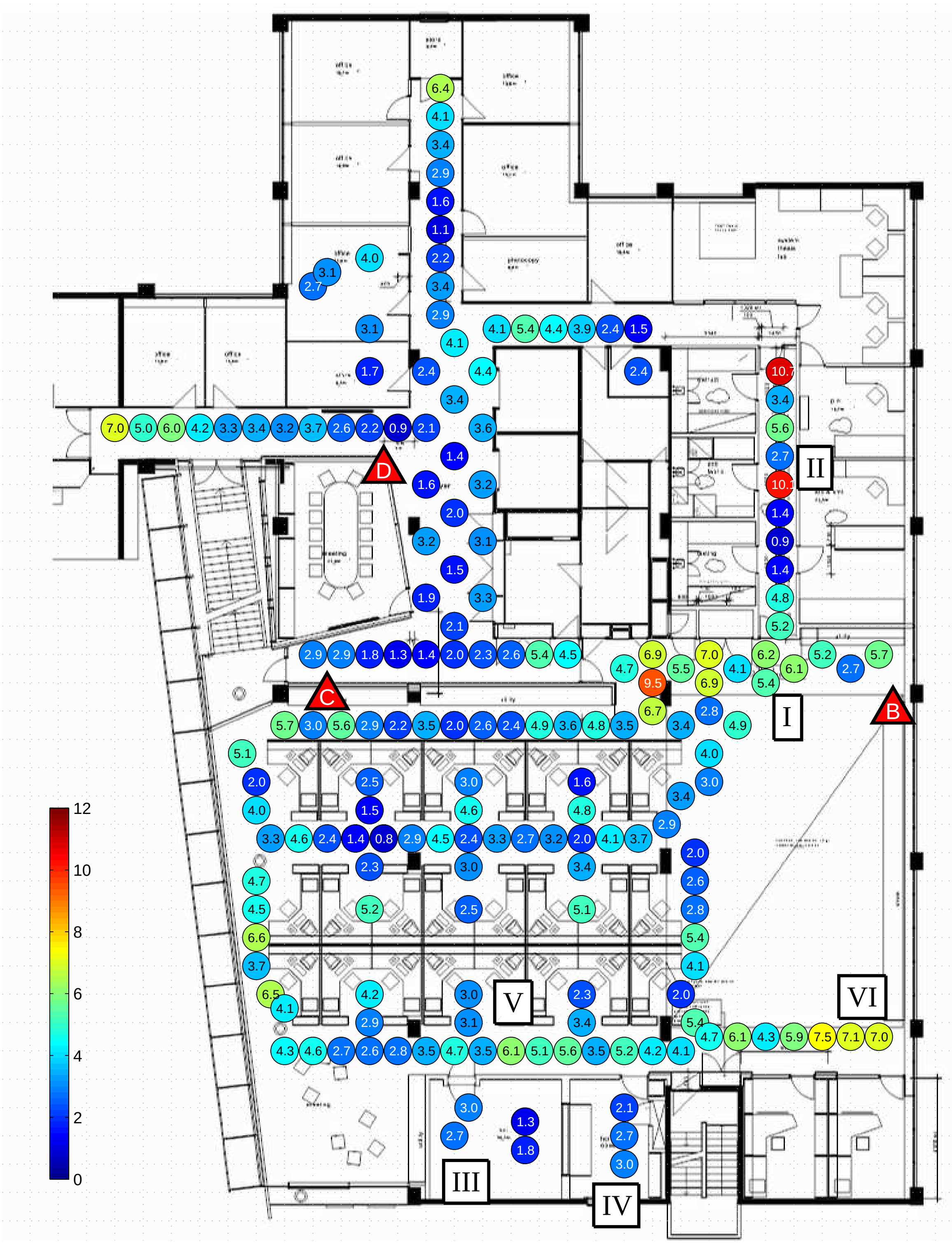


Figure 6.9: *Distance Errors (m) for the test points using access points B, C and D.*

hallway V, which suffers generally poor performance itself because its constituent fingerprints are similar to each other and to fingerprints at VI.

The BCD arrangement is shown in Figure 6.9. It has a lower mean distance errors than CDE, as previously shown in Figure 6.4. This is reflected in the map of positional errors: there are still relatively large errors at I, II and V, but they have been reduced in magnitude, and rooms III and IV are no longer an issue.

The difference between the two signal strength maps is that access point E, present in CDE, was replaced by access point B in BCD, next to the area of poorest accuracy in CDE. As a result, the signal strength map (Figure 6.8) is less ambiguous. At I, the introduction of the nearby access point B means the fingerprints can clearly be distinguished from fingerprints at VI. However, they are still ambiguous between themselves, because the open area produces a low rate of signal strength decay, producing the smaller distance errors. At II, the distance errors are mostly reduced as the signal strength from B provides a gradient from south to north along the hallway. There are still occasionally large errors, the reasons for which are unclear. The neighbouring rooms are PCB fabrication labs; it is possible that they contain metal structures that setup multipath interference patterns. Rooms III and IV are more easily distinguished because their signal strengths from access point B are unique. Aisle V is less of a problem because the decay from access point B improves the difference between neighbouring fingerprints, although area VI is still problematic as the fingerprints there have similar signal strengths to B.

6.4 Conclusions

Increasing the number of access points usually provides some improvement in accuracy. This is due to the increased dimensionality that the fingerprints acquire with the extra access points; the extra dimensions can help disambiguate similar fingerprints. A similar study was carried out in Roos et al. [90], with similar findings.

Access point placement can also have an effect on accuracy. As previously demonstrated, with the same number of access points, some layouts achieve lower mean distance errors than others. These differences in accuracy are due to the differences in signal propagation causing variations in the uniqueness of fingerprints in the signal strength map.

The best placement is dependent on the signal propagation properties and shape of the environment, which makes it difficult to construct a set of general placement heuristics. The only clearly consistent trend identified is that, if the positioning area is restricted, placing the access point on the edge of the environment reduces fingerprint ambiguity. If the access point is on the edge of the area, the number of similar fingerprints can be reduced as the areas of similarity are reduced. This is seen mostly clearly in the house environment examples, where the central access point arrangement produced worse accuracy then when the access point was placed on the edge of the area.

Software that optimised the placement of access points could be helpful to administrators when planning a positioning network. Placement would be informed by signal strength maps of the environment that could be generated empirically, or predictively to save time. The accuracy of the predictive approach would probably be sufficient to provide enough information to inform access point placement. The potential design of this software is discussed further in Chapter 8.

Chapter 7

Infrastructure-side positioning

In the preceding chapters, we have focussed on a scenario in which mobile devices perform positioning using signal strength readings from packets emitted by access points hosting an 802.11 network. This is the most immediately practical approach, as it allows the device to perform positioning without making any modifications to existing networks. However, there are several practical advantages to using the access points to detect signal strength from the mobile devices. In particular, the resource burden of detecting signal strength and estimating position is removed from the mobile device, which may have limited processing power. We refer to this approach as *infrastructure-side positioning*.

Several commercial vendors [18, 25, 2] provide purpose-built access points that provide signal strength monitoring to their proprietary positioning solution. These are expensive, closed systems, making them inappropriate for research purposes. Often they function in tandem with proprietary tags only, rather than consumer mobile devices.

In this chapter, we describe a method for modifying an cheap entry level consumer off the shelf (COTS) access point to provide similar functionality, capable of accurately positioning any 802.11 device, using only free open-source software that can be modified to suit a variety research and commercial situations. The results of accuracy testing using a network of the devices are then presented.

We also show the accuracy of using devices with different transmission power from the surveying device, and describe a method for dynamically compensating for the variations in transmission power to improve accuracy.

7.1 Motivation

Signal strength positioning determines the position of a mobile device by observing the varying signal strength of the link between the mobile device and a set of fixed position access points. The observations can be made by either the mobile device or the access point infrastructure, but it is easier to develop a system in which the mobile device is the observer: Wireless access points, particularly entry level models, are usually unmodifiable black boxes that do not support monitoring of their peers' signal strength; most mobile devices, on the other hand, are programmable - and some (but not all) support a way of observing signal strength.

However, determining positioning using the access points is desirable for a number of reasons:

- Most mobile devices are unable to provide adequate signal strength information. Although many are equipped with wireless chipsets, often there is no mechanism for accessing signal strength information. Many devices that do have such a mechanism have a problematic implementation: some can only report information on the currently associated network or channel, or do not report accurate signal strength; others require the network interface to be disabled during signal strength scanning. These problematic devices cannot be used on a system that uses mobile devices as signal strength observers, but they could be used with an system that used infrastructure-side observation.
- Software only needs to be written once for the infrastructure, rather than for a wide range of mobile device platforms and hardware.
- Calculations can be performed by the infrastructure rather than by mobile devices, which often have strict power consumption, memory, and CPU speed limitations.
- End users and administrators are spared the necessity of installing software packages. People can be positioned immediately upon walking into the mapped area with an 802.11 device.

Note that it is also possible to use personal computers as access points and signal strength observers, which are highly programmable and have considerably more processing power than access points, but they are much more expensive and less practical than a network of access points. Access points are smaller, easier to remotely configure, and have lower power and cabling requirements; they are generally much better suited to acting as nodes in a network as they are designed specifically for that task.

The ability to determine the location of any 802.11 device in the positioning area may raise some privacy issues. The movement of individuals carrying 802.11 devices may be tracked without their knowledge or consent. On ethical grounds, it may be advisable to only track devices belonging to individuals who have actively indicated that they wish to be monitored by the positioning system.

7.2 Implementation

Our system uses Linksys WRT54GL access points running OpenWRT White Russian [79] replacement firmware. OpenWRT is an open source Linux distribution designed for embedded systems. It was originally designed for Linksys WRT series access points but now supports a wide range of different models. Using OpenWRT, entry-level consumer off-the-shelf (COTS) access points gain the feature set and programmability of professional models.

The OpenWRT distribution contains a version of Kismet [54] patched to work on access points. Kismet is the packet sniffer used in prior chapters for experiments; the implementation is described in Appendix A. Hence, it was possible to quickly adapt our implementation to be access point-centric, given the similar data format and signal sampling mechanisms.

It is possible to record signal strength information using Kismet while simultaneously serving network data. This is achieved this by setting up a virtual access point inside the operating system of each AP, and running Kismet on the virtual access point; this allows the main interface to continue to operate normally. This technique may not be possible with all types of hardware.

7.2.1 Network Layout

In this work, we use a centralized server approach to infrastructure positioning. Figure 7.1 shows an example layout using three access points. Positioning is performed using the signal strength retrieved from packets sent by each mobile agent. A central server is used to merge the signal strength data from each access point and calculate the position.

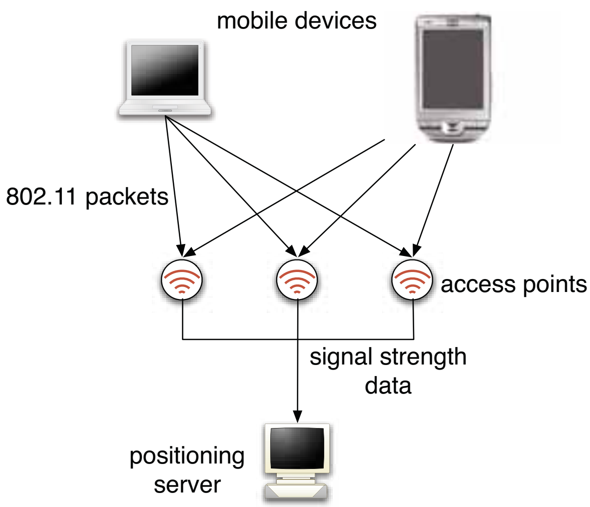


Figure 7.1: *Diagram showing the network layout of a centralized infrastructure-side positioning system.*

The system time of each access point is synchronized to Coordinated Universal Time (UTC) using the Simple Network Time Protocol (SNTP) [40] prior to use, and packets time stamped accordingly. Time stamping the data ensures that the signal strengths from all access points is processed synchronously. The sequence of packets is then processed into a form appropriate for the positioning algorithms using the input averaging process described in Section 4.4.

The limit to the scalability of this approach depends only on the processing requirements involved in positioning multiple clients. If one server has insufficient processing power to compute positions for the number of clients required, the positioning load can be simply distributed among multiple positioning servers by assigning the calculations for each mobile agent to a particular server. Mobile agents could be dynamically assigned to each positioning server as they enter and leave the network.

A large amount of network bandwidth can be consumed by the access points reporting signal strengths. In our work, the access points report every packet they see in significant detail (the fields are described in Section A.1); this could be an excessively large amount of data for a busy network with a high packet rate. If this were a problem, the bandwidth usage could easily be reduced by reporting less fields, or performing some input averaging of signal strengths at the access point (which would ordinarily be done by the server) and reporting at a lower rate.

Instead of a centralised approach, positioning may be possible using the processing power of the access points, but there are few advantages to this approach. The scarce processing power and secondary storage of the access points would make implementing sophisticated algorithms and storing detailed signal strength maps difficult. There is also no obvious way to distribute the algorithm processing among the access points, given that information from all access points is required as input before the position can be reliably estimated.

In this work, the access points were directly connected to the server via wired Ethernet. It is also possible to send the data over the wireless network itself. This might be necessary in the event that the access points cannot be connected via wired network, if for example they are placed in locations that are beneficial to positioning but difficult to connect Ethernet cabling to. In that case, the access points must ignore signal strength information associated with the access points themselves. Otherwise, a feedback loop will result, as the packets containing signal strength data are analysed for their signal strengths, causing new packets containing signal strength data to be sent, which are also analysed, and so on.

7.3 Compensating for Transmission Power

Transmission power varies between devices, and is sometimes adjustable by the end user so that battery life can be extended, or so that interference with neighbouring networks can be reduced. This presents a potential problem for our system: client devices transmitting at different power levels to the original surveying hardware will experience inaccurate results, because the empirical fingerprint map is specific to the transmission power of the surveying device.

The majority of 802.11 wireless chipsets claim to broadcast between 30 and 100mW (14.7 and 20dBm) [95]. The actual transmitted power, as perceived by the access points, varies depending on the quality and directionality of the antenna, and how it is mounted in the casing of the device.

7.3.1 Power offsets

The conventional empirical RF model asserts that, when expressed in logarithmic units of decibels, received power is equal to transmitted power minus path loss [65]. Path loss is a function of free space loss, attenuation and scattering, which vary for different locations, but it is independent of transmission power. Hence, a logarithmic difference in transmission power should result in an identical logarithmic difference in received power.

In our positioning context, this implies that differences in transmission power due to differing client hardware will result in an *identical difference* in received power at the APs, independent of position. For example, device A has a transmission power of 20dBm, and device B has a transmission power of 15dBm. The model suggests that if signal strength recordings are made of emissions from both devices, originating from the same position, the received signal strength from device B will be 5dBm less (that is, the difference in transmission power) than the received signal strength from device A.

This means that if devices with different transmission powers are used with an infrastructure-side positioning system, it should be possible to compensate for the difference by adding a corresponding *offset* to the received power input. If this offset is identical to the difference in transmission power between the surveying and client devices, the new received power should then correspond to the original fingerprints.

7.3.2 Determining the Transmission Power

The problem then lies in determining the transmission power of the client devices relative to the surveying device. Unfortunately, the transmission power of peer devices cannot be reliably determined by the access points using the 802.11 standard. 802.11h transmit power control [49] requires peers to supply power information upon association with the network, but is not a universally implemented standard [38]. For some devices, the transmission power may be documented either in the specifications or in their system settings, but this information cannot be determined automatically by the access point.

It is possible to use a fixed offset, based on prior knowledge of the device based on its specifications. However, this requires either the end user or the network administrator to first find these details in the device's documentation. Ideally, a device should be able to be used immediately with the access point upon entering the area. To accomplish this, we trial an *adaptive* approach to estimating the best offset during online positioning in tandem with position estimation. For each input, instead of calculating only one positional output, a set of separate positional outputs is calculated based on a set of possible offsets, based on the likely range of differences in transmission power among the devices. We apply offsets to the input ranging between whole values -20dBm to +20dBm, including 0. This generates 41 positional outputs, which must then be chosen from on the basis of a previous estimation of the offset.

In this work, we use the sum of the Nearest Neighbour distances to determine the best offset. In the Nearest Neighbour method, a distance is calculated between the input and each fingerprint; this is used to rank the fingerprints in order of their proximity to the mobile agent (Section 3.6.1). These distances are also used to weight the outputs in the weighted output averaging method. A lower Nearest Neighbour distance sum indicates the input is closer (in terms of signal strength) to the signal strength map as a whole. The output with the lowest sum of Nearest Neighbour distances is used as the final output. The sum of distances is accumulated over a sliding window (over time) of previous data to smooth the estimate. This is similar to [43], where the sum of Bayesian confidences is used adaptively to compensate for different signal strength units reported by different devices.

7.4 Experimental Setup

In this section, the procedure used to record data for accuracy testing of the infrastructure-side positioning system is described. Accuracy testing was performed in the house environment using the same procedure to that described in Section 4.2.

Our aim was to determine the accuracy of the system using both the surveying device and other devices with different transmission powers. To achieve this, we first recorded sector fingerprint signal strength maps in the house environment (Section 4.1.1) using a Sony VAIO U8G and its default, maximum transmission power of 18dBm. We then recorded three sets of separate test data. The first set of test data was recorded using the VAIO U8G set to the surveying power of 18dBm, and then to 10dBm, and a Hewlett-Packard iPaq hx4700, using its default power of 15dBm.

Setting the VAIO U8G to 10dBm simulated a device with much lower transmission power than the original, and represented a fairly extreme difference. 18dBm and 10dBm

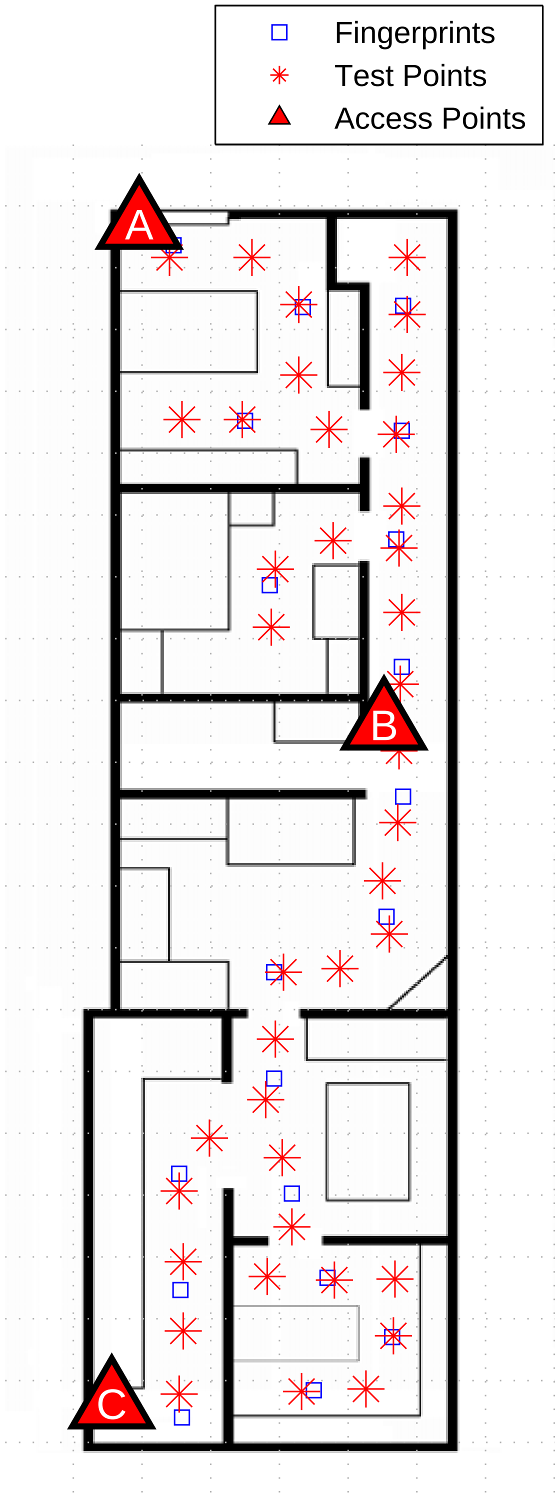


Figure 7.2: *Map showing the house environment with fingerprints, test points, and access point locations for testing the infrastructure-side positioning system.*

were the maximum and minimum signal strengths settings for the wireless Linux drivers we were using. The 8dBm difference represents a larger range than is found between most commercial devices, which as described above, usually transmit at between 14.7dBm and 20dBm [95].

Figure 7.2 is a map of fingerprint locations, test points, and access points. Sector fingerprints were recorded for 20 seconds each; test points were recorded while facing north, east, south and west for 5 seconds per direction. This is the same procedure used to test sector surveying described in Section 4.2, except here we use the access points to record data rather than the mobile device. Three access points were positioned in the environment; one access point at each far end of the house and one in the centre.

During the recording of fingerprint and test points, each device was connected to the live 802.11 network. A small amount of data was broadcast on the network once every 100ms to provide packets for the positioning system to observe the signal strength.

Based on previously good performance described in Chapter 5, we used sector sampling with the Nearest Neighbour method (Section 3.6.1) set to $p = 2$ and $k = 2$. Our aim here was not to perform an exhaustive test of algorithms with the new system, but to provide proof of concept.

7.5 Results

This section contains the results of our experiments using the infrastructure-side positioning system. We first compare the mean differences in mean fingerprint signal strengths for each fingerprint map, and show that the differences are similar to the differences in transmission power used to record each map. We then show the results of accuracy testing using a fixed offset, based on our prior knowledge of the relative transmission powers, and finally show the accuracy results when using the adaptive offset technique.

7.5.1 Mean Offsets

In this section, we show the differences in signal strength between the sets of test data. As described in the previous section, one fingerprint map and three sets of separate test data were recorded. The fingerprint map was recorded using a Sony VAIO U8G at the default transmission power of 18dBm. The three separate sets of testing data were recorded by the access points using the Sony VAIO U8G transmitting at 18dBm and 10dBm (set using Linux Wireless Tools [108]), and an iPaq hx4700 transmitting at 15dBm (according to the model's documentation [46]).

Table 7.1 shows the means and standard deviations of the differences in received signal strength between the U8G transmitting at 18dBm and the two other sets of test data. We also show our predicted differences in received signal strength, based on the transmission

Device	Predicted Difference (dBm)	Actual Mean Difference (dBm)	Standard Deviation (dBm)
U8G 10dBm	-8.00	-8.89	5.13
iPaq	-3.00	-2.21	6.28

Table 7.1: *Means and standard deviations of differences in received signal strength (by the access points), compared to predicted differences. The predicted and actual differences shown are relative to the Sony VAIO U8G transmitting at 18dBm.*

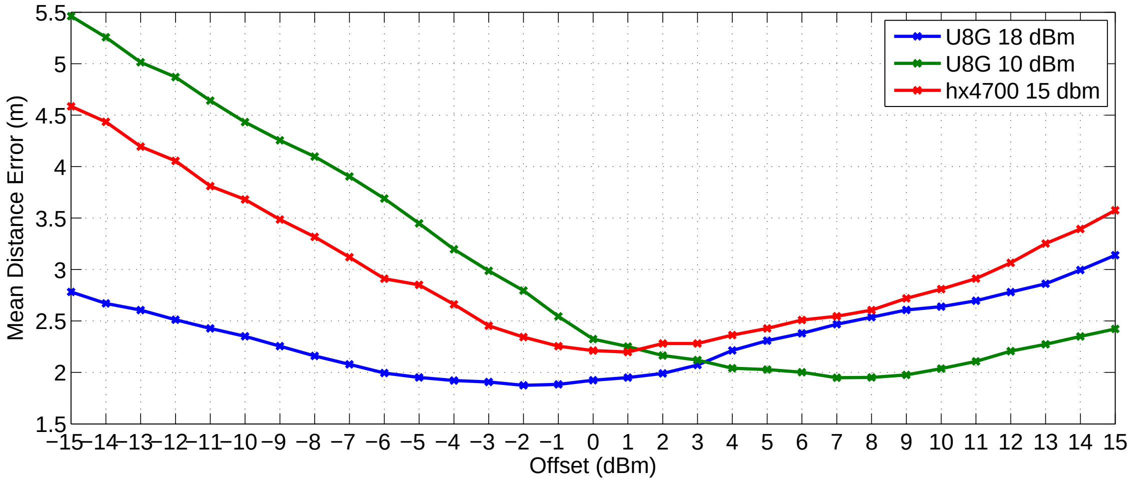


Figure 7.3: *Mean distance errors for input offsets ranging from -15 to 15dBm.*

power used to record each set of tests. The mean offsets closely resemble the predicted offsets. The U8G transmitting at 10dBm resulted in a mean offset of -8.89dBm compared to the same device transmitting at 18dBm, so the actual mean offset differed from the predicted by 0.89dBm. The iPaq had a mean offset of -2.21dBm, different from the predicted by 0.79dBm. There is a large amount of variation in the offset: the standard deviations in offset are 5.13dBm and 6.28dBm for the U8G 10dBm and iPaq fingerprint maps, respectively. This is probably due to transient fluctuations in signal strength and different multipath effects across the data sets due to small surveying positional errors (that is, errors in holding the device at exactly the recorded fingerprint location). Overall, these variations mostly cancel out, and the mean offsets are close to the predicted offsets.

7.5.2 Fixed Offset

In this section, we show the results of accuracy testing using a fixed offset to compensate for the variations in transmission power between the tested devices. According to the model described above, the lowest distance errors should occur for input offsets equal to the difference in transmission power between that used in the fingerprint map and that used to record the test point. In other words, we predicted the best offsets would be 0, 8, and 3dBm for the U8G 18dBm, U8G 10dBm, and hx4700 15dBm tests respectively.

In practice, the minimum distance errors occurred for values close to these predicted offsets, but not exactly the same. Figure 7.3 shows the mean distance errors of accuracy testing for each test set applying input offsets ranging from -15 to 15dBm.

The mean distance errors are relatively similar for each test set, without applying any input offset (that is, the offset is 0). Using no offset, the U8G transmitting at 18dBm (i.e. the same as the surveying configuration) records the best minimum accuracy of 1.92m as expected. This is followed by the U8G transmitting at 10dBm (2.23m) and the hx4700 (2.21m); only a slight degradation in accuracy. This suggests that the system already works reasonably well without applying any offset.

The U8G 18dBm results have a minimum distance error of 1.87m at an offset of -2dBm. We expected the lowest distance errors to occur with no offset, given that this is the test set using the same device and identical transmission power to the surveying device. This result might be due to different transient fluctuations in signal strength between the two test sets. The U8G 10dBm results have a lowest mean distance error of 1.94m at an offset of 7dBm, close to an expected best offset of 8dBm, which had a mean distance error of 1.95m. In contrast, using no offset results in a mean distance error of 2.32m. Therefore, applying the predicted 8dBm offset results in a reduction of distance error of 15%, and reduces the minimum distance error to be similar to the 18dBm result, effectively recovering the lost accuracy due to differing transmission powers.

The hx4700 results have a minimum distance error for an offset of 1dBm, again relatively close to the predicted best offset of 3dBm, but the minimum distance errors are higher than the U8G errors. Without applying an input offset, the U8G 10dBm has a higher mean distance error (2.32m) than the hx4700 (2.21m), but the input offset reduces its distance error to almost the level of the original surveying device. However, adjusting the offset fails to reduce the mean distance errors of the hx4700.

This is probably because the differences in signal strength transmission, as perceived by the access points, are more complex than a simple offset. The iPaq hx4700 is a smaller device, held in a slightly different way by the mobile user, with an antenna in a different relative position to the user. This leads to differences in the way the body of the user attenuates and scatters the signal, which are not modelled by our simple offsetting technique.

7.5.3 Adaptive Offset

To test the effectiveness of the adaptive approach (Section 7.3.2), the same accuracy testing procedure was used, with sums of Nearest Neighbour distances accumulated over consecutive test points. Each full set of test points contains approximately 10 minutes worth of data.

Figure 7.4 shows the effects of accumulating the Nearest Neighbour distance sums for window sizes between 1 and 600 seconds (that is, 10 minutes). The minimum distance

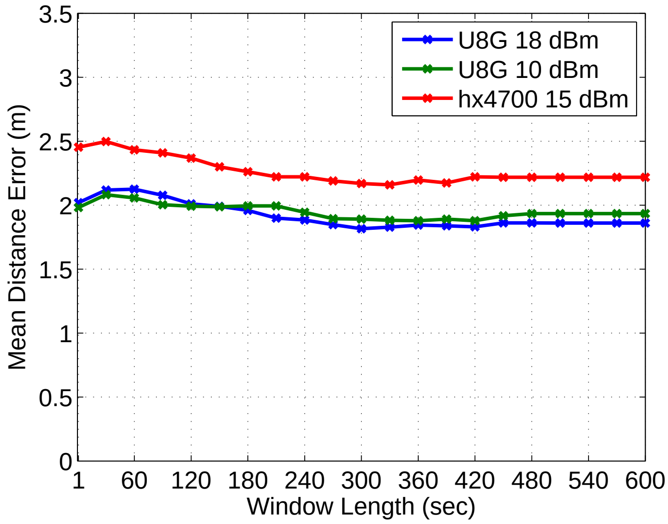


Figure 7.4: *Mean distance errors using an adaptive input offset.*

errors are achieved at relatively long window lengths, greater than 300 seconds. Using lower window lengths leads to a less stable estimate of the offset and poorer accuracy.

7.5.4 Summary

Table 7.2 summarises the mean distance errors for each test set using the different offset methods.

Device	No Offset	Fixed Predicted Offset	Adaptive Offset
U8G 18dBm	1.92	1.92	1.82
U8G 10dBm	2.32	1.95	1.88
hx4700 15dBm	2.21	2.28	2.16

Table 7.2: *Mean distance errors (m) for each test set using different offsets.*

Using no offset, the U8G using its surveying transmission power (18dBm) achieves an accuracy of 1.92m. This is the approximate lower bound of accuracy for the other test sets, as the surveyed fingerprint map is optimal for the U8G in that particular configuration.

The U8G 10dBm tests show reduced distance errors when using either a fixed offset or when using the adaptive offset method: 1.95m and 1.88m respectively, a 16% improvement over the result with no offset. Using an offset effectively removes the increase in distance error due to the difference in transmission power. However, the distance errors of the hx4700 are not significantly reduced using either method. As discussed previously, it is likely that the higher mean distance errors for the hx4700 are due to the difference in relative positions of the antenna and the body of the mobile user for each device. This leads to variations in the signal strength map that cannot be approximated by a simple offset.

7.6 Conclusions

In this chapter, we have described the design and implementation of an infrastructure-side 802.11 positioning system, using modified COTS access points to record signal strength of mobile devices. This system allows any 802.11 device to be positioned based on the signal strengths of any packets it emits, without running any special software on the 802.11 device or modifying it in any way. A position can be estimated as soon as the mobile device enters the positioning area.

Accuracy testing demonstrated that using the original surveying device, the mean distance error was 1.92m. Using the same device and a lower transmission power, mean distance error was slightly higher, but this difference was able to be reduced by offsetting the input signal strength to compensate for the difference. We have demonstrated the effectiveness of using both a fixed offset and using the positioning system to calculate an offset to be applied in parallel with the on-line positioning process.

Using a different device to the surveying model, the mean distance errors were 15% greater, and using an input offset did not improve the difference using either the fixed or adaptive methods. This is probably because the relative position of the antenna from the body of the mobile user is different for each device, leading to slight differences in the signal strength map that cannot be simulated by a simple offset.

Chapter 8

Conclusions and Future Research

This chapter contains overall conclusions on our work, and proposes areas for future research.

8.1 Conclusions

- **The mean distance errors produced by each algorithm are relatively similar** (Section 5.1). In the house environment, the best algorithm was the Bayesian Histogram algorithm, with a mean distance error of 1.25m; the worst algorithm was the p-norm Distribution Distance method, at 1.42m, only 14% higher. In the CSE environment, the best algorithm was the Nearest Neighbour algorithm, with a mean distance error of 2.86m; the worst algorithm was the Bayesian Gaussian algorithm at 3.76m, only 32% higher.

  This suggests that considering more information (as the Bayesian and Distribution Distance algorithms do, compared to the Nearest Neighbour) may not necessarily improve accuracy. This is roughly in agreeance with the findings in Roos et al. [90], who using a similar empirical approach found the Bayesian Histogram method produced only marginally better results (by 6.5%) then the Nearest Neighbour method.

- **Output averaging reduces distance errors**, regardless of the algorithm being used, by approximately 20% using point surveying and 10% using sector surveying (Section 5.3). This result is consistent with Bahl and Padmanabhan [4] and [66], who both found that using output averaging improved accuracy by a similar margin. The improvement in accuracy was greater when using point surveying, because the organisation of point fingerprints allowed a greater number of fingerprints to be taken into account. There was little difference in best case accuracy between the arithmetic and weighted output averaging techniques, except in the house point results where weighted averaging tended to produce poor results using the Bayesian algorithms

- **The best surveying method differs between the two environments.** Sector surveying produced marginally lower mean distance errors (between 2.1% and 21.9% less) than point surveying in the house environment, but similar or marginally greater mean distance errors in the CSE environment (between 8.9% less and 15.0% greater; see Section 5.4). I speculate that the inconsistency in the results may have been due to environmentally specific factors such as the the level of interference. It is also possible that the results were affected by the difference in signal strength sampling rates, and the differences in testing procedures.

- **Access point layout has a substantial effect on accuracy.** Accuracy usually improves with the number of access points, and in some cases it varies according to access point placement (Chapter 6). Any difference in accuracy between access point layouts is due to the differences in underlying signal propagation causing variations in the uniqueness of fingerprints in the signal strength map. How unique each fingerprint is has a large effect on the accuracy of the positioning system; if the positioning algorithm cannot distinguish between fingerprints because their signal strength data is too similar, accuracy will be poor. Our results are consistent with Roos et al. [90], who also found that accuracy increases with the number of access points.

- **Entry-level access points are feasible detectors of signal strength for an 802.11 positioning system**. The effectiveness of a prototype positioning system using modified COTS access points (Chapter 7) has been demonstrated. Results suggest that access point based detection of signal strength has similar accuracy to using client device based detection, and that the system only suffers a relatively small (15%) degradation in accuracy if the device being positioned differs from the surveying device.

8.2 Suggested Future Research

This section makes some suggestions for future research.

8.2.1 Reference Implementations and Test Data

A major problem in the field of 802.11 positioning is the difficulty of comparing results with other research. There are no standard testing procedures, environments, or equipment. There are also no benchmark implementations of the common algorithms. This means that only rough comparisons can be made between studies, which reduces the generality of the work. This is a problem which has been discussed by other authors [4, 90, 61].

Ideally, standard test data and algorithm implementations would be made available and shared between researchers in the field. This would allow more direct comparisons of results, and improve the efficiency of the field overall as the duplication of work would be reduced.

8.2.2 Temporal Tracking

In the absence of motion sensors, it is possible to influence the Bayesian prior probability using a model of human motion. For example, one could make probabilistic assumptions about velocity, turning rates, and likely paths based on historical data. Several authors [43, 61, 14] have explored constraining the positional estimate according to which rooms are physically connected in the environment using the Bayesian prior. Fox et al. [34] explored using a particle filter for tracking human movement with some success using infrared and ultrasonic sensors, but their ”Location Stack” software is designed to integrate with any sensor type and may have potential for 802.11 positioning.

Both Evennou et al. [28] and Seshadri et al. [96] performed 802.11 fingerprint based positioning with a particle filter and human motion model and claim to have acheived reasonable accuracy, but both authors performed empirical testing using only one walk across their respective environments. Further investigation using a similar approach but a more robust series of experiments will prove whether or not the approach holds up. In particular, we recommend that experiments are carried out using a series of walks with humans performing different types of activities - walking, running, standing still, browsing, and so on. The motion model must be appropriate for all of these eventualities for the positioning system to be useful in the general case.

8.3 Final Conclusions

This thesis conducted an experimental study of the effectiveness of indoor 802.11-based positioning. It has been demonstrated that the mean distance error is relatively similar regardless of the positioning algorithm and site surveying method used. In contrast, spatial output averaging always improves the result, regardless of the algorithm used. This thesis has also demonstrated the competitiveness of a new class of algorithm: the Match Distance algorithm, which, based on our results, has similar accuracy to Nearest Neighbour and Bayesian Histogram algorithms.

It was also demonstrated that access point layout can make a substantial difference in accuracy, particularly when using a small number of access points.

Finally, this thesis described the successful implementation and testing of an infrastructure based system for positioning using only cheap entry-level access points, which does not require client device participation in positioning.

Appendix A

Implementation of Experimental Software

This chapter presents two methods of extracting signal strength, one for Linux and one for Windows, suitable for PC and handheld applications. For each platform the implementation details, and the different types of signal strength data extracted, are discussed.

A.1 Linux

On the Linux platform, a modified version of the packet sniffer Kismet [54] was used to extract and interpret packets. Kismet operates by switching the 802.11 driver into monitor mode (also known as passive mode). This mode detaches the wireless drivers from the operating system network stack and allows applications access to the raw data being decoded by the wireless chipset. On some hardware, an extra header is prefixed to the payload of each wireless packet that contains extra information such as channel, encyryption and signal strength data.

The device emits no packets as a result of Kismet's operation. This *passive* scanning approach can be contrasted to an *active* scanning approach, where probe packets are sent to the access points, and the signal strength of the responses analyzed. The passive approach is more scalable than the active approach because it does not use any bandwidth.

Extracting data using Kismet is a more robust approach than writing our own monitor mode software from scratch as the data format varies widely depending on the host's hardware. Kismet is a mature package that is continually updated to support new hardware and drivers, and gave our method the best chance of extracting signal strength data from any given platform.

The Kismet approach was successful with the two types of devices trialled: a Sony VAIO U8G running Fedora Core 4, using its onboard Atheros AR5212 802.11a/b/g wireless NIC, and a Linksys WRT54GL 802.11b/g access point (integrated Broadcom CPU-

/switch) running Linux-based OpenWRT WhiteRussian firmware. The latter required a small modification to Kismet to work; originally it was not reading RSSI from the correct field. Furthermore, problems were experienced in getting the WRT54GL to roam channels during scanning; this might be resolvable with further investigation. Both devices report RSSI in units of dBm.

The Sony U8G and Kismet were used to perform all experimental work in this study. Kismet is an ideal research platform because it allows fine control of the hardware and extracts very detailed information about all packets received by the device.

A.1.1 Kismet Implementation

Kismet contains a TCP server subsystem that uses a simple ASCII line-based protocol to distribute information. Several types of broadcast message exist within this protocol, each containing a set of fields specific to a particular task; for example, monitoring which access points are in view, or spotting suspicious packets to send to an intrusion detection system. Clients register on connection which message types they wish to receive, based on their needs. It was found that although Kismet extracts signal strength from each packet, it does not distribute it on any of the existing message protocols. Subsequent modifications established a custom protocol to acheive this. An example record is shown in table A.1.

Timestamp	Type	Sub-Type	Channel	Beacon Interval	Source Address	Dest Address	SSID	Quality	Signal	Noise Floor
1155262617	0	0	11	100	...	All	Skynet	0	-76	-98

Table A.1: *Example of the packet fields used in our positioning system.*

The timestamp is the system time recorded by Kismet when it processes the packet; accuracy depends on the system clock, usually to the nearest millisecond. Type and Subtype are fields from the frame control field in the 802.11 frame header (in this case indicating a beacon packet). Channel is the 802.11 channel number the packet was received on. Beacon interval is the interval between beacon packets in millseconds. The source and destination addresses are the IEEE MAC identifiers of the station the packet originated from and the station it is destined for, with FF:FF:FF:FF:FF:FF representing a packet broadcast to all stations. MAC examples are omitted from the table due to space constraints. SSID is the network name the packet is associated with.

The final three fields represent the signal strength. The meaning of these values depends on the hardware and how Kismet extracts the data from monitor mode headers. For the U8G's Atheros chipset it was found that quality is always 0, the signal field contains the signal strength of the packet in dBm, and the noise floor contains the level below which packets could not be received (which was always -95). The values were returned as 8-bit signed integers, giving a possible range of 127 to -128dBm. In practice, values greater than -20dBm were never observed, and only a very small number of apparently artifact values below -90dBm.

Note that Kismet generates a new message for each packet seen. The minimum frequency of packets is 10Hz (the frequency of beacon packets); this is higher if the channel is occupied by clients exchanging data. It is possible for signal strength readings to arrive faster than the positioning system can process them, especially on mobile devices lacking processing power. To reduce the amount of data that needs to be processed, only beacon packets are considered, and rather than calculate a new position for each packet received, position is calculated at a fixed rate based on average signal strengths. This averaging process also helps improve the stability of the readings. This is discussed further in section 4.4.

A.1.2 Alternatives

An alternative to Kismet is to obtain signal strength information using Linux Wireless Tools [108]. Wireless Tools is a set of command line utilities that, among other things, can perform scanning of nearby access points. However, the Wireless Tools require looped polling and possibly active scanning (depending on the hardware), as they are designed primarily to perform a single scan for the end user to select which access point to associate with. Active scanning is undesirable as the device transmits extra packets on the channel, which uses some bandwidth overhead.

A more sophisticated solution would be to write a purpose-built monitor mode sniffer. Although the Kismet server is a very lightweight and efficient application, capable of running on embedded devices, a custom application would use fewer resources as it would be contain only functionality specific to signal strength extraction. Note that Kismet supports a wide range of idiosyncratic hardware; it would be wise when writing such an application to make use of the drivers within Kismet, if it was to be generally applicable. For our purposes such an approach was not neccessary; it was far simpler to make a small number of modifications to the existing Kismet server.

A.2 Windows

On Windows XP and Windows Mobile 2003, no monitor mode equivalent exists, so examining the contents of packets directly from the chipset, (including any prefixed headers) is impossible. However, it is possible to request a scan for nearby access points, and their respective signal strengths, using the Windows NDISUIO interface [73]. Signal strength is extracted in Windows by invoking a scan though NDISUIO for nearby access points, which returns data that includes the signal strength to each visible access point. Scanning will be performed either actively or passively, depending on the driver implementation [72].

802.11 chipsets perform *active scanning* for nearby hosts using probe request and probe response packets. Probe request packets are broadcast (i.e. they received by all hosts). Each probe request asks listening access points for specific information; typically the supported access rates and encryption. Although the destination MAC address of a probe request is always the broadcast MAC (FF:FF:FF:FF:FF:FF), the target SSID field depends on the desired response. It can contain either the broadcast SSID, to which all networks will respond, or a specific SSID, to which only access points with that SSID will respond. Multiple access points can share an SSID, which defines a ESS (extended service set), a co-operative network infrastructure. For example, the CSE network SSID is CSE_wireless; it is made up of many base stations that have separate MAC addresses but all share that SSID.

So-called closed networks do not respond to broadcast probe packets. Intended to provide a light level of security, closed access points are invisible to casual scans. They do, however, respond to probe requests containing a matching SSID, so devices pre-configured with that SSID will be able to see the network.

802.11 devices can scan passively by observing beacon packets. Base stations periodically broadcast these packets, which announce the existence of the network and define the service area. Beacon packets contains core mandatory information about the network as well as several optional fields. At the very least, beacons contain the SSID of the network. The interval between beacons is an adjustable parameter, but most equipment broadcasts them once every 100ms.

The scanning mechanism uses ioctls on a specific file handle to request and query data. Scans are invoked by posting the object id OID_802_11_BSSID_LIST_SCAN to the handle, and the results of the scan requested by posting the object id OID_802_11_-BSSID_LIST_SCAN_QUERY. There is no callback raised when the scan has completed, so most calling applications sleep for a few seconds after requesting a scan before calling query.

To resolve the uncertainty in the specification, Kismet was used to observe the actual behaviour of an HP iPaq hx4700 when told to scan. A summary of the important findings:

- The device uses active scanning. When a scan is requested, the device broadcasts a series of probe request packets to all channels, first to all SSIDs, then to just the associated SSID. Each sweep takes about 50ms, followed by a pause while the device waits to receive prove responses packets.

- When unassociated, readings to all access points are updated with a period of 500ms, regardless of how fast scans are requests. This suggests that in a general context, readings should be requested at a maximum rate of twice per second.

- When associated, and scanning without a pause between scans, the update period for the associated access point is mostly distributed on the ranges 0-150ms and 350-650ms; the update period for unassociated access points is distribution on the ranges 0-150ms, 350-650ms, and 950-1150ms. Our theory is that the device uses passive observations of incoming packets to update the readings in addition to active scanning, generating a complex set of inconsistent sampling periods.

The same technique works on Windows XP; and it was possible to get signal strength information from a Intel Centrino 2200GB wireless chipset on a laptop. This chipset had the strange property that access points seen in previous scans were retained in newer scans even when the access point had gone out of range - while maintaining the RSSI last seen. This neccessitated verifying that the timestamps associated with the data were reasonably recent. It is unclear whether this behaviour is specific to the chipset or Windows XP in general, as no testing was performed with any other Windows machines.

Due to the relatively low scanning rates, resource usage is less of a problem on Windows than on Linux. The low scanning rates and information detail also makes it a less useful platform for research. Although one would expect that under NDIS, with functionality tightly specified by Microsoft, signal strength information would be consistent among different types of hardware, this was not the case, with 2 out of the 4 different 802.11 chipsets used being useless for positioning.

Appendix B

Comprehensive Results from the Comparison of Algorithms

This appendix contains comprehensive tables of results from the comparison of algorithms in Chapter 5, provided for the sake of reference. We first show tables summarising the best parameter values (p or b and k) for each combination of algorithm, environment, surveying method, and output averaging method. This is followed by the mean distance errors for all tests performed in their entirety.

B.1 Best Parameter Values

Tables B.1– B.5 show the parameter values that produce the lowest mean distance error using each type of algorithm, for each combination of surveying and output averaging method. For example, the first row of table B.1 shows that in the house environment, using point surveying, and arithmetic output averaging, the best value of p (the p-norm distance parameter, see Section 3.6.1) is 8 and the best value of k (the number of fingerprints to use in output averaging, see Section 3.6.2) is 6; the resulting mean, 50th, 90th, and maximum distance errors are shown in the remaining 4 columns.

Environment	Map	Averaging	p	k	Mean	50th	90th	Max
House	Point	Arithmetic	8	6	1.40	1.30	2.44	5.75
	Point	Weighted	7	6	1.38	1.27	2.44	5.23
	Sector	Arithmetic	1	2	1.37	1.17	2.62	4.53
	Sector	Weighted	1	2	1.35	1.15	2.65	4.52
CSE	Point	Arithmetic	2	7	2.91	2.51	5.24	17.54
	Point	Weighted	2	8	2.86	2.45	5.14	17.45
	Sector	Arithmetic	2	3	3.33	2.69	6.87	13.65
	Sector	Weighted	2	3	3.29	2.57	6.88	13.78

Table B.1: *Parameter combinations with the lowest mean distance errors for the Nearest Neighbour method.*

Environment	Map	Averaging	b	k	Mean	50th	90th	Max
House	Point	Arithmetic	7	6	1.54	1.37	2.80	6.73
	Point	Weighted	21	9	1.60	1.45	2.87	5.72
	Sector	Arithmetic	3	2	1.29	1.07	2.47	5.72
	Sector	Weighted	4	3	1.25	1.12	2.36	5.12
CSE	Point	Arithmetic	6	4	3.36	2.81	6.49	16.93
	Point	Weighted	8	11	3.18	2.61	6.39	14.00
	Sector	Arithmetic	6	3	3.53	3.00	7.09	14.83
	Sector	Weighted	5	9	3.37	2.73	6.87	15.69

Table B.2: *Parameter combinations with the lowest mean distance errors for the Bayesian Histogram method.*

Environment	Map	Averaging	k	Mean	50th	90th	Max
House	Point	Arithmetic	7	1.40	1.31	2.36	4.25
	Point	Weighted	3	1.82	1.70	3.56	5.66
	Sector	Arithmetic	2	1.37	1.13	2.77	5.97
	Sector	Weighted	4	1.47	1.29	2.77	5.66
CSE	Point	Arithmetic	3	3.76	3.10	6.93	21.55
	Point	Weighted	19	3.97	3.04	8.38	20.10
	Sector	Arithmetic	3	3.90	3.07	7.68	21.32
	Sector	Weighted	19	3.89	3.00	7.98	21.11

Table B.3: *Parameter combinations with the lowest mean distance errors for the Bayesian Gaussian method.*

Environment	Map	Averaging	b	k	Mean	50th	90th	Max
House	Point	Arithmetic	16	7	1.56	1.35	2.91	5.41
	Point	Weighted	16	7	1.54	1.34	3.04	4.86
	Sector	Arithmetic	6	2	1.43	1.29	2.66	5.34
	Sector	Weighted	6	2	1.42	1.32	2.67	5.34
CSE	Point	Mean	12	5	3.20	2.84	5.93	13.23
	Point	Weighted	12	6	3.14	2.75	5.72	13.22
	Sector	Mean	12	3	3.47	2.71	7.13	16.87
	Sector	Weighted	12	3	3.43	2.67	7.11	16.43

Table B.4: *Parameter combinations with the lowest mean distance errors for the p-norm Distance method.*

Environment	Map	Averaging	b	k	Mean	50th	90th	Max
House	Point	Mean	1	5	1.40	1.31	2.40	5.97
	Point	Weighted	1	5	1.40	1.29	2.42	5.99
	Sector	Mean	2	2	1.31	1.13	2.46	5.11
	Sector	Weighted	2	2	1.30	1.13	2.47	5.10
CSE	Point	Mean	6	4	2.89	2.50	5.68	19.21
	Point	Weighted	6	7	2.87	2.36	5.29	14.93
	Sector	Mean	4	2	2.96	2.24	6.01	14.29
	Sector	Weighted	4	3	2.95	2.32	5.99	13.94

Table B.5: *Parameter combinations with the lowest mean distance errors for the Match Distance method.*

B.2 Mean Distance Errors for All Tests

In this section, we show the mean distance errors for all tests performed to produce the results in Chapter 5. Each table contains all combinations of the algorithm specific parameter being varied (p or b) and k. Each table represents one combination of environment, algorithm, surveying method and output averaging method. Note that the Bayesian Gaussian results only contain one row as there is no algorithm-specific parameter to vary.

										k										
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
p = 1	1.78	1.57	1.50	1.46	1.46	1.46	1.48	1.46	1.47	1.47	1.47	1.48	1.50	1.52	1.54	1.55	1.57	1.58	1.59	1.60
2	1.76	1.60	1.52	1.48	1.47	1.50	1.52	1.53	1.55	1.54	1.53	1.53	1.54	1.55	1.56	1.57	1.59	1.60	1.62	1.64
3	1.80	1.60	1.51	1.48	1.48	1.50	1.51	1.50	1.51	1.51	1.50	1.52	1.53	1.52	1.52	1.53	1.54	1.55	1.56	1.58
4	1.81	1.59	1.51	1.48	1.49	1.48	1.48	1.47	1.48	1.48	1.48	1.48	1.50	1.49	1.50	1.53	1.54	1.56	1.57	1.58
5	1.80	1.60	1.51	1.48	1.48	1.44	1.44	1.45	1.46	1.46	1.48	1.48	1.48	1.50	1.51	1.53	1.55	1.57	1.58	1.59
6	1.80	1.59	1.50	1.49	1.48	1.42	1.43	1.44	1.44	1.47	1.47	1.48	1.49	1.51	1.52	1.54	1.55	1.57	1.60	1.61
7	1.82	1.58	1.50	1.50	1.47	1.40	1.42	1.43	1.45	1.47	1.48	1.50	1.50	1.52	1.53	1.55	1.56	1.59	1.61	1.63
8	1.82	1.57	1.51	1.50	1.47	1.40	1.42	1.45	1.45	1.48	1.51	1.51	1.51	1.53	1.55	1.56	1.58	1.59	1.62	1.64
9	1.82	1.57	1.51	1.49	1.47	1.41	1.43	1.45	1.46	1.49	1.51	1.52	1.52	1.54	1.56	1.57	1.59	1.60	1.62	1.65
10	1.82	1.57	1.51	1.51	1.46	1.41	1.43	1.45	1.47	1.50	1.50	1.52	1.53	1.54	1.56	1.58	1.59	1.61	1.63	1.66

Table B.6: *Mean distance errors (m) using the* ***Nearest Neighbour*** *algorithm,* ***point*** *surveying and* ***arithmetic*** *output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	1.78	1.56	1.48	1.44	1.43	1.43	1.44	1.42	1.43	1.43	1.43	1.44	1.45	1.47	1.48	1.49	1.50	1.51	1.52	1.52
	2	1.76	1.56	1.47	1.42	1.41	1.43	1.45	1.46	1.48	1.48	1.47	1.47	1.48	1.49	1.50	1.50	1.52	1.53	1.54	1.55
	3	1.80	1.56	1.46	1.42	1.41	1.44	1.44	1.43	1.44	1.45	1.44	1.46	1.47	1.47	1.47	1.47	1.48	1.49	1.50	1.51
	4	1.81	1.56	1.46	1.42	1.42	1.42	1.42	1.42	1.43	1.43	1.43	1.44	1.45	1.45	1.45	1.47	1.48	1.50	1.51	1.51
p	5	1.80	1.56	1.46	1.42	1.43	1.40	1.40	1.40	1.41	1.42	1.44	1.43	1.44	1.45	1.46	1.47	1.49	1.50	1.51	1.52
	6	1.80	1.55	1.46	1.43	1.43	1.38	1.39	1.40	1.41	1.42	1.43	1.43	1.45	1.46	1.47	1.48	1.49	1.51	1.52	1.53
	7	1.82	1.55	1.46	1.44	1.43	1.38	1.39	1.40	1.41	1.43	1.44	1.45	1.45	1.46	1.48	1.49	1.50	1.52	1.53	1.55
	8	1.82	1.54	1.47	1.44	1.43	1.38	1.39	1.41	1.41	1.43	1.45	1.46	1.46	1.47	1.49	1.50	1.51	1.52	1.54	1.56
	9	1.82	1.53	1.47	1.44	1.43	1.39	1.40	1.41	1.42	1.44	1.45	1.46	1.47	1.48	1.50	1.51	1.52	1.53	1.54	1.56
	10	1.82	1.54	1.47	1.45	1.43	1.39	1.40	1.41	1.42	1.45	1.45	1.47	1.47	1.48	1.50	1.51	1.52	1.53	1.55	1.57

Table B.7: *Mean distance errors (m) using the **Nearest Neighbour** algorithm, **point** surveying and **weighted** output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.49	1.37	1.45	1.51	1.62	1.68	1.73	1.86	2.05	2.21	2.45	2.82	3.30	3.84	4.29	4.70	5.09	5.47	5.90
	2	1.69	1.48	1.58	1.61	1.72	1.74	1.79	1.88	2.02	2.17	2.38	2.75	3.25	3.76	4.24	4.66	5.04	5.45	5.90
	3	1.72	1.46	1.60	1.63	1.74	1.73	1.73	1.81	1.94	2.13	2.37	2.76	3.24	3.74	4.23	4.67	5.05	5.46	5.90
	4	1.77	1.54	1.59	1.61	1.71	1.69	1.72	1.80	1.97	2.14	2.38	2.77	3.26	3.75	4.22	4.65	5.04	5.46	5.90
p	5	1.77	1.53	1.59	1.60	1.69	1.70	1.73	1.81	1.97	2.17	2.40	2.81	3.27	3.74	4.21	4.64	5.04	5.47	5.90
	6	1.78	1.51	1.57	1.59	1.71	1.72	1.74	1.82	1.99	2.16	2.41	2.83	3.30	3.74	4.20	4.64	5.04	5.47	5.90
	7	1.79	1.55	1.60	1.61	1.72	1.72	1.75	1.83	1.99	2.16	2.41	2.84	3.30	3.74	4.20	4.64	5.04	5.47	5.90
	8	1.81	1.56	1.63	1.64	1.75	1.73	1.77	1.84	2.00	2.17	2.41	2.85	3.31	3.74	4.20	4.64	5.04	5.47	5.90
	9	1.84	1.57	1.64	1.64	1.75	1.75	1.77	1.85	2.00	2.18	2.42	2.85	3.32	3.74	4.20	4.64	5.04	5.47	5.90
	10	1.86	1.58	1.64	1.64	1.76	1.75	1.78	1.85	1.99	2.18	2.42	2.86	3.32	3.74	4.20	4.64	5.04	5.47	5.90

Table B.8: *Mean distance errors (m) using the **Nearest Neighbour** algorithm, **sector** surveying and **arithmetic** output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.49	1.35	1.38	1.43	1.51	1.54	1.56	1.61	1.67	1.71	1.78	1.88	2.01	2.19	2.37	2.56	2.74	2.93	3.15
	2	1.69	1.47	1.51	1.53	1.60	1.61	1.63	1.65	1.68	1.70	1.76	1.87	2.04	2.23	2.44	2.66	2.84	3.06	3.29
	3	1.72	1.46	1.52	1.55	1.61	1.60	1.57	1.58	1.60	1.67	1.75	1.90	2.08	2.29	2.51	2.73	2.93	3.15	3.38
	4	1.77	1.53	1.52	1.53	1.58	1.55	1.54	1.56	1.62	1.69	1.77	1.92	2.13	2.34	2.55	2.77	2.97	3.20	3.42
p	5	1.77	1.51	1.52	1.51	1.56	1.56	1.55	1.58	1.63	1.71	1.80	1.97	2.16	2.36	2.57	2.79	3.00	3.23	3.45
	6	1.78	1.50	1.50	1.51	1.58	1.57	1.56	1.59	1.65	1.71	1.81	1.99	2.19	2.37	2.59	2.81	3.02	3.24	3.47
	7	1.79	1.54	1.53	1.53	1.59	1.59	1.58	1.61	1.65	1.72	1.82	2.00	2.20	2.39	2.60	2.82	3.03	3.26	3.48
	8	1.81	1.55	1.56	1.55	1.62	1.60	1.60	1.61	1.66	1.74	1.82	2.01	2.21	2.40	2.61	2.83	3.04	3.27	3.49
	9	1.84	1.56	1.57	1.55	1.62	1.61	1.60	1.62	1.67	1.74	1.83	2.02	2.22	2.40	2.61	2.84	3.05	3.27	3.50
	10	1.86	1.57	1.57	1.55	1.63	1.62	1.61	1.62	1.66	1.74	1.84	2.02	2.22	2.40	2.62	2.84	3.05	3.28	3.50

Table B.9: *Mean distance errors (m) using the* ***Nearest Neighbour*** *algorithm,* ***sector*** *surveying and* ***weighted*** *output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	2.21	1.88	1.81	1.80	1.81	1.79	1.81	1.85	1.86	1.88	1.90	1.92	1.97	2.01	2.07	2.13	2.20	2.28	2.36	2.46
	2	2.08	1.86	1.76	1.73	1.74	1.75	1.75	1.76	1.78	1.82	1.84	1.86	1.91	1.94	1.96	2.00	2.03	2.07	2.11	2.17
	3	2.02	1.73	1.70	1.64	1.65	1.67	1.69	1.72	1.73	1.74	1.76	1.78	1.81	1.82	1.87	1.92	1.94	1.97	2.02	2.05
	4	1.97	1.76	1.65	1.64	1.65	1.65	1.64	1.66	1.67	1.68	1.69	1.70	1.73	1.74	1.74	1.77	1.80	1.84	1.89	1.92
	5	1.89	1.67	1.61	1.59	1.63	1.63	1.65	1.66	1.66	1.67	1.69	1.71	1.70	1.73	1.75	1.78	1.80	1.83	1.86	1.92
	6	1.93	1.69	1.55	1.56	1.55	1.54	1.58	1.60	1.62	1.63	1.64	1.65	1.66	1.65	1.71	1.73	1.75	1.78	1.82	1.85
	7	2.05	1.80	1.61	1.55	1.54	1.54	1.56	1.58	1.63	1.63	1.65	1.67	1.68	1.69	1.72	1.74	1.78	1.81	1.82	1.84
	8	1.78	1.66	1.59	1.55	1.57	1.55	1.57	1.56	1.59	1.60	1.61	1.65	1.66	1.67	1.69	1.71	1.74	1.76	1.77	1.81
	9	1.79	1.69	1.60	1.58	1.58	1.58	1.60	1.60	1.62	1.62	1.63	1.64	1.67	1.68	1.69	1.71	1.72	1.74	1.75	1.78
	10	1.83	1.63	1.56	1.57	1.58	1.56	1.57	1.59	1.59	1.62	1.66	1.69	1.70	1.70	1.72	1.75	1.75	1.78	1.80	1.83
	11	1.96	1.80	1.66	1.63	1.65	1.63	1.63	1.63	1.63	1.64	1.64	1.64	1.66	1.70	1.73	1.76	1.78	1.80	1.81	1.83
	12	1.95	1.72	1.63	1.67	1.63	1.61	1.64	1.64	1.64	1.62	1.64	1.67	1.69	1.71	1.70	1.71	1.72	1.74	1.77	1.79
	13	2.11	1.75	1.74	1.71	1.70	1.70	1.64	1.65	1.67	1.65	1.66	1.67	1.69	1.71	1.73	1.75	1.76	1.80	1.83	1.84
	14	1.90	1.81	1.67	1.66	1.68	1.72	1.69	1.75	1.76	1.79	1.81	1.84	1.83	1.85	1.84	1.85	1.86	1.90	1.92	1.91
b	15	1.86	1.80	1.81	1.72	1.71	1.67	1.69	1.68	1.69	1.70	1.72	1.71	1.73	1.73	1.75	1.77	1.76	1.77	1.77	1.80
	16	1.94	1.69	1.57	1.58	1.58	1.57	1.54	1.57	1.59	1.60	1.62	1.64	1.66	1.68	1.70	1.71	1.75	1.79	1.82	1.82
	17	1.99	1.75	1.80	1.77	1.83	1.81	1.81	1.76	1.78	1.76	1.80	1.80	1.81	1.80	1.84	1.84	1.86	1.84	1.87	1.88
	18	2.49	2.13	2.01	1.97	1.95	1.93	1.93	1.90	1.90	1.92	1.89	1.88	1.89	1.91	1.92	1.96	1.98	1.99	2.00	2.01
	19	1.97	1.90	1.88	1.89	1.85	1.83	1.82	1.86	1.84	1.85	1.84	1.86	1.88	1.90	1.91	1.90	1.91	1.93	1.92	1.94
	20	2.00	1.89	1.75	1.68	1.67	1.64	1.69	1.71	1.73	1.74	1.74	1.75	1.77	1.77	1.77	1.78	1.79	1.80	1.80	1.82
	21	2.18	1.85	1.68	1.72	1.66	1.65	1.64	1.63	1.62	1.65	1.67	1.71	1.73	1.74	1.76	1.78	1.79	1.79	1.80	1.79
	22	2.06	1.78	1.76	1.84	1.89	1.91	1.90	1.87	1.89	1.87	1.86	1.87	1.88	1.90	1.90	1.91	1.92	1.94	1.93	1.95
	23	2.29	2.04	1.96	1.90	1.95	1.98	2.04	2.03	1.98	2.04	2.04	2.06	2.05	2.07	2.08	2.10	2.09	2.07	2.12	2.13
	24	2.34	2.14	2.10	2.11	2.09	2.09	2.13	2.16	2.14	2.20	2.19	2.21	2.23	2.22	2.22	2.21	2.22	2.28	2.31	2.33
	25	2.20	2.15	2.15	2.21	2.29	2.29	2.31	2.33	2.33	2.30	2.28	2.32	2.35	2.36	2.40	2.43	2.47	2.53	2.54	2.60
	26	2.35	2.24	2.32	2.17	2.13	2.09	2.11	2.14	2.13	2.11	2.12	2.13	2.14	2.19	2.24	2.23	2.20	2.23	2.24	2.28
	27	2.43	2.19	2.14	2.16	2.08	2.04	2.03	2.06	2.05	2.08	2.04	2.04	2.06	2.08	2.11	2.12	2.16	2.15	2.16	2.19
	28	2.17	2.03	2.07	2.00	1.98	1.95	1.93	1.99	1.97	1.99	1.98	1.99	1.98	2.00	2.01	2.04	2.05	2.07	2.06	2.07
	29	2.09	2.00	1.93	1.91	1.96	1.94	1.88	1.90	1.92	1.92	1.90	1.91	1.91	1.90	1.91	1.91	1.93	1.94	1.94	1.96
	30	2.28	1.90	1.85	1.86	1.85	1.84	1.83	1.81	1.80	1.80	1.79	1.79	1.82	1.82	1.82	1.82	1.81	1.81	1.80	1.81

Table B.10: *Mean distance errors (m) using the* ***Bayesian Histogram*** *algorithm,* ***point*** *surveying and* ***arithmetic*** *output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	2.21	2.07	2.04	2.03	2.03	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02
	2	2.08	1.98	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
	3	2.02	1.91	1.90	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	4	1.97	1.85	1.84	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
	5	1.89	1.79	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77
	6	1.93	1.81	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78
	7	2.05	1.97	1.92	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
	8	1.78	1.74	1.72	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71
	9	1.79	1.76	1.70	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69
	10	1.83	1.68	1.67	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66
	11	1.96	1.85	1.80	1.78	1.77	1.77	1.77	1.77	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
	12	1.95	1.78	1.72	1.72	1.71	1.71	1.71	1.71	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
	13	2.11	1.81	1.79	1.77	1.76	1.76	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74
	14	1.90	1.82	1.72	1.71	1.71	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
b	15	1.86	1.82	1.82	1.80	1.80	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78
	16	1.94	1.69	1.63	1.64	1.62	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61
	17	1.99	1.76	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	1.73	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74
	18	2.49	2.17	2.06	2.02	1.99	1.98	1.98	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
	19	1.97	1.82	1.82	1.81	1.79	1.77	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
	20	2.00	1.95	1.81	1.76	1.74	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72
	21	2.18	1.83	1.68	1.70	1.66	1.62	1.62	1.61	1.60	1.60	1.61	1.61	1.62	1.61	1.61	1.61	1.61	1.61	1.61	1.61
	22	2.06	1.80	1.79	1.84	1.84	1.85	1.84	1.84	1.84	1.84	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
	23	2.29	1.98	1.92	1.89	1.91	1.92	1.92	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
	24	2.34	2.08	2.05	2.03	2.01	2.01	2.01	2.01	2.00	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01
	25	2.20	2.13	2.12	2.16	2.18	2.14	2.12	2.14	2.14	2.13	2.13	2.13	2.13	2.13	2.13	2.14	2.14	2.14	2.14	2.14
	26	2.35	2.23	2.28	2.17	2.11	2.06	2.06	2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.04	2.04	2.04	2.04
	27	2.43	2.21	2.16	2.19	2.12	2.10	2.09	2.12	2.11	2.12	2.10	2.09	2.09	2.08	2.09	2.08	2.08	2.08	2.08	2.08
	28	2.17	2.09	2.10	2.03	1.99	1.96	1.98	1.98	1.97	1.95	1.94	1.94	1.93	1.94	1.93	1.93	1.93	1.93	1.93	1.93
	29	2.09	2.03	1.98	1.97	2.00	1.98	1.95	1.94	1.94	1.95	1.93	1.94	1.94	1.93	1.92	1.91	1.91	1.91	1.91	1.91
	30	2.28	1.96	1.93	1.92	1.90	1.87	1.87	1.86	1.84	1.83	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82

Table B.11: *Mean distance errors (m) using the* ***Bayesian Histogram*** *algorithm,* ***point*** *surveying and* ***weighted*** *output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.49	1.36	1.46	1.53	1.58	1.61	1.75	1.89	2.15	2.48	2.81	3.26	3.72	4.19	4.61	5.00	5.34	5.64	5.90
	2	1.48	1.32	1.41	1.48	1.54	1.60	1.70	1.85	2.12	2.43	2.74	3.17	3.65	4.12	4.55	4.95	5.31	5.65	5.90
	3	1.43	1.29	1.40	1.45	1.53	1.60	1.70	1.86	2.15	2.40	2.72	3.16	3.62	4.09	4.51	4.92	5.28	5.64	5.90
	4	1.42	1.30	1.40	1.48	1.54	1.61	1.71	1.86	2.12	2.37	2.67	3.13	3.60	4.06	4.51	4.91	5.28	5.62	5.90
	5	1.43	1.35	1.38	1.43	1.50	1.60	1.70	1.82	2.06	2.37	2.68	3.11	3.58	4.07	4.46	4.88	5.25	5.62	5.90
	6	1.51	1.32	1.37	1.44	1.52	1.62	1.67	1.85	2.09	2.41	2.71	3.11	3.56	4.06	4.49	4.88	5.25	5.59	5.90
	7	1.55	1.34	1.42	1.48	1.56	1.65	1.75	1.88	2.11	2.34	2.67	3.11	3.57	4.01	4.47	4.89	5.27	5.61	5.90
	8	1.51	1.33	1.44	1.50	1.56	1.67	1.75	1.89	2.12	2.40	2.66	3.10	3.60	4.06	4.51	4.90	5.26	5.63	5.90
	9	1.44	1.42	1.48	1.47	1.62	1.70	1.78	1.88	2.13	2.37	2.68	3.08	3.54	3.99	4.41	4.81	5.22	5.59	5.90
	10	1.58	1.35	1.43	1.47	1.57	1.57	1.66	1.82	2.05	2.33	2.67	3.04	3.50	4.01	4.46	4.83	5.21	5.56	5.90
	11	1.54	1.39	1.49	1.54	1.63	1.67	1.78	1.95	2.13	2.34	2.60	3.01	3.45	3.94	4.42	4.85	5.23	5.58	5.90
	12	1.69	1.43	1.45	1.50	1.67	1.65	1.76	1.82	2.03	2.34	2.65	3.07	3.56	4.04	4.48	4.85	5.20	5.57	5.90
	13	1.59	1.48	1.50	1.49	1.61	1.58	1.70	1.86	2.10	2.29	2.59	3.03	3.46	3.95	4.40	4.80	5.15	5.55	5.90
	14	1.67	1.50	1.56	1.59	1.66	1.75	1.84	1.96	2.18	2.32	2.66	2.98	3.39	3.87	4.38	4.79	5.22	5.61	5.90
b	15	1.65	1.47	1.50	1.56	1.67	1.72	1.79	1.92	2.06	2.30	2.55	2.95	3.40	3.92	4.39	4.78	5.13	5.54	5.90
	16	1.65	1.49	1.52	1.59	1.71	1.71	1.78	1.90	2.07	2.35	2.71	3.18	3.66	4.11	4.50	4.83	5.15	5.56	5.90
	17	1.75	1.66	1.62	1.71	1.74	1.74	1.85	1.96	2.15	2.37	2.62	3.00	3.43	3.84	4.34	4.79	5.18	5.56	5.90
	18	2.06	1.82	1.76	1.77	1.83	1.85	1.92	2.06	2.23	2.50	2.77	3.15	3.54	3.93	4.37	4.79	5.23	5.59	5.90
	19	1.80	1.75	1.73	1.76	1.79	1.83	1.93	2.06	2.17	2.41	2.68	3.07	3.50	3.89	4.39	4.82	5.23	5.57	5.90
	20	1.65	1.58	1.62	1.61	1.72	1.76	1.83	1.92	2.07	2.34	2.59	2.94	3.37	3.85	4.33	4.75	5.09	5.52	5.90
	21	1.65	1.59	1.70	1.68	1.77	1.85	1.89	1.97	2.13	2.41	2.71	3.20	3.67	4.12	4.52	4.83	5.11	5.54	5.90
	22	1.74	1.67	1.72	1.81	1.87	1.94	2.01	2.08	2.28	2.49	2.75	3.19	3.59	4.01	4.45	4.82	5.20	5.58	5.90
	23	2.01	1.81	1.83	1.85	2.01	2.06	2.08	2.18	2.36	2.61	2.87	3.20	3.61	3.98	4.37	4.75	5.16	5.56	5.90
	24	2.40	2.13	2.09	2.12	2.11	2.11	2.19	2.34	2.48	2.75	3.03	3.43	3.91	4.32	4.64	4.95	5.30	5.60	5.90
	25	2.50	2.09	2.12	2.18	2.18	2.27	2.25	2.36	2.53	2.74	2.99	3.45	3.84	4.18	4.59	4.97	5.29	5.60	5.90
	26	2.24	2.11	2.06	2.03	2.08	2.05	2.06	2.20	2.38	2.58	2.82	3.22	3.73	4.12	4.55	4.93	5.21	5.54	5.90
	27	2.37	2.17	2.09	2.05	2.07	2.01	2.11	2.22	2.35	2.54	2.74	3.11	3.42	3.89	4.38	4.79	5.26	5.52	5.90
	28	2.28	2.09	1.98	1.94	1.92	1.97	2.04	2.19	2.30	2.49	2.69	3.09	3.46	3.90	4.41	4.80	5.25	5.56	5.90
	29	2.03	2.05	1.91	1.84	1.90	1.94	2.00	2.05	2.14	2.41	2.69	3.03	3.37	3.80	4.34	4.76	5.11	5.51	5.90
	30	1.98	1.82	1.79	1.77	1.77	1.83	1.86	1.93	2.06	2.28	2.54	2.91	3.36	3.83	4.33	4.72	5.07	5.51	5.90

Table B.12: *Mean distance errors (m) using the **Bayesian Histogram** algorithm, **sector** surveying and **arithmetic** output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.49	1.35	1.34	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
	2	1.48	1.31	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
	3	1.43	1.29	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
	4	1.42	1.27	1.25	1.26	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
	5	1.43	1.32	1.27	1.26	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27
	6	1.51	1.32	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
	7	1.55	1.34	1.32	1.31	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32
	8	1.51	1.32	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
	9	1.44	1.39	1.39	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37
	10	1.58	1.36	1.34	1.34	1.34	1.34	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
	11	1.54	1.36	1.35	1.34	1.34	1.34	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35
	12	1.69	1.45	1.38	1.39	1.40	1.39	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.40
	13	1.59	1.47	1.41	1.39	1.39	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38	1.38
	14	1.67	1.49	1.49	1.48	1.48	1.48	1.48	1.48	1.49	1.49	1.49	1.49	1.49	1.49	1.49	1.49	1.49	1.49	1.49
b	15	1.65	1.47	1.43	1.44	1.44	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45
	16	1.65	1.47	1.43	1.42	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44
	17	1.75	1.63	1.55	1.54	1.53	1.52	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53
	18	2.06	1.83	1.74	1.72	1.70	1.70	1.70	1.70	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71
	19	1.80	1.72	1.69	1.69	1.67	1.66	1.66	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67
	20	1.65	1.57	1.54	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52
	21	1.65	1.57	1.59	1.57	1.57	1.58	1.58	1.58	1.58	1.58	1.58	1.58	1.58	1.58	1.58	1.58	1.58	1.58	1.58
	22	1.74	1.64	1.62	1.64	1.63	1.64	1.64	1.64	1.64	1.64	1.64	1.64	1.64	1.64	1.64	1.64	1.64	1.64	1.64
	23	2.01	1.80	1.78	1.78	1.81	1.82	1.81	1.81	1.80	1.80	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81
	24	2.40	2.15	2.07	2.07	2.03	2.01	2.01	2.02	2.01	2.02	2.01	2.02	2.02	2.02	2.02	2.02	2.02	2.02	2.02
	25	2.50	2.12	2.09	2.11	2.07	2.10	2.09	2.09	2.10	2.11	2.10	2.11	2.11	2.11	2.11	2.11	2.11	2.11	2.11
	26	2.24	2.10	1.97	1.96	1.95	1.95	1.93	1.94	1.93	1.94	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93
	27	2.37	2.15	2.06	2.02	2.00	1.98	1.98	1.98	1.98	1.98	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
	28	2.28	2.06	1.93	1.89	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
	29	2.03	2.04	1.89	1.85	1.85	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
	30	1.98	1.82	1.77	1.75	1.74	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73

Table B.13: *Mean distance errors (m) using the* ***Bayesian Histogram*** *algorithm,* ***sector*** *surveying and* ***weighted*** *output averaging in the house environment.*

k																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.84	1.61	1.51	1.43	1.42	1.40	1.40	1.41	1.42	1.45	1.46	1.47	1.49	1.51	1.55	1.60	1.65	1.72	1.78	1.87

Table B.14: *Mean distance errors (m) using the* ***Bayesian Gaussian*** *algorithm,* ***point*** *surveying and* ***arithmetic*** *output averaging in the house environment.*

k																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.84	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82

Table B.15: *Mean distance errors (m) using the* ***Bayesian Gaussian*** *algorithm,* ***point*** *surveying and* ***weighted*** *output averaging in the house environment.*

k																		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1.54	1.37	1.43	1.51	1.58	1.62	1.67	1.79	2.11	2.47	2.88	3.28	3.72	4.18	4.63	4.97	5.32	5.63	5.90

Table B.16: *Mean distance errors (m) using the* ***Bayesian Gaussian*** *algorithm,* ***sector*** *surveying and* ***arithmetic*** *output averaging in the house environment.*

k																		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1.54	1.48	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47

Table B.17: *Mean distance errors (m) using the **Bayesian Gaussian** algorithm, **sector** surveying and **weighted** output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	2.61	2.25	2.12	2.08	2.02	2.07	2.09	2.13	2.16	2.24	2.28	2.33	2.38	2.44	2.50	2.57	2.65	2.71	2.76	2.84
	2	2.44	2.09	2.00	1.92	1.91	1.90	1.93	1.97	2.02	2.05	2.11	2.17	2.19	2.25	2.29	2.35	2.43	2.49	2.56	2.63
	3	2.26	2.02	1.91	1.85	1.81	1.82	1.84	1.85	1.85	1.89	1.92	1.97	2.01	2.05	2.09	2.13	2.18	2.25	2.31	2.37
	4	2.11	1.89	1.77	1.74	1.73	1.73	1.73	1.76	1.79	1.82	1.84	1.84	1.87	1.89	1.93	1.99	2.06	2.10	2.17	2.23
	5	2.15	1.95	1.82	1.76	1.75	1.79	1.81	1.84	1.80	1.82	1.82	1.81	1.84	1.87	1.89	1.92	1.95	2.02	2.08	2.14
	6	2.07	1.75	1.69	1.65	1.67	1.69	1.71	1.70	1.70	1.72	1.73	1.77	1.79	1.80	1.82	1.85	1.89	1.94	1.98	2.03
	7	1.92	1.70	1.63	1.61	1.61	1.62	1.65	1.67	1.69	1.72	1.75	1.79	1.81	1.82	1.85	1.88	1.90	1.92	1.94	1.98
	8	1.95	1.75	1.65	1.62	1.62	1.65	1.67	1.68	1.67	1.68	1.68	1.69	1.72	1.73	1.74	1.79	1.83	1.85	1.89	1.93
	9	2.02	1.75	1.65	1.61	1.62	1.62	1.60	1.59	1.62	1.64	1.64	1.66	1.67	1.69	1.70	1.74	1.78	1.80	1.82	1.87
	10	1.88	1.72	1.61	1.59	1.60	1.61	1.64	1.65	1.68	1.67	1.68	1.71	1.73	1.77	1.79	1.81	1.83	1.87	1.90	1.93
	11	2.06	1.83	1.72	1.66	1.62	1.60	1.61	1.60	1.62	1.64	1.66	1.68	1.69	1.71	1.72	1.75	1.76	1.78	1.79	1.81
	12	1.97	1.71	1.65	1.66	1.62	1.64	1.66	1.67	1.66	1.65	1.65	1.66	1.68	1.69	1.72	1.74	1.77	1.77	1.79	1.81
	13	1.89	1.69	1.65	1.64	1.64	1.61	1.62	1.65	1.66	1.68	1.67	1.67	1.69	1.69	1.71	1.72	1.74	1.77	1.80	1.83
	14	2.01	1.86	1.80	1.79	1.77	1.75	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.80	1.81	1.82	1.84	1.87	1.88	1.90
b	15	1.95	1.75	1.71	1.68	1.64	1.65	1.64	1.65	1.64	1.65	1.67	1.68	1.69	1.70	1.71	1.73	1.75	1.76	1.78	1.80
	16	1.78	1.64	1.58	1.60	1.59	1.57	1.56	1.59	1.62	1.62	1.63	1.65	1.67	1.71	1.75	1.76	1.79	1.80	1.84	1.86
	17	1.96	1.76	1.81	1.77	1.76	1.74	1.77	1.80	1.79	1.79	1.79	1.81	1.81	1.81	1.81	1.82	1.85	1.87	1.87	1.90
	18	2.53	2.09	2.03	1.96	1.91	1.89	1.86	1.86	1.82	1.82	1.83	1.85	1.86	1.87	1.87	1.86	1.87	1.87	1.89	1.88
	19	2.16	1.96	1.90	1.86	1.84	1.82	1.81	1.82	1.81	1.83	1.85	1.84	1.84	1.84	1.85	1.86	1.88	1.88	1.88	1.90
	20	1.91	1.75	1.73	1.73	1.72	1.69	1.69	1.71	1.73	1.74	1.75	1.76	1.76	1.76	1.76	1.76	1.77	1.78	1.80	1.82
	21	2.04	1.79	1.68	1.67	1.64	1.61	1.61	1.60	1.61	1.65	1.66	1.69	1.72	1.72	1.72	1.71	1.74	1.75	1.78	1.79
	22	2.16	1.83	1.82	1.84	1.87	1.85	1.83	1.85	1.84	1.84	1.86	1.84	1.86	1.84	1.86	1.88	1.90	1.90	1.91	1.93
	23	2.26	2.05	2.03	1.96	2.01	1.99	2.05	2.04	2.02	2.02	2.02	2.03	2.04	2.03	2.04	2.04	2.05	2.09	2.11	2.12
	24	2.30	2.14	2.09	2.07	2.07	2.02	2.01	2.02	2.03	2.05	2.06	2.12	2.14	2.17	2.22	2.23	2.26	2.27	2.29	2.33
	25	2.34	2.26	2.23	2.27	2.31	2.28	2.25	2.28	2.31	2.35	2.38	2.43	2.43	2.45	2.46	2.47	2.51	2.58	2.61	2.66
	26	2.33	2.27	2.33	2.25	2.20	2.16	2.25	2.27	2.30	2.26	2.24	2.22	2.21	2.22	2.22	2.25	2.26	2.28	2.33	2.35
	27	2.30	2.15	2.08	2.09	2.07	2.02	2.03	2.04	2.04	2.06	2.07	2.06	2.06	2.04	2.06	2.08	2.09	2.11	2.11	2.16
	28	2.40	2.03	1.95	1.90	1.92	1.93	1.93	1.93	1.93	1.95	1.97	1.98	1.98	2.01	2.01	2.01	2.02	2.04	2.05	2.07
	29	2.39	2.16	2.03	1.93	1.91	1.88	1.87	1.88	1.90	1.90	1.87	1.86	1.85	1.87	1.87	1.87	1.88	1.90	1.91	1.92
	30	2.36	1.94	1.88	1.89	1.86	1.85	1.84	1.83	1.80	1.80	1.80	1.78	1.77	1.78	1.76	1.76	1.77	1.76	1.77	1.78

Table B.18: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **point** surveying and **arithmetic** output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	2.61	2.24	2.11	2.07	2.01	2.05	2.07	2.11	2.14	2.20	2.24	2.28	2.33	2.39	2.44	2.50	2.58	2.64	2.68	2.76
	2	2.44	2.08	1.99	1.90	1.90	1.88	1.90	1.94	1.99	2.01	2.06	2.12	2.14	2.19	2.23	2.28	2.35	2.41	2.47	2.53
	3	2.26	2.00	1.89	1.83	1.79	1.80	1.81	1.82	1.82	1.86	1.89	1.92	1.96	1.99	2.02	2.06	2.10	2.16	2.21	2.27
	4	2.11	1.88	1.75	1.72	1.71	1.70	1.70	1.73	1.75	1.78	1.79	1.80	1.82	1.84	1.87	1.92	1.97	2.01	2.07	2.11
	5	2.15	1.95	1.81	1.75	1.73	1.76	1.77	1.79	1.76	1.77	1.77	1.77	1.79	1.81	1.82	1.85	1.88	1.93	1.97	2.02
	6	2.07	1.74	1.67	1.63	1.64	1.65	1.68	1.67	1.67	1.68	1.69	1.72	1.73	1.74	1.75	1.78	1.81	1.85	1.88	1.92
	7	1.92	1.70	1.62	1.59	1.59	1.60	1.61	1.63	1.64	1.66	1.69	1.72	1.73	1.74	1.76	1.78	1.79	1.81	1.82	1.85
	8	1.95	1.73	1.64	1.60	1.59	1.61	1.63	1.63	1.63	1.63	1.63	1.64	1.66	1.67	1.67	1.71	1.73	1.75	1.77	1.79
	9	2.02	1.74	1.65	1.60	1.60	1.60	1.58	1.57	1.58	1.60	1.60	1.61	1.61	1.63	1.64	1.67	1.69	1.70	1.72	1.75
	10	1.88	1.70	1.59	1.57	1.56	1.58	1.59	1.61	1.62	1.62	1.63	1.65	1.67	1.69	1.70	1.71	1.73	1.75	1.77	1.78
	11	2.06	1.83	1.74	1.69	1.64	1.61	1.60	1.58	1.60	1.61	1.63	1.64	1.65	1.66	1.67	1.68	1.69	1.70	1.71	1.72
	12	1.97	1.71	1.64	1.63	1.60	1.61	1.62	1.62	1.62	1.61	1.61	1.61	1.63	1.63	1.65	1.67	1.68	1.69	1.70	1.71
	13	1.89	1.70	1.65	1.63	1.62	1.60	1.61	1.61	1.61	1.62	1.61	1.61	1.61	1.61	1.63	1.63	1.64	1.66	1.67	1.69
	14	2.01	1.84	1.77	1.75	1.73	1.72	1.74	1.73	1.74	1.73	1.73	1.73	1.73	1.73	1.74	1.75	1.76	1.78	1.79	1.80
b	15	1.95	1.73	1.69	1.65	1.63	1.63	1.63	1.63	1.62	1.62	1.63	1.63	1.64	1.64	1.64	1.66	1.66	1.67	1.69	1.70
	16	1.78	1.63	1.56	1.56	1.55	1.55	1.54	1.55	1.56	1.56	1.56	1.57	1.59	1.61	1.63	1.63	1.65	1.65	1.67	1.68
	17	1.96	1.76	1.76	1.73	1.70	1.69	1.70	1.71	1.70	1.70	1.70	1.71	1.70	1.70	1.70	1.71	1.73	1.74	1.74	1.76
	18	2.53	2.14	2.05	1.97	1.93	1.92	1.88	1.87	1.85	1.83	1.83	1.84	1.85	1.85	1.86	1.85	1.86	1.85	1.86	1.86
	19	2.16	1.92	1.83	1.79	1.77	1.75	1.74	1.75	1.74	1.74	1.75	1.75	1.75	1.76	1.76	1.77	1.78	1.78	1.78	1.79
	20	1.91	1.72	1.68	1.69	1.67	1.66	1.65	1.66	1.67	1.67	1.68	1.69	1.68	1.68	1.68	1.68	1.68	1.69	1.70	1.70
	21	2.04	1.78	1.65	1.65	1.62	1.59	1.59	1.59	1.59	1.61	1.61	1.62	1.63	1.63	1.63	1.63	1.64	1.65	1.66	1.66
	22	2.16	1.85	1.82	1.78	1.80	1.77	1.76	1.78	1.78	1.78	1.80	1.79	1.80	1.80	1.80	1.81	1.82	1.82	1.82	1.83
	23	2.26	1.98	1.93	1.87	1.87	1.84	1.87	1.86	1.86	1.85	1.86	1.85	1.86	1.86	1.86	1.86	1.86	1.88	1.88	1.89
	24	2.30	2.09	2.03	2.00	2.00	1.97	1.95	1.96	1.96	1.96	1.96	1.97	1.97	1.99	2.00	2.01	2.02	2.03	2.04	2.05
	25	2.34	2.23	2.19	2.20	2.21	2.20	2.18	2.19	2.20	2.21	2.22	2.24	2.24	2.23	2.25	2.26	2.26	2.28	2.30	2.31
	26	2.33	2.22	2.25	2.22	2.20	2.17	2.18	2.19	2.18	2.16	2.15	2.16	2.16	2.16	2.16	2.17	2.18	2.19	2.21	2.21
	27	2.30	2.19	2.10	2.11	2.09	2.05	2.05	2.05	2.04	2.05	2.05	2.04	2.04	2.03	2.03	2.04	2.04	2.05	2.04	2.06
	28	2.40	2.02	1.92	1.88	1.89	1.90	1.89	1.89	1.89	1.90	1.90	1.90	1.90	1.91	1.91	1.91	1.91	1.92	1.93	1.93
	29	2.39	2.16	2.04	1.93	1.91	1.87	1.87	1.87	1.88	1.87	1.85	1.85	1.84	1.85	1.84	1.84	1.84	1.84	1.85	1.85
	30	2.36	1.97	1.91	1.89	1.87	1.84	1.82	1.81	1.79	1.78	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.76	1.76	1.76

Table B.19: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **point** surveying and **weighted** output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.58	1.47	1.58	1.64	1.70	1.79	2.00	2.21	2.52	2.88	3.28	3.70	4.14	4.49	4.84	5.16	5.42	5.66	5.90
	2	1.54	1.47	1.54	1.61	1.68	1.78	1.98	2.20	2.50	2.87	3.25	3.65	4.09	4.46	4.81	5.12	5.40	5.65	5.90
	3	1.50	1.45	1.53	1.62	1.68	1.79	1.96	2.17	2.48	2.83	3.22	3.63	4.08	4.48	4.82	5.12	5.39	5.64	5.90
	4	1.60	1.50	1.53	1.61	1.66	1.72	1.95	2.15	2.50	2.83	3.25	3.68	4.08	4.48	4.83	5.14	5.41	5.66	5.90
	5	1.60	1.47	1.51	1.58	1.67	1.76	1.95	2.16	2.47	2.78	3.19	3.61	4.02	4.43	4.79	5.12	5.39	5.65	5.90
	6	1.59	1.43	1.51	1.57	1.63	1.74	1.90	2.14	2.46	2.82	3.22	3.60	4.02	4.41	4.73	5.08	5.37	5.65	5.90
	7	1.63	1.53	1.52	1.57	1.66	1.79	1.98	2.21	2.47	2.79	3.15	3.56	3.99	4.41	4.78	5.09	5.39	5.65	5.90
	8	1.64	1.55	1.61	1.58	1.70	1.82	1.97	2.17	2.49	2.77	3.14	3.58	4.02	4.45	4.83	5.13	5.42	5.68	5.90
	9	1.62	1.57	1.60	1.62	1.67	1.79	1.92	2.14	2.44	2.78	3.12	3.54	3.96	4.40	4.76	5.08	5.38	5.67	5.90
	10	1.64	1.48	1.55	1.61	1.67	1.85	2.05	2.22	2.48	2.74	3.12	3.56	4.00	4.39	4.73	5.06	5.37	5.67	5.90
	11	1.56	1.48	1.57	1.63	1.71	1.76	1.90	2.09	2.45	2.80	3.13	3.49	3.87	4.25	4.66	4.97	5.27	5.59	5.90
	12	1.79	1.55	1.56	1.62	1.71	1.82	1.97	2.15	2.42	2.72	3.09	3.55	4.03	4.44	4.80	5.14	5.41	5.69	5.90
	13	1.63	1.62	1.64	1.68	1.71	1.77	2.01	2.21	2.53	2.80	3.11	3.47	3.81	4.17	4.62	5.00	5.32	5.61	5.90
	14	1.78	1.65	1.70	1.79	1.82	1.84	1.96	2.13	2.38	2.72	3.15	3.54	3.97	4.33	4.72	5.07	5.38	5.65	5.90
b	15	1.70	1.58	1.62	1.67	1.76	1.82	1.91	2.07	2.41	2.75	3.13	3.61	4.08	4.47	4.88	5.19	5.44	5.68	5.90
	16	1.69	1.55	1.56	1.66	1.78	1.85	2.07	2.25	2.49	2.72	2.92	3.28	3.71	4.13	4.54	4.94	5.28	5.63	5.90
	17	1.85	1.67	1.67	1.68	1.76	1.87	2.09	2.31	2.59	2.80	3.00	3.27	3.63	4.04	4.44	4.81	5.19	5.56	5.90
	18	2.05	1.87	1.86	1.81	1.82	1.84	1.97	2.21	2.58	2.94	3.33	3.73	4.10	4.45	4.77	5.11	5.40	5.65	5.90
	19	1.84	1.84	1.86	1.82	1.82	1.88	2.02	2.22	2.56	2.99	3.41	3.87	4.23	4.62	4.91	5.22	5.44	5.70	5.90
	20	1.67	1.64	1.71	1.71	1.72	1.79	1.88	2.04	2.32	2.63	3.03	3.50	3.94	4.40	4.80	5.15	5.41	5.64	5.90
	21	1.68	1.64	1.71	1.74	1.77	1.82	1.90	2.04	2.27	2.47	2.78	3.19	3.63	4.09	4.50	4.91	5.26	5.60	5.90
	22	1.82	1.74	1.83	1.95	1.94	1.98	2.15	2.28	2.41	2.65	2.94	3.16	3.54	3.96	4.37	4.80	5.20	5.54	5.90
	23	2.00	1.78	1.87	2.01	2.07	2.12	2.28	2.41	2.62	2.85	3.01	3.23	3.55	4.00	4.41	4.81	5.20	5.55	5.90
	24	2.33	2.12	2.09	2.14	2.21	2.32	2.55	2.74	2.92	3.15	3.47	3.78	4.12	4.44	4.78	5.07	5.37	5.65	5.90
	25	2.44	2.28	2.18	2.26	2.34	2.47	2.66	2.95	3.27	3.58	3.83	4.12	4.45	4.75	4.96	5.20	5.44	5.66	5.90
	26	2.41	2.12	2.18	2.08	2.23	2.37	2.52	2.71	2.89	3.32	3.64	4.00	4.31	4.59	4.95	5.26	5.48	5.68	5.90
	27	2.31	2.14	2.07	2.10	2.10	2.21	2.38	2.51	2.80	3.14	3.49	3.89	4.19	4.54	4.86	5.15	5.39	5.67	5.90
	28	2.21	2.07	2.08	2.06	2.09	2.14	2.24	2.35	2.64	2.94	3.34	3.78	4.14	4.48	4.84	5.16	5.38	5.67	5.90
	29	2.05	2.06	1.94	1.96	1.96	1.99	2.02	2.09	2.38	2.74	3.18	3.54	3.92	4.37	4.75	5.01	5.34	5.63	5.90
	30	2.00	1.82	1.84	1.79	1.80	1.85	1.90	1.99	2.24	2.55	2.93	3.38	3.78	4.26	4.69	5.02	5.31	5.59	5.90

Table B.20: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **sector** surveying and **arithmetic** output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.58	1.47	1.57	1.62	1.68	1.76	1.94	2.13	2.41	2.74	3.11	3.50	3.91	4.24	4.58	4.88	5.13	5.37	5.60
	2	1.54	1.47	1.53	1.60	1.65	1.74	1.91	2.11	2.37	2.70	3.04	3.41	3.81	4.16	4.48	4.78	5.05	5.29	5.53
	3	1.50	1.44	1.52	1.59	1.65	1.75	1.88	2.07	2.33	2.63	2.97	3.34	3.74	4.10	4.42	4.70	4.96	5.20	5.44
	4	1.60	1.49	1.52	1.58	1.63	1.67	1.86	2.03	2.31	2.59	2.94	3.31	3.67	4.03	4.35	4.64	4.89	5.13	5.35
	5	1.60	1.47	1.49	1.55	1.63	1.69	1.85	2.02	2.26	2.52	2.86	3.21	3.56	3.92	4.25	4.54	4.79	5.04	5.27
	6	1.59	1.42	1.49	1.53	1.59	1.67	1.79	1.97	2.22	2.50	2.82	3.14	3.49	3.81	4.10	4.40	4.67	4.93	5.16
	7	1.63	1.52	1.50	1.54	1.61	1.71	1.84	2.02	2.22	2.46	2.74	3.06	3.41	3.75	4.07	4.35	4.61	4.86	5.08
	8	1.64	1.54	1.58	1.55	1.65	1.74	1.85	2.00	2.22	2.41	2.68	3.01	3.35	3.70	4.02	4.28	4.53	4.77	4.97
	9	1.62	1.55	1.57	1.58	1.62	1.72	1.80	1.95	2.14	2.37	2.60	2.90	3.22	3.55	3.85	4.12	4.38	4.63	4.83
	10	1.64	1.47	1.51	1.56	1.61	1.73	1.87	1.98	2.16	2.36	2.62	2.93	3.25	3.56	3.83	4.10	4.36	4.61	4.81
	11	1.56	1.46	1.53	1.57	1.62	1.65	1.73	1.85	2.06	2.29	2.50	2.74	3.01	3.28	3.58	3.83	4.07	4.34	4.59
	12	1.79	1.54	1.53	1.58	1.64	1.72	1.81	1.91	2.06	2.24	2.48	2.76	3.08	3.37	3.63	3.90	4.12	4.34	4.52
	13	1.63	1.59	1.60	1.62	1.63	1.66	1.80	1.92	2.12	2.28	2.47	2.69	2.91	3.16	3.47	3.74	3.99	4.21	4.43
	14	1.78	1.64	1.66	1.74	1.76	1.76	1.83	1.92	2.01	2.19	2.43	2.66	2.92	3.16	3.41	3.67	3.89	4.11	4.30
b	15	1.70	1.56	1.58	1.62	1.67	1.71	1.75	1.83	2.00	2.18	2.38	2.65	2.93	3.18	3.44	3.66	3.85	4.03	4.20
	16	1.69	1.54	1.52	1.58	1.66	1.71	1.83	1.94	2.08	2.21	2.33	2.54	2.76	2.99	3.24	3.49	3.72	3.95	4.14
	17	1.85	1.65	1.62	1.62	1.66	1.73	1.86	1.97	2.13	2.26	2.35	2.48	2.67	2.89	3.12	3.35	3.60	3.85	4.08
	18	2.05	1.86	1.83	1.79	1.78	1.78	1.85	1.95	2.11	2.28	2.45	2.65	2.85	3.06	3.26	3.48	3.68	3.86	4.04
	19	1.84	1.82	1.83	1.79	1.78	1.82	1.88	1.98	2.11	2.28	2.48	2.70	2.88	3.09	3.26	3.45	3.60	3.77	3.90
	20	1.67	1.62	1.67	1.66	1.66	1.71	1.75	1.83	1.95	2.07	2.23	2.42	2.61	2.82	3.02	3.20	3.36	3.50	3.66
	21	1.68	1.61	1.66	1.68	1.68	1.73	1.77	1.83	1.89	1.97	2.08	2.23	2.39	2.56	2.73	2.93	3.11	3.30	3.46
	22	1.82	1.72	1.78	1.87	1.84	1.85	1.94	2.01	2.06	2.17	2.30	2.38	2.52	2.71	2.90	3.11	3.32	3.51	3.70
	23	2.00	1.76	1.81	1.90	1.94	1.97	2.07	2.13	2.25	2.38	2.46	2.58	2.75	2.98	3.20	3.43	3.64	3.84	4.05
	24	2.33	2.10	2.04	2.06	2.11	2.17	2.29	2.42	2.54	2.69	2.87	3.06	3.27	3.47	3.68	3.87	4.07	4.25	4.41
	25	2.44	2.26	2.14	2.18	2.20	2.28	2.39	2.55	2.74	2.91	3.07	3.24	3.44	3.63	3.77	3.93	4.10	4.26	4.43
	26	2.41	2.12	2.14	2.03	2.10	2.19	2.27	2.37	2.49	2.69	2.85	3.02	3.20	3.35	3.55	3.72	3.85	3.98	4.11
	27	2.31	2.14	2.06	2.07	2.06	2.12	2.20	2.24	2.30	2.39	2.49	2.60	2.70	2.82	2.95	3.07	3.18	3.32	3.44
	28	2.21	2.06	2.06	2.05	2.06	2.10	2.13	2.13	2.14	2.16	2.19	2.23	2.29	2.36	2.44	2.52	2.60	2.70	2.80
	29	2.05	2.03	1.94	1.95	1.95	1.96	1.95	1.93	1.94	1.96	1.99	2.02	2.07	2.14	2.21	2.27	2.37	2.45	2.55
	30	2.00	1.81	1.82	1.78	1.78	1.80	1.80	1.81	1.81	1.83	1.88	1.93	1.98	2.06	2.15	2.24	2.33	2.43	2.54

Table B.21: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **sector** surveying and **weighted** output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	1.75	1.57	1.47	1.44	1.40	1.41	1.41	1.41	1.43	1.43	1.45	1.44	1.46	1.46	1.49	1.51	1.54	1.56	1.58	1.58
	2	1.76	1.60	1.49	1.45	1.42	1.42	1.43	1.43	1.43	1.43	1.44	1.45	1.46	1.46	1.50	1.52	1.53	1.55	1.56	1.57
	3	1.79	1.58	1.49	1.45	1.43	1.43	1.43	1.44	1.44	1.44	1.45	1.45	1.46	1.48	1.50	1.52	1.54	1.56	1.57	1.58
	4	1.81	1.55	1.48	1.47	1.44	1.44	1.44	1.44	1.44	1.45	1.47	1.48	1.48	1.50	1.52	1.54	1.55	1.57	1.57	1.59
	5	1.86	1.59	1.53	1.51	1.49	1.48	1.47	1.47	1.47	1.47	1.47	1.48	1.48	1.49	1.51	1.52	1.55	1.57	1.59	1.60
	6	1.73	1.50	1.48	1.44	1.46	1.45	1.47	1.47	1.48	1.46	1.46	1.48	1.50	1.50	1.51	1.53	1.54	1.55	1.56	1.59
	7	1.77	1.56	1.47	1.45	1.45	1.45	1.45	1.46	1.46	1.48	1.48	1.50	1.51	1.53	1.54	1.56	1.58	1.60	1.60	1.62
	8	1.86	1.62	1.54	1.50	1.49	1.47	1.48	1.49	1.51	1.52	1.53	1.53	1.55	1.56	1.57	1.58	1.59	1.61	1.63	1.64
	9	1.95	1.64	1.58	1.53	1.51	1.47	1.47	1.49	1.50	1.51	1.53	1.55	1.55	1.58	1.60	1.60	1.62	1.62	1.63	1.63
	10	1.75	1.59	1.49	1.49	1.48	1.49	1.50	1.51	1.52	1.53	1.53	1.55	1.55	1.57	1.57	1.59	1.62	1.63	1.65	1.65
	11	1.94	1.73	1.64	1.57	1.57	1.56	1.55	1.57	1.57	1.58	1.59	1.59	1.59	1.60	1.62	1.61	1.63	1.64	1.65	1.67
	12	1.88	1.65	1.60	1.58	1.56	1.57	1.58	1.59	1.59	1.59	1.59	1.59	1.58	1.60	1.61	1.62	1.64	1.66	1.67	1.68
	13	1.83	1.69	1.60	1.57	1.53	1.52	1.54	1.56	1.54	1.55	1.55	1.58	1.58	1.59	1.59	1.59	1.61	1.63	1.65	1.67
	14	2.00	1.81	1.75	1.74	1.74	1.71	1.68	1.69	1.69	1.69	1.70	1.71	1.72	1.74	1.75	1.75	1.75	1.77	1.78	1.78
b	15	1.95	1.74	1.69	1.66	1.62	1.64	1.65	1.64	1.62	1.64	1.66	1.66	1.68	1.67	1.68	1.68	1.69	1.70	1.71	1.72
	16	1.78	1.62	1.57	1.56	1.56	1.55	1.56	1.58	1.55	1.58	1.58	1.59	1.58	1.60	1.62	1.61	1.63	1.64	1.66	1.68
	17	1.92	1.78	1.73	1.74	1.72	1.70	1.70	1.70	1.70	1.69	1.69	1.69	1.69	1.67	1.66	1.69	1.71	1.72	1.74	1.76
	18	2.49	2.12	2.02	1.97	1.92	1.90	1.88	1.88	1.86	1.86	1.84	1.84	1.84	1.83	1.84	1.83	1.84	1.84	1.85	1.85
	19	2.02	1.87	1.84	1.81	1.79	1.79	1.80	1.78	1.80	1.82	1.82	1.81	1.81	1.80	1.80	1.81	1.82	1.83	1.83	1.83
	20	1.94	1.78	1.75	1.72	1.71	1.69	1.69	1.71	1.74	1.74	1.75	1.76	1.75	1.74	1.73	1.74	1.74	1.76	1.77	1.78
	21	2.00	1.77	1.67	1.66	1.64	1.61	1.61	1.61	1.61	1.64	1.66	1.68	1.69	1.68	1.69	1.70	1.72	1.74	1.76	1.77
	22	2.14	1.81	1.83	1.87	1.87	1.85	1.82	1.82	1.82	1.82	1.82	1.81	1.81	1.82	1.83	1.85	1.86	1.86	1.89	1.91
	23	2.09	2.00	2.01	1.95	2.00	2.04	2.06	2.03	1.97	1.99	1.99	1.99	1.97	1.97	2.00	2.00	2.01	2.02	2.02	2.03
	24	2.36	2.17	2.08	1.99	2.00	1.97	1.93	1.95	1.97	2.01	1.99	2.02	2.04	2.07	2.10	2.11	2.14	2.13	2.16	2.17
	25	2.51	2.26	2.21	2.22	2.21	2.21	2.14	2.18	2.17	2.19	2.19	2.19	2.24	2.22	2.23	2.21	2.22	2.25	2.27	2.29
	26	2.28	2.28	2.31	2.23	2.17	2.13	2.13	2.16	2.17	2.14	2.11	2.10	2.07	2.07	2.09	2.11	2.12	2.13	2.13	2.17
	27	2.36	2.23	2.10	2.08	2.07	2.04	2.04	2.05	2.05	2.07	2.05	2.04	2.04	2.03	2.05	2.05	2.06	2.08	2.09	2.11
	28	2.45	2.08	1.99	1.93	1.94	1.96	1.96	1.97	1.96	1.95	1.97	1.99	1.99	2.02	2.01	2.01	2.02	2.04	2.05	2.05
	29	2.43	2.18	2.05	1.97	1.95	1.92	1.92	1.92	1.93	1.91	1.89	1.88	1.86	1.87	1.86	1.86	1.86	1.88	1.90	1.89
	30	2.40	1.99	1.92	1.93	1.91	1.87	1.87	1.87	1.84	1.83	1.83	1.81	1.79	1.79	1.78	1.79	1.79	1.78	1.79	1.80

Table B.22: *Mean distance errors (m) using the* ***Match Distribution Distance*** *algorithm,* ***point*** *surveying and* ***arithmetic*** *output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	1.75	1.57	1.46	1.44	1.40	1.40	1.40	1.40	1.42	1.42	1.43	1.43	1.44	1.45	1.48	1.49	1.51	1.53	1.56	1.56
	2	1.76	1.60	1.49	1.44	1.41	1.41	1.42	1.41	1.41	1.42	1.42	1.44	1.45	1.45	1.47	1.49	1.50	1.52	1.53	1.53
	3	1.79	1.58	1.49	1.44	1.42	1.42	1.42	1.42	1.42	1.42	1.43	1.44	1.44	1.45	1.47	1.49	1.51	1.52	1.53	1.54
	4	1.81	1.55	1.47	1.45	1.42	1.42	1.42	1.42	1.42	1.43	1.44	1.45	1.46	1.47	1.48	1.50	1.51	1.52	1.53	1.54
	5	1.86	1.59	1.52	1.50	1.48	1.47	1.46	1.45	1.45	1.45	1.46	1.46	1.46	1.47	1.48	1.49	1.51	1.53	1.54	1.55
	6	1.73	1.50	1.46	1.42	1.43	1.42	1.44	1.44	1.45	1.44	1.44	1.45	1.47	1.47	1.48	1.49	1.50	1.51	1.52	1.53
	7	1.77	1.55	1.46	1.44	1.43	1.43	1.43	1.44	1.44	1.45	1.45	1.46	1.47	1.49	1.49	1.51	1.53	1.54	1.55	1.56
	8	1.86	1.61	1.53	1.48	1.46	1.44	1.45	1.46	1.48	1.48	1.50	1.50	1.51	1.52	1.53	1.54	1.55	1.56	1.57	1.58
	9	1.95	1.65	1.57	1.52	1.50	1.46	1.46	1.48	1.48	1.49	1.49	1.51	1.52	1.54	1.55	1.55	1.56	1.57	1.58	1.58
	10	1.75	1.57	1.47	1.46	1.45	1.46	1.47	1.47	1.48	1.49	1.49	1.51	1.50	1.52	1.52	1.53	1.55	1.56	1.57	1.58
	11	1.94	1.74	1.65	1.58	1.56	1.55	1.54	1.55	1.55	1.56	1.56	1.57	1.57	1.57	1.58	1.58	1.59	1.59	1.60	1.61
	12	1.88	1.65	1.59	1.56	1.54	1.54	1.55	1.56	1.56	1.56	1.56	1.56	1.55	1.56	1.57	1.58	1.59	1.60	1.61	1.62
	13	1.83	1.67	1.58	1.56	1.53	1.52	1.51	1.52	1.51	1.51	1.52	1.53	1.53	1.54	1.54	1.54	1.55	1.55	1.56	1.58
	14	2.00	1.79	1.73	1.72	1.71	1.68	1.66	1.65	1.65	1.65	1.66	1.66	1.67	1.68	1.69	1.69	1.69	1.70	1.71	1.71
b	15	1.95	1.73	1.67	1.64	1.61	1.61	1.62	1.62	1.60	1.61	1.62	1.62	1.63	1.62	1.63	1.63	1.64	1.65	1.65	1.66
	16	1.78	1.60	1.54	1.53	1.52	1.52	1.54	1.54	1.52	1.53	1.54	1.54	1.53	1.54	1.55	1.54	1.55	1.56	1.56	1.58
	17	1.92	1.75	1.69	1.69	1.67	1.64	1.64	1.64	1.64	1.64	1.63	1.63	1.62	1.61	1.61	1.62	1.64	1.65	1.65	1.66
	18	2.49	2.12	2.01	1.96	1.92	1.90	1.88	1.88	1.86	1.85	1.83	1.83	1.83	1.82	1.82	1.82	1.82	1.82	1.82	1.82
	19	2.02	1.82	1.79	1.75	1.73	1.73	1.74	1.72	1.72	1.73	1.73	1.72	1.72	1.72	1.73	1.73	1.74	1.74	1.74	1.75
	20	1.94	1.75	1.70	1.69	1.67	1.66	1.65	1.66	1.67	1.68	1.68	1.69	1.68	1.67	1.66	1.67	1.67	1.68	1.69	1.69
	21	2.00	1.76	1.64	1.64	1.61	1.59	1.59	1.59	1.58	1.60	1.60	1.62	1.62	1.62	1.62	1.63	1.64	1.65	1.66	1.66
	22	2.14	1.83	1.82	1.80	1.81	1.79	1.78	1.77	1.78	1.78	1.78	1.78	1.78	1.78	1.79	1.79	1.80	1.80	1.81	1.83
	23	2.09	1.94	1.91	1.87	1.87	1.88	1.89	1.88	1.85	1.86	1.85	1.85	1.85	1.84	1.86	1.86	1.86	1.86	1.86	1.87
	24	2.36	2.12	2.02	1.95	1.95	1.92	1.91	1.91	1.92	1.93	1.91	1.91	1.92	1.94	1.95	1.95	1.96	1.96	1.97	1.98
	25	2.51	2.25	2.19	2.16	2.13	2.14	2.09	2.11	2.11	2.09	2.09	2.10	2.10	2.09	2.10	2.08	2.09	2.10	2.10	2.10
	26	2.28	2.23	2.23	2.19	2.16	2.13	2.10	2.11	2.09	2.07	2.07	2.05	2.04	2.04	2.05	2.05	2.06	2.06	2.06	2.07
	27	2.36	2.25	2.13	2.10	2.08	2.05	2.04	2.04	2.04	2.04	2.04	2.03	2.02	2.01	2.02	2.02	2.03	2.03	2.03	2.04
	28	2.45	2.06	1.95	1.90	1.90	1.93	1.92	1.92	1.91	1.91	1.90	1.91	1.91	1.92	1.92	1.92	1.92	1.92	1.93	1.93
	29	2.43	2.17	2.04	1.94	1.93	1.90	1.90	1.90	1.90	1.89	1.87	1.85	1.85	1.85	1.84	1.83	1.83	1.83	1.83	1.83
	30	2.40	2.01	1.93	1.92	1.90	1.86	1.85	1.84	1.82	1.81	1.81	1.80	1.80	1.79	1.78	1.79	1.78	1.78	1.77	1.77

Table B.23: *Mean distance errors (m) using the* ***Match Distribution Distance*** *algorithm,* ***point*** *surveying and* ***weighted*** *output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.43	1.32	1.41	1.47	1.56	1.63	1.73	1.85	2.05	2.21	2.42	2.78	3.29	3.81	4.28	4.68	5.07	5.48	5.90
	2	1.45	1.31	1.41	1.48	1.57	1.63	1.73	1.86	2.04	2.20	2.44	2.81	3.29	3.81	4.28	4.68	5.07	5.47	5.90
	3	1.44	1.31	1.45	1.48	1.57	1.62	1.71	1.86	2.04	2.20	2.43	2.80	3.30	3.82	4.28	4.68	5.06	5.46	5.90
	4	1.48	1.32	1.44	1.50	1.57	1.63	1.74	1.85	2.04	2.21	2.43	2.79	3.30	3.80	4.27	4.69	5.08	5.48	5.90
	5	1.44	1.32	1.42	1.51	1.58	1.67	1.73	1.85	2.03	2.21	2.45	2.81	3.31	3.81	4.28	4.68	5.07	5.47	5.90
	6	1.49	1.32	1.42	1.50	1.59	1.64	1.74	1.87	2.05	2.23	2.43	2.79	3.29	3.81	4.28	4.68	5.04	5.45	5.90
	7	1.49	1.34	1.46	1.53	1.60	1.65	1.73	1.88	2.07	2.22	2.46	2.82	3.34	3.82	4.29	4.67	5.05	5.46	5.90
	8	1.48	1.38	1.49	1.57	1.62	1.69	1.77	1.89	2.06	2.21	2.47	2.84	3.35	3.82	4.29	4.68	5.05	5.47	5.90
	9	1.48	1.43	1.50	1.57	1.61	1.67	1.78	1.90	2.05	2.21	2.48	2.83	3.29	3.82	4.29	4.65	5.02	5.45	5.90
	10	1.58	1.41	1.49	1.56	1.62	1.69	1.78	1.93	2.09	2.22	2.45	2.82	3.32	3.81	4.27	4.68	5.04	5.45	5.90
	11	1.51	1.37	1.53	1.58	1.63	1.70	1.78	1.90	2.11	2.28	2.54	2.86	3.34	3.82	4.31	4.73	5.11	5.52	5.90
	12	1.65	1.48	1.53	1.61	1.63	1.72	1.81	1.92	2.06	2.21	2.42	2.81	3.34	3.86	4.30	4.69	5.04	5.44	5.90
	13	1.47	1.43	1.49	1.54	1.62	1.68	1.70	1.87	2.12	2.31	2.56	2.87	3.31	3.78	4.28	4.68	5.09	5.48	5.90
	14	1.69	1.62	1.65	1.67	1.73	1.78	1.84	1.97	2.12	2.29	2.52	2.94	3.44	3.88	4.31	4.67	5.05	5.46	5.90
b	15	1.66	1.57	1.59	1.64	1.70	1.75	1.82	1.91	2.02	2.28	2.50	2.81	3.34	3.86	4.29	4.70	5.07	5.52	5.90
	16	1.66	1.48	1.48	1.57	1.67	1.71	1.79	1.89	2.13	2.40	2.58	2.96	3.37	3.82	4.30	4.74	5.13	5.52	5.90
	17	1.79	1.61	1.61	1.64	1.72	1.77	1.87	2.03	2.24	2.43	2.62	2.93	3.35	3.86	4.32	4.67	5.06	5.47	5.90
	18	2.02	1.78	1.81	1.79	1.81	1.83	1.89	2.00	2.19	2.37	2.59	3.00	3.49	3.90	4.26	4.66	5.04	5.44	5.90
	19	1.84	1.80	1.85	1.81	1.81	1.88	1.91	2.04	2.17	2.45	2.79	3.11	3.48	3.98	4.38	4.76	5.16	5.54	5.90
	20	1.67	1.65	1.75	1.74	1.74	1.78	1.82	1.94	2.14	2.29	2.59	3.08	3.55	3.94	4.44	4.87	5.20	5.55	5.90
	21	1.66	1.61	1.70	1.68	1.75	1.78	1.84	1.96	2.20	2.39	2.70	2.98	3.45	3.95	4.44	4.86	5.25	5.56	5.90
	22	1.80	1.69	1.80	1.89	1.93	1.94	2.05	2.14	2.31	2.55	2.79	3.09	3.49	3.91	4.38	4.76	5.15	5.50	5.90
	23	1.88	1.75	1.86	2.00	1.99	2.06	2.12	2.27	2.49	2.68	2.80	3.10	3.47	3.99	4.39	4.74	5.09	5.50	5.90
	24	2.22	2.02	1.99	2.03	2.08	2.16	2.33	2.36	2.55	2.71	2.86	3.19	3.69	4.15	4.53	4.81	5.13	5.50	5.90
	25	2.37	2.13	2.06	2.05	2.15	2.17	2.24	2.36	2.52	2.72	3.13	3.42	3.86	4.19	4.50	4.89	5.17	5.51	5.90
	26	2.23	2.03	2.09	1.96	1.99	2.09	2.26	2.29	2.50	2.85	3.25	3.50	3.81	4.13	4.53	4.90	5.17	5.48	5.90
	27	2.33	2.20	2.08	2.10	2.09	2.13	2.25	2.39	2.64	2.96	3.10	3.53	3.93	4.18	4.61	4.99	5.23	5.59	5.90
	28	2.27	2.14	2.11	2.08	2.05	2.09	2.19	2.28	2.55	2.86	3.27	3.70	3.97	4.43	4.79	5.10	5.37	5.66	5.90
	29	2.10	2.10	1.94	1.98	1.98	2.01	2.03	2.09	2.38	2.71	3.11	3.51	3.89	4.36	4.75	5.01	5.35	5.63	5.90
	30	2.04	1.88	1.90	1.84	1.85	1.89	1.93	1.99	2.26	2.55	2.93	3.36	3.77	4.28	4.69	5.02	5.32	5.60	5.90

Table B.24: *Mean distance errors (m) using the* ***Match Distribution Distance*** *algorithm,* ***sector*** *surveying and* ***arithmetic*** *output averaging in the house environment.*

		k																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	1	1.43	1.31	1.38	1.44	1.52	1.57	1.65	1.73	1.85	1.95	2.07	2.27	2.55	2.86	3.18	3.47	3.75	4.04	4.35
	2	1.45	1.30	1.38	1.44	1.51	1.56	1.63	1.71	1.80	1.88	2.00	2.17	2.40	2.67	2.94	3.20	3.45	3.72	4.01
	3	1.44	1.30	1.41	1.44	1.50	1.55	1.61	1.70	1.78	1.85	1.95	2.11	2.33	2.59	2.85	3.09	3.32	3.58	3.86
	4	1.48	1.31	1.40	1.45	1.51	1.55	1.62	1.68	1.78	1.84	1.94	2.08	2.29	2.53	2.79	3.03	3.26	3.52	3.78
	5	1.44	1.31	1.38	1.46	1.51	1.58	1.61	1.68	1.76	1.83	1.94	2.08	2.28	2.51	2.76	2.99	3.22	3.46	3.72
	6	1.49	1.31	1.38	1.45	1.52	1.55	1.62	1.68	1.77	1.84	1.92	2.06	2.26	2.49	2.74	2.97	3.18	3.43	3.69
	7	1.49	1.33	1.42	1.47	1.53	1.56	1.61	1.69	1.78	1.84	1.93	2.07	2.27	2.48	2.72	2.94	3.15	3.39	3.66
	8	1.48	1.37	1.45	1.51	1.55	1.60	1.65	1.70	1.78	1.84	1.94	2.07	2.27	2.48	2.72	2.94	3.15	3.39	3.65
	9	1.48	1.41	1.46	1.52	1.54	1.58	1.66	1.72	1.78	1.83	1.93	2.05	2.23	2.46	2.68	2.89	3.10	3.34	3.60
	10	1.58	1.40	1.44	1.50	1.54	1.59	1.65	1.73	1.79	1.84	1.92	2.06	2.26	2.48	2.71	2.94	3.14	3.38	3.64
	11	1.51	1.36	1.48	1.52	1.55	1.60	1.65	1.70	1.79	1.86	1.96	2.07	2.24	2.44	2.68	2.91	3.12	3.35	3.58
	12	1.65	1.46	1.48	1.55	1.56	1.63	1.69	1.73	1.78	1.83	1.90	2.05	2.25	2.47	2.70	2.91	3.11	3.34	3.60
	13	1.47	1.41	1.45	1.48	1.54	1.58	1.59	1.67	1.78	1.87	1.97	2.09	2.26	2.45	2.69	2.92	3.14	3.37	3.60
	14	1.69	1.60	1.61	1.62	1.66	1.70	1.72	1.78	1.83	1.89	1.96	2.10	2.28	2.46	2.66	2.85	3.05	3.27	3.50
b	15	1.66	1.55	1.55	1.59	1.62	1.65	1.70	1.73	1.76	1.85	1.93	2.03	2.23	2.45	2.66	2.87	3.08	3.33	3.55
	16	1.66	1.46	1.44	1.50	1.57	1.61	1.64	1.68	1.79	1.91	1.99	2.14	2.30	2.48	2.71	2.94	3.15	3.37	3.58
	17	1.79	1.60	1.58	1.58	1.63	1.67	1.71	1.80	1.88	1.97	2.04	2.16	2.31	2.51	2.72	2.91	3.11	3.33	3.56
	18	2.02	1.78	1.78	1.76	1.77	1.77	1.80	1.84	1.90	1.97	2.04	2.17	2.34	2.51	2.69	2.88	3.08	3.29	3.53
	19	1.84	1.78	1.82	1.78	1.78	1.81	1.82	1.87	1.90	1.99	2.09	2.18	2.31	2.52	2.71	2.90	3.11	3.31	3.52
	20	1.67	1.63	1.70	1.69	1.68	1.71	1.72	1.78	1.81	1.85	1.95	2.10	2.25	2.40	2.61	2.81	2.99	3.18	3.37
	21	1.66	1.58	1.65	1.63	1.67	1.70	1.73	1.77	1.84	1.88	2.00	2.07	2.20	2.37	2.56	2.75	2.94	3.11	3.28
	22	1.80	1.67	1.75	1.81	1.83	1.83	1.87	1.91	1.98	2.07	2.17	2.28	2.42	2.57	2.75	2.93	3.12	3.29	3.49
	23	1.88	1.74	1.80	1.89	1.88	1.93	1.96	2.04	2.15	2.24	2.30	2.43	2.58	2.81	3.00	3.19	3.37	3.57	3.77
	24	2.22	2.00	1.94	1.95	1.98	2.05	2.14	2.15	2.26	2.35	2.43	2.59	2.84	3.09	3.30	3.48	3.65	3.86	4.08
	25	2.37	2.12	2.03	1.99	2.04	2.05	2.10	2.14	2.22	2.33	2.51	2.65	2.90	3.09	3.29	3.53	3.70	3.91	4.13
	26	2.23	2.02	2.05	1.92	1.91	1.97	2.06	2.08	2.17	2.33	2.47	2.58	2.73	2.90	3.12	3.32	3.47	3.64	3.88
	27	2.33	2.19	2.08	2.08	2.06	2.07	2.11	2.16	2.17	2.26	2.28	2.37	2.49	2.58	2.73	2.89	3.01	3.18	3.34
	28	2.27	2.13	2.08	2.07	2.03	2.05	2.08	2.08	2.09	2.09	2.13	2.18	2.23	2.32	2.42	2.51	2.61	2.72	2.83
	29	2.10	2.08	1.94	1.97	1.97	1.98	1.95	1.94	1.94	1.94	1.97	2.00	2.05	2.14	2.22	2.29	2.39	2.48	2.58
	30	2.04	1.86	1.88	1.83	1.84	1.84	1.84	1.84	1.83	1.85	1.89	1.92	1.98	2.08	2.18	2.26	2.36	2.46	2.58

Table B.25: *Mean distance errors (m) using the* ***Match Distribution Distance*** *algorithm,* ***sector*** *surveying and* ***weighted*** *output averaging in the house environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	3.48	3.13	3.02	2.99	2.95	2.93	2.93	2.96	2.96	2.95	2.98	3.00	3.04	3.08	3.11	3.14	3.16	3.20	3.23	3.28
	2	3.61	3.17	2.99	2.99	2.96	2.94	2.91	2.91	2.93	2.97	2.96	2.97	2.99	3.01	3.05	3.06	3.10	3.12	3.15	3.17
	3	3.59	3.20	3.01	2.96	2.93	2.93	2.95	2.93	2.94	2.95	2.95	2.97	2.98	3.02	3.04	3.07	3.10	3.14	3.16	3.19
	4	3.58	3.23	3.02	2.99	2.95	2.93	2.93	2.93	2.92	2.94	2.97	2.97	3.01	3.03	3.05	3.10	3.12	3.15	3.17	3.21
p	5	3.57	3.21	3.05	3.00	2.96	2.93	2.95	2.94	2.92	2.93	2.96	2.99	3.02	3.06	3.08	3.11	3.13	3.16	3.19	3.22
	6	3.57	3.23	3.07	2.99	2.95	2.96	2.95	2.95	2.94	2.93	2.97	2.99	3.03	3.07	3.09	3.12	3.14	3.16	3.20	3.24
	7	3.59	3.20	3.07	2.98	2.95	2.95	2.97	2.95	2.95	2.95	2.97	2.99	3.04	3.08	3.10	3.12	3.14	3.16	3.21	3.24
	8	3.58	3.21	3.07	2.98	2.96	2.95	2.96	2.96	2.96	2.96	2.98	3.00	3.04	3.08	3.10	3.13	3.15	3.18	3.22	3.26
	9	3.56	3.22	3.07	3.00	2.97	2.97	2.97	2.97	2.97	2.97	2.99	3.00	3.04	3.09	3.11	3.13	3.15	3.18	3.22	3.27
	10	3.57	3.21	3.08	2.98	2.97	2.98	2.99	2.97	2.97	2.98	2.99	3.00	3.04	3.09	3.11	3.13	3.16	3.19	3.22	3.27

Table B.26: *Mean distance errors (m) using the* ***Nearest Neighbour*** *algorithm,* ***point*** *surveying and* ***arithmetic*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	3.48	3.13	3.02	2.98	2.92	2.89	2.88	2.90	2.89	2.88	2.90	2.92	2.94	2.97	2.99	3.01	3.02	3.05	3.07	3.10
	2	3.61	3.17	2.99	2.96	2.93	2.90	2.86	2.86	2.87	2.90	2.89	2.90	2.91	2.92	2.95	2.96	2.99	3.00	3.02	3.03
	3	3.59	3.20	3.01	2.94	2.90	2.90	2.90	2.88	2.89	2.89	2.88	2.90	2.91	2.94	2.95	2.97	3.00	3.02	3.03	3.06
	4	3.58	3.23	3.01	2.97	2.93	2.89	2.88	2.88	2.87	2.89	2.90	2.91	2.93	2.95	2.96	2.99	3.01	3.03	3.04	3.07
p	5	3.57	3.21	3.04	2.98	2.93	2.89	2.90	2.89	2.87	2.88	2.90	2.92	2.94	2.97	2.99	3.01	3.02	3.04	3.06	3.09
	6	3.57	3.22	3.05	2.97	2.92	2.91	2.90	2.90	2.88	2.88	2.91	2.92	2.95	2.98	2.99	3.02	3.03	3.04	3.07	3.10
	7	3.59	3.19	3.06	2.96	2.92	2.91	2.92	2.90	2.90	2.89	2.91	2.92	2.96	2.99	3.00	3.02	3.03	3.05	3.08	3.11
	8	3.58	3.20	3.06	2.96	2.94	2.91	2.92	2.91	2.91	2.90	2.92	2.93	2.96	2.99	3.01	3.02	3.04	3.06	3.09	3.12
	9	3.56	3.21	3.06	2.98	2.93	2.92	2.92	2.92	2.92	2.91	2.93	2.93	2.96	3.00	3.01	3.03	3.04	3.06	3.09	3.13
	10	3.57	3.20	3.07	2.97	2.94	2.94	2.94	2.92	2.91	2.92	2.93	2.93	2.96	3.00	3.01	3.03	3.05	3.07	3.09	3.13

Table B.27: *Mean distance errors (m) using the **Nearest Neighbour** algorithm, **point** surveying and **weighted** output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
p	1	3.61	3.37	3.35	3.43	3.58	3.72	3.86	3.89	3.99	4.10	4.18	4.24	4.32	4.40	4.50	4.56	4.62	4.67	4.73	4.77
	2	3.62	3.38	3.33	3.37	3.44	3.56	3.68	3.75	3.81	3.90	3.96	4.03	4.12	4.18	4.26	4.35	4.42	4.49	4.54	4.61
	3	3.68	3.35	3.38	3.40	3.45	3.53	3.66	3.71	3.78	3.84	3.94	4.03	4.09	4.16	4.24	4.31	4.38	4.43	4.50	4.56
	4	3.77	3.42	3.37	3.42	3.48	3.59	3.67	3.72	3.78	3.85	3.93	4.02	4.11	4.17	4.25	4.32	4.37	4.43	4.49	4.57
	5	3.79	3.45	3.42	3.47	3.52	3.59	3.68	3.72	3.79	3.86	3.94	4.04	4.11	4.18	4.25	4.33	4.38	4.44	4.50	4.58
	6	3.77	3.45	3.45	3.48	3.52	3.60	3.68	3.73	3.81	3.88	3.95	4.03	4.11	4.18	4.24	4.33	4.39	4.46	4.52	4.58
	7	3.75	3.48	3.48	3.50	3.53	3.60	3.68	3.73	3.82	3.89	3.95	4.03	4.11	4.18	4.25	4.32	4.39	4.46	4.53	4.58
	8	3.76	3.48	3.50	3.52	3.53	3.61	3.68	3.75	3.82	3.89	3.95	4.04	4.11	4.19	4.26	4.32	4.38	4.47	4.54	4.59
	9	3.80	3.49	3.53	3.53	3.55	3.62	3.68	3.76	3.83	3.88	3.96	4.05	4.11	4.19	4.25	4.32	4.38	4.47	4.55	4.59
	10	3.81	3.50	3.54	3.53	3.55	3.62	3.68	3.75	3.84	3.89	3.96	4.05	4.12	4.19	4.25	4.32	4.40	4.48	4.55	4.60

Table B.28: *Mean distance errors (m) using the* ***Nearest Neighbour*** *algorithm,* ***sector*** *surveying and* ***arithmetic*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	3.61	3.35	3.31	3.34	3.42	3.50	3.59	3.62	3.69	3.78	3.84	3.89	3.96	4.03	4.10	4.15	4.20	4.24	4.29	4.33
	2	3.62	3.37	3.29	3.29	3.33	3.40	3.48	3.53	3.58	3.64	3.69	3.75	3.82	3.87	3.93	4.00	4.06	4.11	4.16	4.21
	3	3.68	3.34	3.34	3.33	3.35	3.40	3.49	3.52	3.57	3.62	3.69	3.76	3.81	3.87	3.93	3.99	4.04	4.08	4.14	4.18
	4	3.77	3.42	3.33	3.35	3.39	3.45	3.51	3.54	3.59	3.64	3.70	3.76	3.83	3.88	3.95	4.00	4.04	4.09	4.14	4.19
p	5	3.79	3.44	3.37	3.40	3.42	3.46	3.52	3.55	3.60	3.66	3.71	3.79	3.84	3.90	3.95	4.01	4.05	4.10	4.15	4.21
	6	3.77	3.44	3.40	3.41	3.43	3.47	3.53	3.57	3.62	3.68	3.73	3.79	3.85	3.90	3.95	4.01	4.06	4.12	4.17	4.21
	7	3.75	3.47	3.44	3.43	3.44	3.47	3.53	3.57	3.64	3.69	3.73	3.79	3.85	3.90	3.96	4.01	4.06	4.12	4.17	4.21
	8	3.76	3.47	3.45	3.45	3.44	3.49	3.54	3.58	3.64	3.69	3.74	3.80	3.86	3.91	3.97	4.01	4.06	4.13	4.18	4.22
	9	3.80	3.48	3.47	3.46	3.45	3.49	3.54	3.59	3.65	3.69	3.75	3.81	3.86	3.91	3.97	4.01	4.07	4.14	4.19	4.23
	10	3.81	3.48	3.48	3.46	3.46	3.50	3.54	3.60	3.65	3.70	3.75	3.81	3.87	3.92	3.96	4.02	4.08	4.14	4.19	4.23

Table B.29: *Mean distance errors (m) using the* ***Nearest Neighbour*** *algorithm,* ***sector*** *surveying and* ***weighted*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	4.87	4.30	4.25	4.24	4.23	4.21	4.26	4.30	4.36	4.40	4.47	4.50	4.50	4.53	4.60	4.64	4.66	4.71	4.76	4.77
	2	4.17	3.90	3.75	3.74	3.72	3.76	3.82	3.84	3.92	3.98	4.01	4.07	4.12	4.16	4.22	4.24	4.27	4.28	4.31	4.34
	3	4.05	3.65	3.56	3.57	3.58	3.59	3.64	3.65	3.74	3.77	3.76	3.82	3.88	3.91	3.94	3.97	4.01	4.04	4.09	4.12
	4	3.85	3.61	3.46	3.45	3.39	3.44	3.47	3.55	3.58	3.63	3.70	3.73	3.80	3.82	3.86	3.90	3.92	3.97	3.99	4.02
	5	3.80	3.57	3.50	3.37	3.37	3.38	3.38	3.39	3.42	3.49	3.54	3.58	3.61	3.66	3.73	3.75	3.79	3.83	3.86	3.89
	6	3.79	3.47	3.42	3.36	3.38	3.43	3.41	3.45	3.47	3.50	3.52	3.57	3.61	3.64	3.66	3.70	3.74	3.75	3.79	3.84
	7	3.81	3.56	3.52	3.43	3.41	3.45	3.47	3.48	3.53	3.51	3.56	3.57	3.62	3.65	3.70	3.71	3.75	3.81	3.84	3.87
	8	3.81	3.49	3.42	3.41	3.36	3.37	3.42	3.42	3.45	3.47	3.51	3.56	3.59	3.60	3.63	3.67	3.67	3.71	3.75	3.78
	9	3.98	3.69	3.56	3.55	3.55	3.53	3.51	3.50	3.49	3.54	3.58	3.61	3.62	3.63	3.66	3.71	3.73	3.77	3.81	3.85
	10	4.19	3.86	3.69	3.63	3.62	3.61	3.56	3.59	3.62	3.66	3.68	3.67	3.70	3.73	3.73	3.77	3.81	3.83	3.83	3.86
	11	4.20	4.03	3.81	3.63	3.64	3.62	3.67	3.67	3.65	3.67	3.69	3.70	3.73	3.75	3.78	3.82	3.86	3.88	3.89	3.92
	12	4.41	3.84	3.79	3.71	3.75	3.75	3.70	3.70	3.70	3.76	3.81	3.87	3.90	3.90	3.91	3.94	3.94	3.95	3.99	4.02
	13	4.27	3.90	3.77	3.80	3.71	3.77	3.73	3.70	3.69	3.73	3.76	3.78	3.82	3.87	3.90	3.91	3.94	3.97	4.00	4.03
	14	4.37	4.24	4.10	4.05	4.01	3.96	3.95	3.99	4.03	4.00	4.04	4.03	4.08	4.11	4.11	4.14	4.16	4.17	4.19	4.22
b	15	4.64	4.08	4.10	3.94	3.95	3.94	3.86	3.88	3.93	3.96	3.93	3.97	3.98	3.98	3.99	4.03	4.04	4.07	4.12	4.17
	16	4.39	4.14	4.08	4.00	3.99	3.92	3.91	3.94	3.91	3.92	3.92	3.95	3.95	3.96	4.00	4.03	4.04	4.04	4.08	4.10
	17	5.10	4.39	4.28	4.16	4.11	4.09	4.11	4.15	4.15	4.18	4.17	4.18	4.24	4.26	4.29	4.28	4.28	4.32	4.34	4.35
	18	4.46	4.79	4.58	4.45	4.42	4.43	4.40	4.43	4.39	4.39	4.41	4.39	4.38	4.37	4.40	4.43	4.47	4.48	4.51	4.57
	19	4.97	4.70	4.84	4.72	4.57	4.57	4.57	4.55	4.49	4.50	4.47	4.49	4.53	4.54	4.56	4.54	4.58	4.60	4.63	4.66
	20	4.51	4.50	4.20	4.11	4.20	4.22	4.25	4.21	4.22	4.23	4.20	4.20	4.21	4.22	4.22	4.25	4.28	4.31	4.33	4.35
	21	5.02	4.53	4.39	4.34	4.36	4.26	4.24	4.22	4.24	4.25	4.25	4.27	4.30	4.33	4.36	4.37	4.41	4.41	4.41	4.44
	22	4.86	4.69	4.48	4.27	4.32	4.21	4.20	4.22	4.25	4.24	4.20	4.24	4.26	4.28	4.31	4.36	4.37	4.40	4.42	4.43
	23	5.17	4.91	4.79	4.69	4.65	4.67	4.71	4.75	4.72	4.72	4.74	4.71	4.73	4.80	4.84	4.85	4.86	4.87	4.89	4.91
	24	5.81	5.11	5.37	5.07	4.88	4.94	4.97	5.00	4.96	4.95	5.00	4.97	4.93	4.95	4.96	4.98	4.97	4.96	4.99	5.05
	25	5.86	5.45	5.32	5.30	5.48	5.45	5.26	5.20	5.19	5.15	5.11	5.18	5.14	5.14	5.16	5.17	5.18	5.21	5.20	5.23
	26	5.96	5.88	5.56	5.47	5.54	5.40	5.37	5.28	5.28	5.26	5.31	5.26	5.20	5.19	5.23	5.24	5.27	5.26	5.28	5.29
	27	6.43	5.39	5.46	5.29	5.20	5.11	5.12	5.10	5.09	5.08	5.08	5.10	5.14	5.12	5.15	5.20	5.21	5.21	5.23	5.24
	28	5.57	5.65	5.47	5.37	5.42	5.34	5.28	5.25	5.21	5.17	5.15	5.15	5.18	5.18	5.21	5.23	5.23	5.23	5.23	5.23
	29	5.58	5.09	5.01	4.98	4.94	5.01	4.94	4.92	4.88	4.86	4.87	4.86	4.88	4.89	4.89	4.90	4.92	4.93	4.92	4.91
	30	5.18	4.75	4.70	4.72	4.67	4.64	4.67	4.63	4.68	4.66	4.66	4.66	4.70	4.74	4.77	4.77	4.79	4.85	4.85	4.84

Table B.30: *Mean distance errors (m) using the **Bayesian Histogram** algorithm, **point** surveying and **arithmetic** output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	4.87	4.36	4.24	4.19	4.15	4.13	4.11	4.11	4.10	4.09	4.09	4.08	4.08	4.08	4.08	4.08	4.07	4.07	4.07	4.07
	2	4.17	3.85	3.72	3.66	3.64	3.62	3.62	3.62	3.62	3.61	3.61	3.61	3.61	3.61	3.61	3.61	3.60	3.60	3.60	3.60
	3	4.05	3.66	3.56	3.51	3.47	3.46	3.45	3.44	3.44	3.44	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43	3.43
	4	3.85	3.54	3.43	3.37	3.33	3.31	3.29	3.29	3.28	3.28	3.28	3.28	3.28	3.28	3.28	3.27	3.27	3.27	3.27	3.27
	5	3.80	3.47	3.38	3.30	3.28	3.26	3.25	3.25	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24
	6	3.79	3.48	3.37	3.31	3.27	3.25	3.24	3.23	3.23	3.22	3.21	3.21	3.21	3.21	3.21	3.20	3.20	3.20	3.20	3.20
	7	3.81	3.53	3.42	3.34	3.29	3.28	3.27	3.26	3.25	3.24	3.24	3.23	3.23	3.23	3.23	3.23	3.22	3.22	3.22	3.22
	8	3.81	3.45	3.32	3.27	3.22	3.20	3.20	3.19	3.19	3.18	3.18	3.19	3.19	3.19	3.19	3.19	3.18	3.18	3.18	3.18
	9	3.98	3.66	3.53	3.50	3.46	3.43	3.40	3.38	3.36	3.34	3.34	3.34	3.34	3.33	3.33	3.33	3.33	3.33	3.33	3.33
	10	4.19	3.79	3.61	3.53	3.50	3.46	3.42	3.41	3.40	3.39	3.39	3.38	3.38	3.39	3.38	3.38	3.38	3.38	3.37	3.38
	11	4.20	3.95	3.80	3.61	3.56	3.52	3.52	3.51	3.49	3.48	3.47	3.46	3.46	3.45	3.45	3.45	3.44	3.44	3.44	3.44
	12	4.41	3.82	3.73	3.64	3.62	3.61	3.56	3.55	3.54	3.53	3.53	3.53	3.54	3.54	3.54	3.53	3.53	3.52	3.53	3.53
	13	4.27	3.86	3.75	3.73	3.64	3.63	3.60	3.58	3.56	3.56	3.56	3.55	3.56	3.55	3.54	3.54	3.54	3.54	3.54	3.53
	14	4.37	4.21	4.04	3.98	3.93	3.87	3.84	3.84	3.82	3.80	3.79	3.78	3.79	3.79	3.78	3.77	3.77	3.77	3.77	3.76
b	15	4.64	4.01	3.97	3.79	3.79	3.77	3.73	3.73	3.73	3.73	3.72	3.73	3.73	3.72	3.71	3.71	3.71	3.71	3.70	3.71
	16	4.39	4.10	4.04	3.92	3.89	3.83	3.82	3.81	3.78	3.76	3.75	3.75	3.74	3.73	3.73	3.73	3.73	3.72	3.73	3.72
	17	5.10	4.33	4.21	4.10	4.06	4.02	4.01	4.02	4.01	4.01	3.99	3.99	3.99	3.99	3.99	3.98	3.98	3.98	3.97	3.97
	18	4.46	4.76	4.50	4.37	4.30	4.28	4.25	4.24	4.21	4.20	4.18	4.17	4.15	4.14	4.14	4.14	4.15	4.13	4.13	4.13
	19	4.97	4.65	4.74	4.61	4.48	4.48	4.45	4.42	4.38	4.38	4.36	4.36	4.37	4.35	4.35	4.33	4.33	4.32	4.32	4.32
	20	4.51	4.43	4.17	4.08	4.10	4.09	4.09	4.06	4.07	4.07	4.06	4.05	4.06	4.06	4.05	4.06	4.06	4.07	4.08	4.08
	21	5.02	4.53	4.39	4.29	4.25	4.16	4.15	4.12	4.12	4.10	4.09	4.10	4.11	4.11	4.12	4.12	4.14	4.12	4.11	4.12
	22	4.86	4.66	4.47	4.24	4.27	4.16	4.16	4.14	4.13	4.13	4.09	4.11	4.12	4.12	4.12	4.12	4.12	4.12	4.12	4.12
	23	5.17	4.86	4.74	4.63	4.61	4.61	4.64	4.65	4.59	4.59	4.60	4.57	4.57	4.59	4.61	4.60	4.60	4.61	4.62	4.62
	24	5.81	5.09	5.29	4.98	4.80	4.80	4.79	4.79	4.76	4.74	4.76	4.74	4.72	4.72	4.71	4.70	4.70	4.70	4.70	4.71
	25	5.86	5.43	5.25	5.19	5.34	5.31	5.09	5.06	5.04	5.02	4.97	5.02	4.98	4.96	4.94	4.94	4.93	4.93	4.92	4.93
	26	5.96	5.81	5.46	5.34	5.41	5.28	5.24	5.14	5.15	5.11	5.13	5.08	5.04	5.02	5.01	4.98	4.98	4.96	4.96	4.94
	27	6.43	5.36	5.39	5.20	5.11	5.04	5.05	5.03	5.01	5.00	4.99	5.00	5.00	4.98	4.99	5.00	5.00	4.98	4.99	4.99
	28	5.57	5.60	5.42	5.34	5.38	5.31	5.24	5.23	5.20	5.17	5.14	5.12	5.12	5.11	5.12	5.14	5.13	5.13	5.12	5.12
	29	5.58	5.09	5.00	4.95	4.92	4.95	4.88	4.85	4.82	4.81	4.82	4.81	4.82	4.83	4.83	4.82	4.82	4.83	4.82	4.81
	30	5.18	4.74	4.69	4.71	4.63	4.59	4.59	4.55	4.58	4.56	4.55	4.54	4.56	4.60	4.62	4.63	4.63	4.65	4.65	4.65

Table B.31: *Mean distance errors (m) using the* ***Bayesian Histogram*** *algorithm,* ***point*** *surveying and* ***weighted*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	4.20	3.97	3.99	4.13	4.24	4.39	4.54	4.71	4.83	4.97	5.12	5.22	5.31	5.43	5.51	5.59	5.67	5.79	5.87	5.96
	2	3.98	3.75	3.73	3.89	4.02	4.11	4.22	4.34	4.45	4.59	4.71	4.83	4.93	5.06	5.17	5.26	5.37	5.45	5.52	5.61
	3	3.79	3.73	3.71	3.87	3.91	4.02	4.11	4.23	4.32	4.44	4.56	4.68	4.77	4.87	4.96	5.07	5.18	5.28	5.38	5.48
	4	3.72	3.66	3.63	3.69	3.86	3.96	4.04	4.11	4.22	4.31	4.40	4.50	4.58	4.71	4.83	4.92	5.03	5.16	5.24	5.36
	5	3.85	3.58	3.68	3.73	3.82	3.90	4.08	4.17	4.26	4.34	4.41	4.49	4.58	4.64	4.75	4.86	4.96	5.06	5.16	5.26
	6	3.88	3.60	3.53	3.66	3.75	3.89	4.03	4.12	4.24	4.34	4.42	4.49	4.61	4.67	4.77	4.87	4.96	5.04	5.15	5.27
	7	4.03	3.73	3.72	3.88	3.97	4.04	4.11	4.23	4.32	4.42	4.53	4.63	4.71	4.80	4.86	4.93	5.00	5.09	5.19	5.27
	8	4.03	3.77	3.70	3.81	3.84	3.94	4.04	4.13	4.24	4.35	4.39	4.48	4.56	4.63	4.76	4.86	4.94	5.05	5.11	5.18
	9	4.14	4.09	3.91	3.93	4.03	4.06	4.12	4.22	4.30	4.36	4.48	4.54	4.62	4.71	4.81	4.85	4.92	5.00	5.09	5.16
	10	4.46	4.03	4.17	4.10	4.13	4.21	4.28	4.35	4.41	4.44	4.47	4.52	4.59	4.71	4.80	4.85	4.92	4.99	5.08	5.17
	11	4.43	4.10	4.22	4.18	4.24	4.27	4.38	4.44	4.54	4.57	4.65	4.68	4.73	4.79	4.88	4.95	4.98	5.06	5.12	5.17
	12	4.49	4.12	4.01	4.18	4.20	4.21	4.27	4.39	4.48	4.57	4.62	4.65	4.72	4.79	4.87	4.95	5.02	5.11	5.21	5.30
	13	4.38	4.19	4.27	4.23	4.31	4.33	4.42	4.42	4.47	4.56	4.57	4.63	4.67	4.78	4.86	4.91	4.96	5.03	5.11	5.13
	14	5.05	4.86	4.70	4.75	4.82	4.85	4.81	4.87	4.89	4.88	4.90	4.98	5.02	5.09	5.17	5.22	5.27	5.32	5.37	5.43
b	15	4.69	4.33	4.47	4.55	4.61	4.59	4.74	4.74	4.79	4.83	4.90	4.90	4.95	4.96	4.97	5.04	5.08	5.14	5.20	5.27
	16	4.80	4.53	4.43	4.48	4.48	4.44	4.46	4.44	4.46	4.50	4.53	4.58	4.66	4.73	4.81	4.88	4.97	5.03	5.08	5.14
	17	5.11	4.92	4.91	5.01	5.08	4.99	5.00	5.00	5.12	5.29	5.33	5.36	5.34	5.40	5.48	5.54	5.59	5.63	5.66	5.71
	18	5.46	5.23	5.13	5.05	5.18	5.23	5.19	5.18	5.25	5.31	5.36	5.35	5.42	5.49	5.56	5.58	5.61	5.68	5.74	5.78
	19	5.53	5.36	5.57	5.54	5.47	5.36	5.31	5.31	5.35	5.33	5.43	5.45	5.47	5.47	5.49	5.49	5.49	5.58	5.62	5.63
	20	4.98	4.90	4.71	4.81	4.78	4.70	4.69	4.69	4.70	4.69	4.70	4.77	4.83	4.86	4.91	4.95	4.97	5.05	5.10	5.15
	21	5.32	4.94	4.88	4.85	4.73	4.69	4.68	4.72	4.71	4.76	4.82	4.88	4.88	4.91	4.97	5.02	5.09	5.16	5.18	5.24
	22	5.57	5.30	5.05	5.06	5.02	5.07	5.13	5.09	5.10	5.19	5.21	5.24	5.30	5.34	5.44	5.48	5.48	5.50	5.51	5.57
	23	5.65	5.73	6.08	6.00	5.81	5.75	5.83	5.88	5.91	5.94	5.91	5.90	5.91	5.95	5.97	6.04	6.09	6.14	6.21	6.27
	24	6.10	6.07	6.26	6.04	5.97	5.85	5.85	5.91	5.86	5.94	5.92	5.94	5.96	5.98	6.00	6.04	6.08	6.13	6.12	6.20
	25	6.09	5.94	6.01	6.03	6.04	6.00	5.94	5.94	5.98	6.07	6.10	6.06	6.13	6.16	6.18	6.16	6.23	6.23	6.26	6.34
	26	6.83	6.11	6.18	6.18	6.10	5.99	5.92	5.98	5.98	6.00	6.03	6.05	6.04	6.10	6.07	6.06	6.07	6.09	6.14	6.17
	27	6.50	6.52	6.24	5.81	5.69	5.75	5.82	5.75	5.76	5.78	5.84	5.81	5.82	5.84	5.86	5.88	5.90	5.94	5.96	5.99
	28	6.98	6.95	6.03	6.02	5.74	5.60	5.54	5.59	5.64	5.67	5.69	5.72	5.75	5.77	5.80	5.80	5.83	5.86	5.89	5.93
	29	6.35	6.15	5.54	5.72	5.62	5.63	5.65	5.66	5.67	5.65	5.66	5.67	5.69	5.71	5.76	5.84	5.84	5.86	5.94	6.00
	30	7.06	6.49	6.02	5.71	5.67	5.54	5.49	5.61	5.53	5.61	5.63	5.68	5.69	5.71	5.65	5.68	5.73	5.79	5.85	5.93

Table B.32: *Mean distance errors (m) using the* ***Bayesian Histogram*** *algorithm,* ***sector*** *surveying and* ***arithmetic*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	4.20	3.94	3.85	3.83	3.82	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83
	2	3.98	3.72	3.62	3.59	3.56	3.54	3.54	3.54	3.54	3.54	3.54	3.54	3.54	3.55	3.55	3.55	3.55	3.55	3.55	3.55
	3	3.79	3.65	3.54	3.50	3.46	3.45	3.44	3.44	3.44	3.44	3.44	3.45	3.44	3.45	3.45	3.45	3.45	3.45	3.45	3.45
	4	3.72	3.59	3.49	3.43	3.43	3.39	3.39	3.38	3.38	3.38	3.38	3.38	3.38	3.39	3.39	3.39	3.39	3.39	3.40	3.40
	5	3.85	3.53	3.49	3.42	3.40	3.37	3.38	3.37	3.37	3.37	3.37	3.37	3.37	3.37	3.37	3.38	3.38	3.38	3.38	3.38
	6	3.88	3.60	3.46	3.40	3.39	3.39	3.40	3.39	3.40	3.41	3.41	3.41	3.41	3.42	3.42	3.42	3.42	3.42	3.42	3.42
	7	4.03	3.72	3.65	3.63	3.60	3.57	3.56	3.56	3.56	3.56	3.57	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58	3.58
	8	4.03	3.78	3.67	3.65	3.60	3.58	3.58	3.58	3.56	3.57	3.57	3.57	3.57	3.57	3.58	3.58	3.58	3.58	3.58	3.59
	9	4.14	3.99	3.80	3.74	3.72	3.69	3.68	3.68	3.67	3.66	3.67	3.66	3.66	3.67	3.68	3.68	3.68	3.68	3.68	3.68
	10	4.46	4.03	4.01	3.95	3.94	3.93	3.91	3.91	3.89	3.86	3.83	3.83	3.82	3.83	3.84	3.83	3.83	3.82	3.83	3.83
	11	4.43	4.12	4.07	4.00	3.99	3.93	3.93	3.91	3.90	3.90	3.91	3.91	3.91	3.92	3.92	3.92	3.92	3.92	3.93	3.93
	12	4.49	4.14	3.97	3.97	3.94	3.91	3.91	3.90	3.90	3.91	3.91	3.90	3.90	3.90	3.90	3.90	3.90	3.90	3.90	3.91
	13	4.38	4.15	4.07	3.96	4.00	3.99	3.98	3.94	3.92	3.92	3.92	3.93	3.93	3.94	3.94	3.95	3.95	3.95	3.96	3.96
	14	5.05	4.78	4.57	4.54	4.48	4.44	4.39	4.37	4.35	4.33	4.33	4.33	4.32	4.31	4.32	4.32	4.32	4.33	4.33	4.33
b	15	4.69	4.28	4.22	4.26	4.21	4.20	4.22	4.16	4.13	4.13	4.14	4.12	4.11	4.09	4.08	4.08	4.08	4.08	4.08	4.07
	16	4.80	4.49	4.36	4.31	4.30	4.28	4.27	4.22	4.20	4.20	4.19	4.19	4.19	4.19	4.19	4.19	4.20	4.20	4.21	4.21
	17	5.11	4.88	4.75	4.70	4.71	4.67	4.66	4.64	4.65	4.65	4.66	4.66	4.65	4.65	4.66	4.66	4.67	4.67	4.67	4.67
	18	5.46	5.15	4.96	4.86	4.90	4.84	4.81	4.78	4.75	4.73	4.72	4.69	4.70	4.69	4.72	4.70	4.70	4.70	4.71	4.70
	19	5.53	5.31	5.32	5.25	5.17	5.12	5.11	5.11	5.11	5.08	5.10	5.08	5.08	5.07	5.06	5.05	5.05	5.05	5.06	5.05
	20	4.98	4.87	4.68	4.71	4.67	4.63	4.57	4.50	4.49	4.48	4.47	4.47	4.48	4.48	4.49	4.48	4.48	4.49	4.49	4.49
	21	5.32	4.95	4.90	4.80	4.70	4.67	4.65	4.65	4.62	4.61	4.60	4.60	4.58	4.58	4.58	4.58	4.58	4.57	4.57	4.57
	22	5.57	5.47	5.15	5.05	5.02	5.04	5.02	4.95	4.91	4.91	4.88	4.86	4.84	4.84	4.85	4.86	4.84	4.86	4.84	4.84
	23	5.65	5.64	5.77	5.53	5.45	5.48	5.50	5.47	5.46	5.44	5.40	5.37	5.36	5.36	5.37	5.38	5.38	5.41	5.44	5.46
	24	6.10	5.94	5.91	5.71	5.68	5.61	5.62	5.58	5.51	5.48	5.43	5.40	5.40	5.37	5.36	5.34	5.35	5.34	5.33	5.33
	25	6.09	5.84	5.83	5.91	5.94	5.92	5.90	5.89	5.87	5.87	5.88	5.83	5.84	5.84	5.84	5.82	5.81	5.81	5.80	5.80
	26	6.83	6.07	6.02	5.99	5.98	6.00	5.98	6.01	6.01	5.99	5.96	5.97	5.97	5.96	5.95	5.95	5.94	5.93	5.94	5.94
	27	6.50	6.44	6.11	5.92	5.86	5.88	5.87	5.80	5.80	5.81	5.83	5.80	5.80	5.78	5.78	5.77	5.76	5.77	5.77	5.78
	28	6.98	6.86	6.13	6.04	5.93	5.86	5.83	5.83	5.83	5.80	5.80	5.77	5.77	5.76	5.76	5.75	5.76	5.75	5.75	5.75
	29	6.35	6.19	5.70	5.74	5.70	5.66	5.66	5.64	5.59	5.54	5.52	5.50	5.49	5.49	5.48	5.47	5.45	5.45	5.45	5.46
	30	7.06	6.63	6.18	5.78	5.63	5.57	5.51	5.57	5.45	5.49	5.41	5.40	5.39	5.37	5.31	5.25	5.26	5.23	5.22	5.22

Table B.33: *Mean distance errors (m) using the* ***Bayesian Histogram*** *algorithm,* ***sector*** *surveying and* ***weighted*** *output averaging in the CSE environment.*

k																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4.21	3.91	3.76	3.88	3.85	3.93	3.93	3.96	4.05	4.07	4.16	4.17	4.24	4.31	4.37	4.42	4.48	4.54	4.58	4.63

Table B.34: *Mean distance errors (m) using the* ***Bayesian Gaussian*** *algorithm,* ***point*** *surveying and* ***arithmetic*** *output averaging in the CSE environment.*

k																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4.21	4.02	3.98	3.98	3.98	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97

Table B.35: *Mean distance errors (m) using the **Bayesian Gaussian** algorithm, **point** surveying and **weighted** output averaging in the CSE environment.*

k																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4.21	3.93	3.90	3.98	4.05	4.16	4.28	4.39	4.52	4.61	4.71	4.79	4.87	4.94	5.00	5.06	5.13	5.17	5.22	5.27

Table B.36: *Mean distance errors (m) using the* ***Bayesian Gaussian*** *algorithm,* ***sector*** *surveying and* ***arithmetic*** *output averaging in the CSE environment.*

k																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4.21	4.01	3.95	3.93	3.91	3.90	3.90	3.89	3.89	3.89	3.89	3.89	3.89	3.89	3.89	3.89	3.89	3.89	3.89	3.89

Table B.37: *Mean distance errors (m) using the* ***Bayesian Gaussian*** *algorithm,* ***sector*** *surveying and* ***weighted*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	6.05	5.54	5.42	5.30	5.32	5.27	5.29	5.26	5.26	5.25	5.28	5.36	5.39	5.43	5.48	5.50	5.53	5.57	5.60	5.64
	2	4.49	4.23	4.15	4.14	4.17	4.19	4.21	4.23	4.26	4.31	4.40	4.44	4.45	4.47	4.53	4.55	4.57	4.60	4.63	4.67
	3	4.42	3.93	3.89	3.88	3.87	3.90	3.88	3.90	3.93	3.98	4.00	4.03	4.06	4.11	4.13	4.15	4.18	4.22	4.24	4.26
	4	4.17	3.94	3.78	3.71	3.73	3.71	3.78	3.84	3.86	3.93	3.98	4.02	4.07	4.11	4.14	4.20	4.22	4.26	4.28	4.32
	5	4.03	3.67	3.63	3.57	3.56	3.63	3.62	3.64	3.66	3.71	3.75	3.79	3.82	3.84	3.89	3.94	3.99	4.02	4.07	4.12
	6	3.96	3.68	3.55	3.58	3.55	3.58	3.65	3.69	3.73	3.76	3.78	3.82	3.85	3.87	3.91	3.99	4.01	4.06	4.11	4.16
	7	3.78	3.51	3.40	3.41	3.39	3.41	3.46	3.49	3.52	3.56	3.64	3.67	3.71	3.77	3.82	3.84	3.88	3.93	4.00	4.03
	8	3.63	3.34	3.34	3.32	3.34	3.35	3.40	3.43	3.48	3.48	3.53	3.55	3.62	3.67	3.72	3.75	3.79	3.84	3.89	3.92
	9	3.75	3.39	3.35	3.37	3.35	3.38	3.41	3.44	3.49	3.57	3.60	3.63	3.68	3.71	3.74	3.79	3.85	3.87	3.91	3.94
	10	3.68	3.44	3.36	3.35	3.31	3.36	3.44	3.49	3.52	3.54	3.56	3.60	3.61	3.62	3.64	3.68	3.72	3.77	3.81	3.85
	11	3.54	3.29	3.21	3.23	3.25	3.28	3.30	3.35	3.37	3.38	3.41	3.46	3.49	3.54	3.58	3.62	3.67	3.73	3.78	3.82
	12	3.69	3.37	3.30	3.21	3.20	3.21	3.24	3.28	3.29	3.35	3.39	3.43	3.48	3.51	3.56	3.62	3.68	3.73	3.76	3.82
	13	3.55	3.41	3.35	3.36	3.38	3.36	3.41	3.47	3.50	3.54	3.57	3.60	3.62	3.65	3.68	3.72	3.76	3.82	3.87	3.91
	14	3.93	3.51	3.39	3.34	3.32	3.33	3.35	3.35	3.37	3.42	3.46	3.51	3.55	3.60	3.66	3.72	3.76	3.79	3.84	3.86
b	15	3.88	3.51	3.52	3.53	3.51	3.52	3.56	3.60	3.63	3.64	3.65	3.69	3.73	3.76	3.79	3.81	3.85	3.89	3.91	3.94
	16	3.82	3.56	3.47	3.43	3.45	3.46	3.45	3.46	3.43	3.45	3.49	3.50	3.54	3.58	3.62	3.66	3.70	3.74	3.77	3.81
	17	3.99	3.74	3.64	3.63	3.64	3.60	3.65	3.67	3.69	3.72	3.73	3.74	3.75	3.80	3.84	3.87	3.89	3.93	3.97	4.00
	18	4.18	3.90	3.84	3.85	3.86	3.88	3.91	3.92	3.95	3.99	4.03	4.07	4.10	4.11	4.13	4.17	4.22	4.24	4.26	4.30
	19	4.28	3.97	3.88	3.85	3.86	3.85	3.86	3.86	3.87	3.87	3.89	3.91	3.94	3.96	3.99	4.02	4.05	4.08	4.13	4.15
	20	4.13	3.73	3.65	3.58	3.59	3.60	3.56	3.60	3.63	3.65	3.65	3.68	3.71	3.73	3.75	3.79	3.81	3.84	3.87	3.91
	21	4.26	3.83	3.80	3.76	3.67	3.62	3.62	3.59	3.57	3.60	3.65	3.68	3.71	3.74	3.77	3.80	3.85	3.88	3.89	3.90
	22	4.49	4.00	3.83	3.76	3.73	3.75	3.76	3.75	3.78	3.76	3.79	3.80	3.84	3.90	3.95	3.99	4.01	4.03	4.06	4.11
	23	4.56	4.07	4.00	4.00	3.98	4.07	4.13	4.18	4.23	4.28	4.32	4.33	4.36	4.40	4.43	4.48	4.54	4.59	4.63	4.67
	24	4.60	4.29	4.06	4.02	4.06	4.07	4.11	4.11	4.13	4.19	4.23	4.25	4.27	4.28	4.33	4.37	4.39	4.42	4.48	4.53
	25	4.69	4.22	4.18	4.26	4.24	4.23	4.19	4.18	4.20	4.21	4.25	4.27	4.27	4.28	4.33	4.35	4.40	4.43	4.45	4.47
	26	4.92	4.56	4.32	4.21	4.20	4.16	4.17	4.13	4.14	4.15	4.16	4.19	4.22	4.27	4.31	4.36	4.39	4.43	4.48	4.51
	27	4.77	4.35	4.14	4.10	4.10	4.11	4.13	4.17	4.19	4.25	4.26	4.30	4.32	4.35	4.38	4.43	4.48	4.51	4.56	4.60
	28	5.16	4.51	4.38	4.34	4.37	4.36	4.45	4.44	4.43	4.43	4.43	4.43	4.48	4.49	4.50	4.52	4.52	4.56	4.58	4.61
	29	4.77	4.41	4.25	4.28	4.27	4.20	4.19	4.18	4.17	4.16	4.19	4.24	4.26	4.27	4.31	4.34	4.37	4.40	4.42	4.45
	30	4.63	4.21	4.05	3.97	4.00	3.99	4.05	4.10	4.14	4.16	4.19	4.23	4.25	4.29	4.33	4.36	4.38	4.41	4.43	4.46

Table B.38: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **point** surveying and **arithmetic** output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	6.05	5.52	5.40	5.27	5.28	5.24	5.25	5.22	5.22	5.21	5.24	5.32	5.35	5.39	5.43	5.45	5.49	5.53	5.55	5.59
	2	4.49	4.22	4.12	4.11	4.14	4.15	4.17	4.18	4.21	4.26	4.34	4.38	4.39	4.41	4.46	4.49	4.50	4.54	4.56	4.60
	3	4.42	3.92	3.86	3.84	3.82	3.85	3.83	3.85	3.88	3.92	3.94	3.97	4.00	4.04	4.06	4.09	4.11	4.14	4.16	4.19
	4	4.17	3.91	3.74	3.67	3.69	3.66	3.72	3.78	3.79	3.86	3.91	3.95	3.98	4.02	4.05	4.10	4.12	4.16	4.18	4.22
	5	4.03	3.66	3.60	3.53	3.52	3.56	3.56	3.58	3.59	3.63	3.67	3.71	3.74	3.75	3.80	3.84	3.89	3.91	3.95	4.00
	6	3.96	3.64	3.51	3.53	3.50	3.52	3.57	3.61	3.64	3.67	3.69	3.72	3.75	3.77	3.80	3.87	3.89	3.93	3.98	4.02
	7	3.78	3.48	3.36	3.35	3.33	3.34	3.38	3.41	3.43	3.46	3.53	3.56	3.59	3.64	3.68	3.70	3.74	3.79	3.84	3.87
	8	3.63	3.31	3.27	3.25	3.26	3.27	3.30	3.33	3.37	3.37	3.41	3.43	3.49	3.54	3.58	3.60	3.63	3.68	3.71	3.74
	9	3.75	3.36	3.30	3.30	3.27	3.30	3.32	3.34	3.38	3.44	3.47	3.50	3.54	3.57	3.59	3.63	3.68	3.69	3.72	3.75
	10	3.68	3.39	3.30	3.29	3.24	3.28	3.34	3.38	3.41	3.42	3.45	3.47	3.49	3.49	3.50	3.54	3.57	3.60	3.63	3.66
	11	3.54	3.27	3.18	3.18	3.18	3.19	3.20	3.24	3.25	3.26	3.28	3.32	3.34	3.39	3.41	3.45	3.49	3.53	3.57	3.60
	12	3.69	3.33	3.24	3.15	3.14	3.14	3.16	3.19	3.19	3.23	3.26	3.30	3.33	3.36	3.39	3.44	3.48	3.52	3.54	3.58
	13	3.55	3.36	3.27	3.26	3.27	3.25	3.28	3.32	3.35	3.38	3.41	3.43	3.44	3.47	3.49	3.52	3.55	3.59	3.63	3.66
	14	3.93	3.50	3.37	3.30	3.26	3.27	3.27	3.28	3.28	3.32	3.35	3.38	3.41	3.45	3.49	3.54	3.56	3.58	3.62	3.64
b	15	3.90	3.48	3.43	3.45	3.42	3.43	3.46	3.48	3.50	3.51	3.53	3.55	3.58	3.60	3.63	3.64	3.67	3.69	3.72	3.74
	16	3.81	3.58	3.46	3.40	3.39	3.40	3.39	3.40	3.40	3.41	3.42	3.42	3.44	3.47	3.49	3.50	3.54	3.56	3.59	3.62
	17	3.98	3.71	3.61	3.58	3.58	3.55	3.58	3.58	3.61	3.63	3.63	3.64	3.64	3.67	3.69	3.72	3.73	3.76	3.78	3.80
	18	4.25	3.87	3.77	3.79	3.77	3.78	3.80	3.81	3.83	3.86	3.91	3.92	3.93	3.94	3.97	3.99	4.02	4.04	4.06	4.09
	19	NaN	3.93	3.79	3.75	3.77	3.75	3.75	3.76	3.77	3.79	3.78	3.79	3.80	3.82	3.84	3.85	3.86	3.88	3.91	3.93
	20	NaN	3.73	3.62	3.55	3.54	3.54	3.52	3.53	3.56	3.57	3.57	3.61	3.61	3.63	3.63	3.65	3.66	3.68	3.70	3.72
	21	4.21	3.75	3.71	3.69	3.62	3.58	3.57	3.56	3.56	3.57	3.58	3.60	3.61	3.62	3.64	3.66	3.69	3.70	3.72	3.73
	22	4.47	3.97	3.84	3.75	3.70	3.70	3.71	3.70	3.70	3.69	3.71	3.71	3.73	3.76	3.78	3.79	3.81	3.83	3.85	3.87
	23	4.54	4.11	4.03	4.00	3.99	4.02	4.06	4.09	4.11	4.15	4.16	4.17	4.19	4.22	4.24	4.27	4.31	4.33	4.36	4.38
	24	NaN	4.23	4.08	4.02	4.01	4.00	4.01	4.01	4.02	4.06	4.10	4.11	4.13	4.15	4.18	4.21	4.22	4.24	4.27	4.31
	25	NaN	4.21	4.14	4.14	4.10	4.10	4.08	4.07	4.09	4.10	4.13	4.13	4.13	4.15	4.17	4.18	4.20	4.22	4.24	4.26
	26	NaN	4.49	4.31	4.21	4.17	4.15	4.11	4.10	4.10	4.10	4.09	4.10	4.10	4.12	4.14	4.17	4.18	4.20	4.21	4.23
	27	NaN	4.52	4.17	4.10	4.09	4.06	4.05	4.08	4.10	4.12	4.12	4.15	4.17	4.19	4.19	4.22	4.24	4.26	4.28	4.30
	28	NaN	4.56	4.39	4.29	4.29	4.28	4.31	4.31	4.31	4.31	4.31	4.31	4.34	4.34	4.35	4.35	4.37	4.38	4.40	4.41
	29	NaN	4.36	4.24	4.23	4.21	4.14	4.12	4.09	4.09	4.09	4.09	4.11	4.13	4.15	4.16	4.17	4.19	4.21	4.22	4.23
	30	4.63	4.23	4.08	4.00	3.98	3.96	3.95	3.98	3.99	4.00	4.01	4.03	4.05	4.06	4.07	4.08	4.10	4.11	4.12	4.13

Table B.39: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **point** surveying and **weighted** output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	5.31	5.10	5.11	5.20	5.30	5.42	5.53	5.62	5.71	5.78	5.86	5.98	6.07	6.13	6.19	6.28	6.38	6.46	6.55	6.63
	2	4.48	4.28	4.26	4.37	4.57	4.65	4.80	4.92	5.01	5.15	5.25	5.35	5.41	5.50	5.60	5.69	5.77	5.85	5.94	6.02
	3	4.14	4.10	4.14	4.21	4.35	4.50	4.62	4.76	4.87	4.96	5.09	5.18	5.27	5.37	5.47	5.55	5.65	5.74	5.82	5.91
	4	4.03	3.85	3.94	3.97	4.10	4.23	4.31	4.44	4.59	4.70	4.79	4.89	4.98	5.07	5.18	5.25	5.35	5.45	5.55	5.66
	5	4.30	4.11	4.04	4.13	4.27	4.37	4.53	4.65	4.74	4.85	4.91	4.99	5.08	5.18	5.27	5.37	5.47	5.54	5.67	5.76
	6	4.14	3.91	3.80	4.00	4.11	4.24	4.37	4.50	4.59	4.69	4.81	4.92	4.97	5.05	5.13	5.20	5.27	5.38	5.47	5.55
	7	4.09	3.80	3.83	3.93	4.03	4.18	4.27	4.37	4.46	4.60	4.70	4.82	4.90	4.99	5.09	5.18	5.28	5.37	5.46	5.56
	8	3.90	3.72	3.71	3.78	3.92	4.10	4.24	4.34	4.47	4.60	4.69	4.78	4.88	4.94	5.01	5.08	5.14	5.24	5.33	5.43
	9	3.97	3.75	3.71	3.78	3.97	4.09	4.24	4.35	4.44	4.55	4.66	4.76	4.86	4.98	5.06	5.14	5.24	5.32	5.39	5.47
	10	4.06	3.93	3.87	3.84	3.99	4.12	4.25	4.34	4.44	4.53	4.60	4.70	4.79	4.85	4.93	5.02	5.12	5.20	5.28	5.39
	11	3.82	3.71	3.74	3.89	4.03	4.20	4.30	4.46	4.52	4.61	4.68	4.76	4.83	4.93	5.02	5.08	5.15	5.21	5.30	5.41
	12	3.70	3.55	3.47	3.67	3.84	3.98	4.13	4.24	4.38	4.52	4.62	4.73	4.84	4.92	4.97	5.07	5.12	5.19	5.25	5.30
	13	4.06	3.89	3.84	4.00	4.05	4.18	4.32	4.46	4.55	4.63	4.71	4.79	4.85	4.92	4.99	5.06	5.11	5.21	5.31	5.38
	14	4.05	3.84	3.74	3.97	4.15	4.28	4.45	4.53	4.64	4.72	4.79	4.86	4.93	5.00	5.07	5.14	5.20	5.25	5.31	5.36
b	15	4.13	3.90	3.88	3.97	4.07	4.13	4.23	4.34	4.42	4.49	4.58	4.69	4.77	4.84	4.91	4.96	5.04	5.13	5.22	5.32
	16	4.14	4.01	3.97	4.01	4.09	4.14	4.21	4.23	4.31	4.40	4.45	4.52	4.58	4.66	4.72	4.81	4.86	4.92	4.97	5.03
	17	3.96	3.83	3.87	4.10	4.32	4.46	4.53	4.63	4.73	4.87	4.95	4.99	5.05	5.09	5.16	5.21	5.28	5.32	5.36	5.41
	18	4.84	4.38	4.30	4.29	4.45	4.56	4.67	4.79	4.88	4.98	5.04	5.13	5.18	5.25	5.32	5.41	5.44	5.54	5.62	5.70
	19	4.92	4.67	4.52	4.46	4.57	4.52	4.59	4.61	4.66	4.70	4.77	4.85	4.90	4.99	5.06	5.12	5.19	5.26	5.35	5.41
	20	4.15	3.96	3.96	3.93	4.04	4.11	4.17	4.24	4.26	4.33	4.39	4.46	4.51	4.59	4.68	4.74	4.81	4.85	4.90	4.98
	21	4.04	3.86	3.79	3.88	3.95	4.04	4.10	4.20	4.29	4.36	4.45	4.52	4.59	4.67	4.75	4.82	4.90	4.96	4.99	5.04
	22	4.55	4.29	4.28	4.44	4.56	4.74	4.80	4.93	5.06	5.12	5.18	5.23	5.27	5.31	5.36	5.39	5.44	5.48	5.51	5.57
	23	5.10	4.92	4.88	5.09	5.16	5.27	5.39	5.52	5.57	5.64	5.71	5.77	5.83	5.88	5.94	6.00	6.07	6.12	6.16	6.20
	24	4.99	4.71	4.76	4.79	4.92	5.01	5.12	5.23	5.32	5.40	5.47	5.53	5.57	5.61	5.69	5.75	5.81	5.87	5.93	5.97
	25	5.11	4.82	4.89	4.89	4.94	5.08	5.17	5.24	5.30	5.37	5.43	5.46	5.50	5.57	5.60	5.68	5.79	5.84	5.95	6.05
	26	5.56	5.20	5.07	4.99	5.07	5.27	5.35	5.33	5.36	5.46	5.52	5.60	5.63	5.65	5.66	5.71	5.78	5.84	5.86	5.88
	27	5.18	5.07	5.01	5.03	5.07	5.10	5.17	5.18	5.20	5.27	5.32	5.36	5.37	5.40	5.44	5.49	5.53	5.53	5.53	5.56
	28	5.34	4.76	4.60	4.56	4.63	4.71	4.76	4.80	4.85	4.87	4.93	4.98	5.01	5.03	5.05	5.10	5.14	5.21	5.29	5.34
	29	4.76	4.48	4.49	4.53	4.63	4.65	4.70	4.77	4.78	4.78	4.82	4.89	4.94	5.03	5.06	5.11	5.16	5.26	5.34	5.43
	30	4.96	4.98	4.89	4.68	4.61	4.70	4.76	4.87	4.97	5.03	5.09	5.12	5.22	5.30	5.36	5.41	5.48	5.57	5.65	5.74

Table B.40: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **sector** surveying and **arithmetic** output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	5.31	5.09	5.09	5.17	5.27	5.39	5.50	5.58	5.67	5.74	5.81	5.93	6.02	6.08	6.14	6.23	6.32	6.40	6.49	6.56
	2	4.48	4.26	4.24	4.33	4.52	4.60	4.74	4.85	4.94	5.07	5.17	5.27	5.33	5.41	5.50	5.59	5.66	5.74	5.83	5.90
	3	4.14	4.08	4.11	4.16	4.29	4.43	4.54	4.67	4.78	4.86	4.98	5.07	5.15	5.25	5.34	5.42	5.51	5.59	5.67	5.75
	4	4.03	3.83	3.89	3.91	4.02	4.14	4.21	4.34	4.47	4.57	4.66	4.75	4.84	4.93	5.02	5.09	5.18	5.28	5.36	5.46
	5	4.30	4.10	4.01	4.08	4.19	4.28	4.42	4.53	4.62	4.72	4.78	4.85	4.93	5.03	5.11	5.20	5.29	5.36	5.47	5.55
	6	4.14	3.88	3.75	3.91	4.00	4.12	4.23	4.35	4.43	4.52	4.63	4.73	4.78	4.85	4.93	4.99	5.06	5.15	5.22	5.30
	7	4.09	3.79	3.78	3.85	3.93	4.05	4.13	4.22	4.30	4.42	4.51	4.62	4.70	4.78	4.87	4.95	5.03	5.12	5.19	5.28
	8	3.90	3.70	3.66	3.71	3.81	3.96	4.07	4.16	4.27	4.38	4.46	4.54	4.63	4.68	4.75	4.81	4.86	4.94	5.02	5.10
	9	3.97	3.72	3.66	3.70	3.85	3.93	4.06	4.15	4.23	4.32	4.41	4.50	4.59	4.70	4.77	4.84	4.93	5.00	5.06	5.13
	10	4.06	3.89	3.81	3.78	3.87	3.97	4.07	4.15	4.23	4.30	4.36	4.45	4.52	4.58	4.65	4.72	4.81	4.87	4.95	5.03
	11	3.82	3.68	3.67	3.78	3.87	4.00	4.08	4.21	4.27	4.35	4.42	4.48	4.55	4.63	4.70	4.76	4.82	4.86	4.92	5.00
	12	3.70	3.52	3.43	3.56	3.68	3.79	3.91	4.00	4.11	4.23	4.32	4.41	4.50	4.58	4.62	4.71	4.75	4.81	4.87	4.91
	13	4.06	3.84	3.78	3.89	3.91	4.01	4.12	4.23	4.31	4.38	4.45	4.52	4.58	4.64	4.70	4.76	4.80	4.88	4.96	5.02
	14	4.05	3.82	3.71	3.85	3.97	4.07	4.21	4.28	4.37	4.44	4.50	4.56	4.62	4.69	4.75	4.81	4.86	4.90	4.95	4.99
b	15	4.13	3.88	3.80	3.87	3.91	3.94	4.01	4.10	4.17	4.23	4.30	4.39	4.45	4.51	4.57	4.62	4.68	4.75	4.81	4.88
	16	4.14	3.98	3.92	3.94	3.99	4.02	4.06	4.07	4.13	4.19	4.22	4.28	4.31	4.37	4.41	4.47	4.51	4.54	4.58	4.62
	17	3.96	3.80	3.79	3.95	4.09	4.20	4.26	4.34	4.42	4.54	4.61	4.65	4.70	4.74	4.81	4.85	4.91	4.94	4.97	5.01
	18	4.84	4.39	4.26	4.22	4.30	4.35	4.43	4.51	4.59	4.67	4.72	4.79	4.84	4.89	4.95	5.02	5.05	5.13	5.18	5.24
	19	4.92	4.63	4.43	4.35	4.43	4.39	4.44	4.45	4.49	4.53	4.58	4.64	4.69	4.75	4.80	4.84	4.89	4.93	4.99	5.04
	20	4.15	3.93	3.91	3.86	3.92	3.96	4.01	4.05	4.06	4.12	4.16	4.21	4.24	4.29	4.34	4.37	4.41	4.44	4.47	4.51
	21	4.04	3.83	3.75	3.78	3.82	3.87	3.90	3.97	4.03	4.08	4.15	4.20	4.25	4.32	4.38	4.43	4.49	4.53	4.56	4.59
	22	4.55	4.27	4.23	4.30	4.36	4.46	4.50	4.59	4.68	4.74	4.78	4.83	4.87	4.91	4.95	4.99	5.03	5.06	5.09	5.14
	23	5.10	4.86	4.78	4.85	4.91	4.98	5.06	5.16	5.20	5.27	5.33	5.39	5.44	5.49	5.54	5.59	5.65	5.69	5.73	5.76
	24	4.99	4.68	4.68	4.66	4.74	4.79	4.87	4.94	5.00	5.06	5.12	5.17	5.20	5.24	5.29	5.33	5.38	5.43	5.47	5.50
	25	5.11	4.78	4.81	4.80	4.82	4.90	4.94	4.99	5.03	5.08	5.13	5.15	5.19	5.24	5.27	5.33	5.41	5.45	5.53	5.61
	26	5.56	5.19	5.06	4.98	4.99	5.12	5.18	5.17	5.19	5.26	5.30	5.35	5.38	5.40	5.41	5.46	5.51	5.55	5.57	5.59
	27	5.18	5.06	4.98	4.97	4.99	4.99	5.03	5.02	5.04	5.09	5.13	5.16	5.17	5.20	5.22	5.25	5.28	5.29	5.29	5.32
	28	5.34	4.77	4.61	4.51	4.52	4.58	4.60	4.62	4.66	4.68	4.72	4.75	4.78	4.79	4.80	4.83	4.86	4.90	4.94	4.97
	29	4.76	4.44	4.37	4.37	4.42	4.42	4.45	4.50	4.51	4.51	4.54	4.59	4.62	4.68	4.70	4.73	4.76	4.82	4.87	4.91
	30	4.96	4.91	4.72	4.57	4.48	4.49	4.51	4.56	4.63	4.67	4.72	4.75	4.83	4.88	4.93	4.97	5.01	5.07	5.13	5.19

Table B.41: *Mean distance errors (m) using the p-**norm Distribution Distance** algorithm, **sector** surveying and **weighted** output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	3.66	3.36	3.22	3.14	3.12	3.13	3.16	3.20	3.21	3.24	3.27	3.32	3.34	3.40	3.46	3.50	3.54	3.58	3.61	3.64
	2	3.58	3.24	3.06	3.02	3.04	3.03	3.05	3.05	3.05	3.08	3.13	3.19	3.22	3.26	3.29	3.33	3.37	3.40	3.43	3.48
	3	3.50	3.10	3.01	2.90	2.92	2.95	2.98	3.00	3.01	3.04	3.07	3.10	3.13	3.16	3.19	3.22	3.26	3.30	3.34	3.40
	4	3.51	3.12	3.00	2.92	2.90	2.91	2.94	2.97	2.98	3.02	3.06	3.10	3.12	3.13	3.18	3.21	3.25	3.29	3.34	3.39
	5	3.48	3.22	3.08	2.93	2.90	2.96	3.00	3.00	3.03	3.03	3.08	3.12	3.13	3.16	3.19	3.21	3.26	3.29	3.32	3.36
	6	3.44	3.08	2.96	2.89	2.92	2.93	2.91	2.93	2.95	2.98	2.97	3.00	3.05	3.09	3.11	3.13	3.17	3.22	3.26	3.30
	7	3.41	3.21	3.04	2.99	2.94	2.92	2.93	2.94	2.94	2.95	2.99	3.02	3.05	3.09	3.12	3.16	3.19	3.21	3.26	3.30
	8	3.42	3.12	3.03	2.97	2.91	2.94	2.97	2.97	2.99	3.00	3.03	3.05	3.08	3.13	3.16	3.20	3.24	3.29	3.32	3.35
	9	3.62	3.23	3.07	3.00	3.02	2.97	2.98	2.99	3.03	3.07	3.08	3.09	3.13	3.14	3.16	3.19	3.21	3.25	3.29	3.32
	10	3.37	3.27	3.07	3.01	2.96	2.99	3.01	3.04	3.05	3.07	3.11	3.13	3.15	3.16	3.18	3.20	3.23	3.26	3.30	3.33
	11	3.52	3.19	3.08	3.02	2.99	3.00	3.03	3.02	3.06	3.08	3.12	3.15	3.18	3.20	3.22	3.26	3.29	3.31	3.34	3.38
	12	3.42	3.16	3.12	3.07	3.04	3.05	3.05	3.06	3.10	3.13	3.14	3.16	3.16	3.20	3.23	3.25	3.28	3.32	3.35	3.40
	13	3.69	3.26	3.13	3.10	3.10	3.12	3.11	3.15	3.17	3.21	3.22	3.23	3.28	3.31	3.33	3.36	3.39	3.42	3.45	3.48
	14	3.77	3.34	3.21	3.13	3.09	3.09	3.08	3.11	3.10	3.12	3.17	3.20	3.23	3.28	3.32	3.34	3.37	3.39	3.42	3.46
b	15	3.83	3.47	3.44	3.35	3.29	3.34	3.36	3.36	3.36	3.37	3.40	3.41	3.43	3.47	3.50	3.54	3.57	3.60	3.63	3.64
	16	3.74	3.37	3.30	3.25	3.22	3.21	3.21	3.21	3.22	3.25	3.27	3.29	3.30	3.33	3.37	3.40	3.43	3.46	3.50	3.52
	17	3.95	3.64	3.52	3.46	3.52	3.49	3.46	3.47	3.49	3.50	3.51	3.53	3.54	3.56	3.60	3.63	3.67	3.70	3.74	3.75
	18	4.08	3.82	3.73	3.67	3.71	3.72	3.76	3.75	3.76	3.78	3.78	3.80	3.84	3.84	3.85	3.86	3.90	3.91	3.94	3.97
	19	4.28	3.90	3.71	3.71	3.69	3.71	3.70	3.67	3.67	3.67	3.67	3.70	3.75	3.77	3.79	3.81	3.83	3.86	3.91	3.94
	20	4.03	3.68	3.59	3.51	3.52	3.50	3.46	3.47	3.46	3.47	3.50	3.52	3.55	3.57	3.59	3.60	3.63	3.65	3.68	3.71
	21	4.06	3.80	3.69	3.65	3.60	3.56	3.55	3.54	3.55	3.57	3.61	3.63	3.64	3.66	3.69	3.71	3.74	3.75	3.77	3.79
	22	4.43	4.01	3.80	3.73	3.70	3.66	3.68	3.68	3.70	3.71	3.72	3.75	3.76	3.79	3.83	3.86	3.89	3.92	3.95	3.99
	23	4.34	4.05	3.92	3.93	3.93	3.99	4.02	4.05	4.07	4.11	4.14	4.16	4.20	4.23	4.25	4.27	4.33	4.38	4.43	4.46
	24	4.46	4.08	3.89	3.88	3.90	3.88	3.88	3.91	3.97	3.99	4.02	4.03	4.05	4.08	4.12	4.16	4.20	4.24	4.30	4.32
	25	4.63	4.22	4.13	4.13	4.10	4.10	4.07	4.06	4.06	4.07	4.07	4.12	4.12	4.13	4.14	4.17	4.19	4.20	4.22	4.26
	26	4.81	4.41	4.21	4.14	4.08	4.08	4.08	4.07	4.08	4.10	4.10	4.12	4.16	4.18	4.21	4.23	4.23	4.26	4.28	4.30
	27	4.92	4.27	4.21	4.11	4.08	4.08	4.10	4.10	4.13	4.13	4.16	4.15	4.17	4.19	4.24	4.28	4.31	4.32	4.36	4.39
	28	5.14	4.59	4.35	4.28	4.28	4.30	4.33	4.30	4.30	4.31	4.30	4.30	4.33	4.35	4.38	4.38	4.40	4.43	4.44	4.45
	29	4.60	4.32	4.19	4.17	4.14	4.15	4.15	4.11	4.15	4.12	4.13	4.18	4.19	4.21	4.23	4.26	4.27	4.31	4.33	4.35
	30	4.64	4.22	4.05	4.01	4.00	3.99	4.04	4.07	4.09	4.11	4.15	4.17	4.19	4.21	4.22	4.26	4.27	4.28	4.31	4.34

Table B.42: *Mean distance errors (m) using the* ***Match Distribution Distance*** *algorithm,* ***point*** *surveying and* ***arithmetic*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	3.66	3.34	3.19	3.12	3.09	3.09	3.11	3.15	3.15	3.17	3.20	3.24	3.26	3.31	3.36	3.39	3.43	3.46	3.49	3.51
	2	3.58	3.22	3.04	3.00	3.01	2.99	3.00	3.00	3.00	3.02	3.06	3.11	3.14	3.17	3.20	3.23	3.26	3.28	3.31	3.35
	3	3.50	3.09	2.99	2.89	2.89	2.91	2.93	2.95	2.95	2.98	3.00	3.03	3.05	3.07	3.09	3.12	3.15	3.19	3.22	3.27
	4	3.51	3.11	2.98	2.90	2.87	2.87	2.89	2.91	2.92	2.95	2.98	3.02	3.03	3.05	3.09	3.11	3.14	3.17	3.21	3.25
	5	3.48	3.20	3.06	2.92	2.87	2.93	2.95	2.95	2.97	2.97	3.01	3.04	3.05	3.07	3.10	3.11	3.15	3.18	3.21	3.23
	6	3.44	3.07	2.93	2.87	2.88	2.88	2.87	2.88	2.89	2.91	2.91	2.93	2.97	3.00	3.02	3.03	3.06	3.10	3.13	3.17
	7	3.41	3.17	3.01	2.96	2.91	2.88	2.88	2.89	2.89	2.89	2.92	2.94	2.96	3.00	3.03	3.06	3.08	3.10	3.14	3.17
	8	3.42	3.11	2.99	2.93	2.87	2.89	2.91	2.91	2.92	2.93	2.95	2.97	2.99	3.03	3.06	3.09	3.12	3.16	3.18	3.20
	9	3.62	3.21	3.05	2.97	2.98	2.93	2.93	2.93	2.96	2.99	3.01	3.01	3.04	3.06	3.07	3.09	3.11	3.14	3.16	3.19
	10	3.37	3.24	3.05	2.98	2.93	2.95	2.96	2.98	2.99	3.00	3.03	3.05	3.07	3.08	3.09	3.11	3.13	3.15	3.18	3.20
	11	3.52	3.18	3.06	2.99	2.96	2.95	2.97	2.96	2.99	3.01	3.04	3.06	3.08	3.10	3.12	3.15	3.17	3.19	3.21	3.24
	12	3.42	3.12	3.08	3.03	2.99	2.99	2.99	3.00	3.03	3.04	3.06	3.07	3.07	3.09	3.11	3.13	3.16	3.18	3.21	3.24
	13	3.69	3.24	3.10	3.06	3.05	3.05	3.04	3.07	3.08	3.11	3.12	3.13	3.16	3.18	3.20	3.22	3.24	3.26	3.29	3.31
	14	3.77	3.34	3.20	3.12	3.07	3.05	3.04	3.06	3.05	3.06	3.09	3.11	3.14	3.17	3.21	3.22	3.24	3.25	3.27	3.30
b	15	3.82	3.46	3.37	3.30	3.27	3.29	3.31	3.28	3.29	3.31	3.32	3.33	3.35	3.37	3.39	3.41	3.42	3.45	3.48	3.50
	16	3.73	3.41	3.31	3.25	3.21	3.20	3.19	3.18	3.18	3.19	3.20	3.21	3.22	3.23	3.25	3.27	3.30	3.32	3.35	3.37
	17	3.94	3.62	3.49	3.43	3.43	3.41	3.40	3.41	3.44	3.44	3.44	3.46	3.48	3.48	3.50	3.53	3.55	3.58	3.60	3.61
	18	4.11	3.85	3.69	3.64	3.64	3.65	3.67	3.67	3.67	3.68	3.70	3.72	3.74	3.74	3.75	3.77	3.78	3.80	3.81	3.84
	19	NaN	3.86	3.68	3.66	3.61	3.59	3.61	3.59	3.59	3.60	3.59	3.61	3.65	3.66	3.68	3.69	3.70	3.73	3.75	3.77
	20	NaN	3.67	3.49	3.46	3.46	3.47	3.43	3.42	3.42	3.41	3.42	3.44	3.45	3.46	3.48	3.49	3.51	3.53	3.55	3.56
	21	4.06	3.73	3.63	3.60	3.53	3.50	3.49	3.50	3.50	3.50	3.52	3.53	3.54	3.56	3.58	3.59	3.60	3.61	3.62	3.63
	22	4.37	4.02	3.81	3.71	3.67	3.64	3.65	3.64	3.64	3.64	3.65	3.66	3.66	3.68	3.70	3.71	3.72	3.74	3.76	3.79
	23	4.38	4.03	3.92	3.88	3.90	3.92	3.94	3.96	3.98	3.99	4.00	4.02	4.02	4.05	4.07	4.11	4.14	4.16	4.19	4.21
	24	NaN	4.04	3.91	3.87	3.85	3.84	3.83	3.84	3.86	3.89	3.90	3.92	3.95	3.97	4.00	4.02	4.05	4.07	4.10	4.12
	25	NaN	4.22	4.06	4.05	4.01	4.01	3.98	3.98	3.98	4.00	4.00	4.02	4.03	4.04	4.05	4.06	4.08	4.08	4.09	4.10
	26	NaN	4.36	4.20	4.10	4.05	4.01	4.02	4.02	4.04	4.05	4.04	4.03	4.05	4.05	4.07	4.08	4.07	4.09	4.10	4.11
	27	NaN	4.28	4.21	4.10	4.03	4.03	4.04	4.02	4.03	4.05	4.07	4.07	4.08	4.09	4.11	4.13	4.15	4.17	4.20	4.21
	28	NaN	4.63	4.37	4.23	4.22	4.22	4.23	4.20	4.20	4.21	4.22	4.22	4.24	4.25	4.27	4.27	4.26	4.28	4.30	4.30
	29	NaN	4.29	4.17	4.17	4.10	4.09	4.07	4.06	4.06	4.06	4.06	4.06	4.08	4.10	4.10	4.12	4.13	4.14	4.16	4.18
	30	4.63	4.25	4.10	4.03	3.98	3.97	3.97	4.00	3.99	4.00	4.00	4.01	4.03	4.03	4.04	4.06	4.07	4.09	4.08	4.09

Table B.43: *Mean distance errors (m) using the **Match Distribution Distance** algorithm, **point** surveying and **weighted** output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	1	4.07	3.89	3.93	4.02	4.08	4.17	4.25	4.33	4.36	4.43	4.49	4.54	4.58	4.61	4.66	4.69	4.72	4.77	4.82	4.89
	2	3.23	3.14	3.11	3.27	3.37	3.56	3.66	3.80	3.89	3.96	4.03	4.11	4.17	4.22	4.29	4.35	4.40	4.49	4.57	4.63
	3	3.12	2.99	2.97	3.13	3.22	3.40	3.54	3.69	3.80	3.86	3.94	3.99	4.07	4.14	4.21	4.29	4.37	4.43	4.50	4.57
	4	3.08	2.96	2.97	3.08	3.21	3.41	3.56	3.68	3.77	3.85	3.93	4.02	4.08	4.16	4.23	4.31	4.38	4.43	4.52	4.61
	5	3.29	3.12	3.06	3.18	3.25	3.40	3.56	3.69	3.80	3.86	3.94	4.02	4.10	4.17	4.25	4.33	4.40	4.47	4.54	4.63
	6	3.31	3.05	3.05	3.13	3.26	3.39	3.54	3.66	3.77	3.85	3.93	4.03	4.11	4.19	4.28	4.35	4.41	4.47	4.55	4.62
	7	3.35	3.16	3.08	3.17	3.29	3.46	3.63	3.73	3.85	3.92	3.97	4.05	4.14	4.21	4.29	4.36	4.42	4.49	4.55	4.62
	8	3.25	3.06	3.07	3.13	3.27	3.46	3.59	3.70	3.77	3.88	3.98	4.05	4.15	4.24	4.31	4.39	4.46	4.53	4.58	4.66
	9	3.38	3.21	3.16	3.20	3.33	3.44	3.60	3.71	3.78	3.87	3.97	4.06	4.16	4.26	4.32	4.38	4.46	4.54	4.63	4.70
	10	3.42	3.23	3.25	3.30	3.37	3.51	3.64	3.73	3.85	3.92	4.02	4.08	4.17	4.24	4.30	4.37	4.42	4.49	4.58	4.66
	11	3.33	3.26	3.27	3.38	3.53	3.67	3.81	3.95	4.05	4.14	4.20	4.24	4.33	4.39	4.47	4.53	4.60	4.67	4.73	4.79
	12	3.47	3.28	3.30	3.35	3.45	3.59	3.70	3.78	3.85	3.96	4.05	4.17	4.27	4.34	4.39	4.47	4.53	4.61	4.67	4.71
	13	3.37	3.28	3.39	3.46	3.61	3.76	3.88	3.96	4.01	4.10	4.18	4.27	4.35	4.40	4.46	4.54	4.58	4.65	4.71	4.78
	14	3.81	3.62	3.48	3.59	3.67	3.86	3.99	4.08	4.13	4.21	4.29	4.35	4.41	4.49	4.58	4.65	4.71	4.75	4.80	4.83
	15	3.80	3.57	3.49	3.52	3.64	3.78	3.87	3.96	4.07	4.10	4.18	4.24	4.31	4.38	4.45	4.51	4.56	4.62	4.67	4.74
	16	3.84	3.66	3.67	3.65	3.73	3.80	3.92	3.98	4.03	4.11	4.15	4.21	4.26	4.34	4.42	4.49	4.56	4.62	4.70	4.75
	17	3.83	3.67	3.64	3.80	3.97	4.11	4.22	4.31	4.40	4.48	4.56	4.66	4.74	4.79	4.85	4.89	4.95	5.02	5.07	5.10
	18	4.50	4.02	3.98	4.03	4.11	4.21	4.28	4.41	4.48	4.56	4.62	4.67	4.73	4.76	4.82	4.87	4.91	4.97	5.03	5.08
	19	4.36	4.19	4.05	4.02	4.03	4.13	4.22	4.24	4.28	4.33	4.43	4.47	4.51	4.53	4.57	4.60	4.66	4.73	4.79	4.83
	20	3.88	3.78	3.72	3.77	3.79	3.86	3.98	4.02	4.09	4.14	4.19	4.23	4.28	4.35	4.42	4.50	4.56	4.61	4.69	4.74
	21	4.08	3.76	3.74	3.81	3.86	3.88	4.01	4.08	4.16	4.22	4.30	4.39	4.42	4.49	4.56	4.64	4.69	4.74	4.80	4.84
	22	4.37	4.11	4.06	4.27	4.32	4.50	4.58	4.66	4.76	4.81	4.87	4.94	4.97	5.00	5.04	5.09	5.13	5.16	5.19	5.22
	23	4.92	4.68	4.59	4.74	4.81	4.94	5.09	5.15	5.20	5.27	5.30	5.33	5.36	5.39	5.43	5.47	5.51	5.54	5.55	5.58
	24	4.82	4.60	4.54	4.59	4.68	4.75	4.82	4.90	4.99	5.06	5.11	5.16	5.18	5.22	5.32	5.37	5.41	5.49	5.53	5.57
	25	5.10	4.74	4.69	4.70	4.75	4.86	4.97	4.99	5.01	5.02	5.06	5.10	5.13	5.19	5.26	5.33	5.43	5.50	5.58	5.65
	26	5.23	4.95	4.82	4.81	4.91	4.96	4.99	5.07	5.09	5.14	5.19	5.26	5.27	5.29	5.30	5.37	5.45	5.53	5.59	5.63
	27	5.53	5.02	4.95	4.91	4.84	4.88	4.96	5.06	5.13	5.16	5.19	5.22	5.24	5.31	5.36	5.40	5.43	5.42	5.45	5.48
	28	5.16	4.68	4.54	4.50	4.57	4.62	4.69	4.72	4.80	4.82	4.86	4.91	4.92	4.97	5.02	5.06	5.10	5.15	5.19	5.20
	29	4.87	4.55	4.46	4.45	4.49	4.58	4.57	4.60	4.61	4.65	4.70	4.78	4.82	4.89	4.91	4.96	5.00	5.07	5.11	5.14
	30	5.10	4.89	4.72	4.51	4.44	4.47	4.50	4.58	4.63	4.66	4.70	4.74	4.77	4.85	4.89	4.92	4.95	4.98	5.05	5.08

Table B.44: *Mean distance errors (m) using the* ***Match Distribution Distance*** *algorithm,* ***sector*** *surveying and* ***arithmetic*** *output averaging in the CSE environment.*

		k																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	4.07	3.86	3.89	3.98	4.02	4.11	4.19	4.26	4.29	4.36	4.41	4.46	4.50	4.53	4.58	4.61	4.64	4.68	4.73	4.80
	2	3.23	3.13	3.09	3.22	3.32	3.48	3.58	3.70	3.79	3.86	3.93	4.01	4.07	4.11	4.18	4.23	4.29	4.36	4.44	4.49
	3	3.12	2.98	2.95	3.09	3.17	3.32	3.45	3.59	3.69	3.75	3.82	3.87	3.95	4.01	4.08	4.16	4.23	4.29	4.35	4.42
	4	3.08	2.95	2.95	3.04	3.15	3.32	3.46	3.56	3.65	3.72	3.80	3.89	3.95	4.02	4.09	4.16	4.22	4.27	4.35	4.43
	5	3.29	3.12	3.04	3.13	3.19	3.32	3.46	3.58	3.68	3.73	3.81	3.88	3.96	4.02	4.10	4.17	4.23	4.29	4.36	4.43
	6	3.31	3.05	3.03	3.09	3.20	3.30	3.43	3.54	3.64	3.71	3.79	3.88	3.95	4.03	4.11	4.17	4.22	4.28	4.35	4.41
	7	3.35	3.15	3.05	3.13	3.22	3.37	3.51	3.59	3.70	3.77	3.82	3.89	3.97	4.04	4.11	4.18	4.23	4.29	4.35	4.40
	8	3.25	3.05	3.04	3.09	3.19	3.35	3.46	3.55	3.62	3.72	3.81	3.88	3.97	4.05	4.11	4.18	4.24	4.31	4.36	4.42
	9	3.38	3.20	3.13	3.16	3.26	3.34	3.49	3.57	3.63	3.71	3.80	3.88	3.97	4.05	4.11	4.17	4.24	4.31	4.39	4.45
	10	3.42	3.22	3.21	3.25	3.30	3.40	3.51	3.59	3.69	3.76	3.85	3.90	3.98	4.05	4.10	4.16	4.21	4.27	4.34	4.42
	11	3.33	3.24	3.23	3.32	3.42	3.53	3.65	3.77	3.86	3.94	4.00	4.04	4.12	4.18	4.24	4.30	4.36	4.42	4.48	4.52
	12	3.47	3.27	3.26	3.30	3.36	3.47	3.56	3.63	3.68	3.78	3.86	3.97	4.05	4.11	4.16	4.23	4.29	4.35	4.40	4.45
	13	3.37	3.26	3.34	3.40	3.51	3.63	3.73	3.79	3.85	3.92	4.00	4.07	4.14	4.19	4.24	4.31	4.35	4.41	4.46	4.52
	14	3.81	3.61	3.46	3.53	3.58	3.73	3.83	3.91	3.96	4.03	4.10	4.15	4.21	4.28	4.35	4.41	4.46	4.50	4.55	4.58
b	15	3.80	3.56	3.45	3.46	3.54	3.64	3.71	3.78	3.87	3.90	3.97	4.02	4.08	4.14	4.20	4.26	4.30	4.35	4.39	4.45
	16	3.84	3.65	3.64	3.61	3.67	3.72	3.81	3.85	3.88	3.94	3.99	4.03	4.07	4.13	4.20	4.25	4.30	4.35	4.41	4.45
	17	3.83	3.66	3.61	3.71	3.83	3.94	4.04	4.12	4.20	4.27	4.35	4.43	4.49	4.54	4.59	4.63	4.68	4.74	4.79	4.82
	18	4.50	4.03	3.96	3.97	4.02	4.07	4.11	4.21	4.26	4.33	4.38	4.43	4.48	4.52	4.57	4.61	4.65	4.70	4.75	4.80
	19	4.36	4.16	4.00	3.95	3.94	4.01	4.08	4.10	4.14	4.18	4.27	4.29	4.33	4.35	4.37	4.40	4.44	4.49	4.53	4.56
	20	3.88	3.77	3.69	3.72	3.73	3.76	3.84	3.88	3.93	3.98	4.01	4.04	4.08	4.13	4.18	4.23	4.28	4.31	4.36	4.40
	21	4.08	3.75	3.71	3.73	3.76	3.77	3.86	3.92	3.98	4.02	4.09	4.16	4.18	4.24	4.30	4.35	4.40	4.43	4.48	4.52
	22	4.37	4.10	4.02	4.16	4.17	4.29	4.35	4.42	4.50	4.54	4.60	4.66	4.69	4.72	4.76	4.80	4.83	4.87	4.89	4.91
	23	4.92	4.66	4.55	4.61	4.65	4.73	4.85	4.90	4.95	5.01	5.05	5.08	5.11	5.14	5.17	5.21	5.24	5.26	5.28	5.31
	24	4.82	4.57	4.49	4.50	4.56	4.61	4.66	4.72	4.78	4.83	4.88	4.92	4.93	4.97	5.03	5.07	5.11	5.17	5.20	5.24
	25	5.10	4.72	4.65	4.65	4.69	4.75	4.81	4.82	4.84	4.84	4.87	4.90	4.94	4.98	5.04	5.08	5.16	5.22	5.29	5.34
	26	5.23	4.95	4.81	4.79	4.83	4.87	4.90	4.95	4.97	5.00	5.04	5.10	5.11	5.13	5.13	5.19	5.25	5.31	5.35	5.39
	27	5.53	5.02	4.94	4.89	4.81	4.83	4.87	4.95	5.00	5.02	5.05	5.07	5.09	5.14	5.19	5.21	5.24	5.23	5.25	5.27
	28	5.16	4.69	4.53	4.45	4.49	4.52	4.55	4.58	4.64	4.66	4.69	4.73	4.74	4.78	4.81	4.84	4.87	4.90	4.92	4.93
	29	4.87	4.50	4.37	4.34	4.35	4.41	4.40	4.43	4.43	4.46	4.50	4.55	4.58	4.64	4.65	4.68	4.71	4.74	4.77	4.79
	30	5.10	4.86	4.61	4.45	4.36	4.34	4.34	4.38	4.41	4.43	4.46	4.49	4.52	4.56	4.59	4.62	4.64	4.66	4.71	4.74

Table B.45: *Mean distance errors (m) using the* ***Match Distribution Distance*** *algorithm,* ***sector*** *surveying and* ***weighted*** *output averaging in the CSE environment.*

Appendix C

Signal Strength Maps

This chapter contains illustrations of the signal strength maps used in our experiments. These map were generated using the average received signal strength in the signal strength maps. For the directional signal strength map, each figure shows the signal strength from a single access point facing each direction; by their nature sector maps are omnidirectional.

Visualizing the signal strength map is a good way to ensure it's integrity prior to positioning use, and it provides a demonstration of how the fingerprinting technique provides unique identifiers of each point. Intuitively, locations with fingerprints containing distinct colour sets are easily differentiated by the positioning algorithm, while those with similar colour sets are not.

C.1 House Environment

Figures C.1-C.6 show the directional maps, and Figures C.7-C.8 show the sector maps.

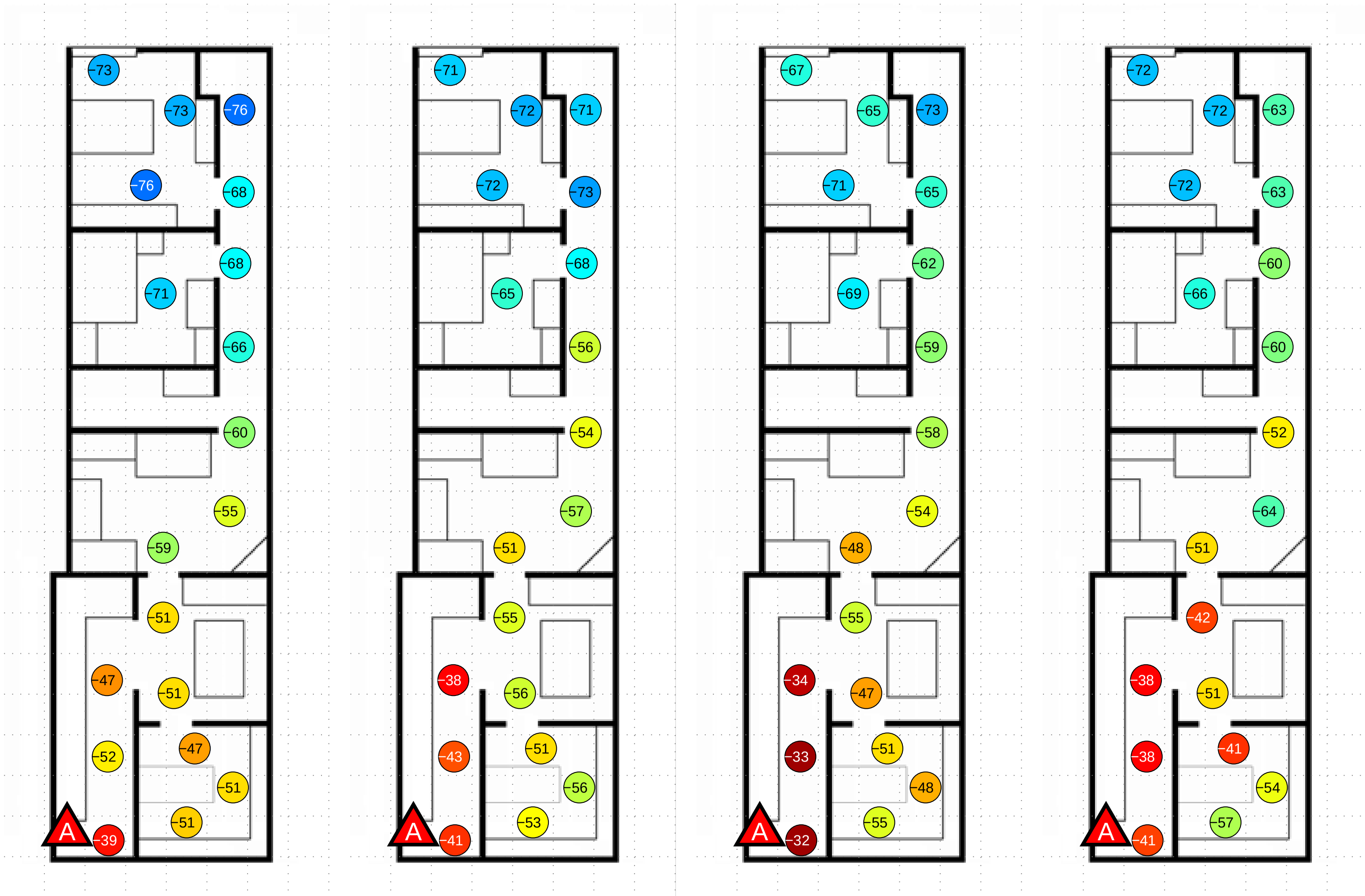
There is a reasonable degree of directional variation due to the attenuating effects of the human body, especially at locations close to the access point where facing some directions results in a direct, unobstructed path between mobile device and access points, and some directions result in a path obstructed by the surveyor.

It is also apparent that some of the access points were more effective transmitters than their peers. In particular, access point B (Figure C.2) seems to be a weak transmitter, relative to peers C and D which are identical models. This might be due to a faulty antenna or an unfavourable location leading to multipath fading. variation from the manufacturing process. Positioning performance should not be greatly affected by this problem, as the rate of decay remains unaffected; maximum throughput would however be degraded and the coverage area for both data and positioning would be reduced.

The effect of direction faced on signal strength is occasionally dramatic (for example, for access point B, the fingerprint immediately west of the access point), but often the received signal strength is similar over all headings. The effect is most obvious in positions closest to the access point; at these positions the surveying person's body blocks a wider angular range, reducing the number of possible reflected paths which might provide similar signal strength to the direct path.

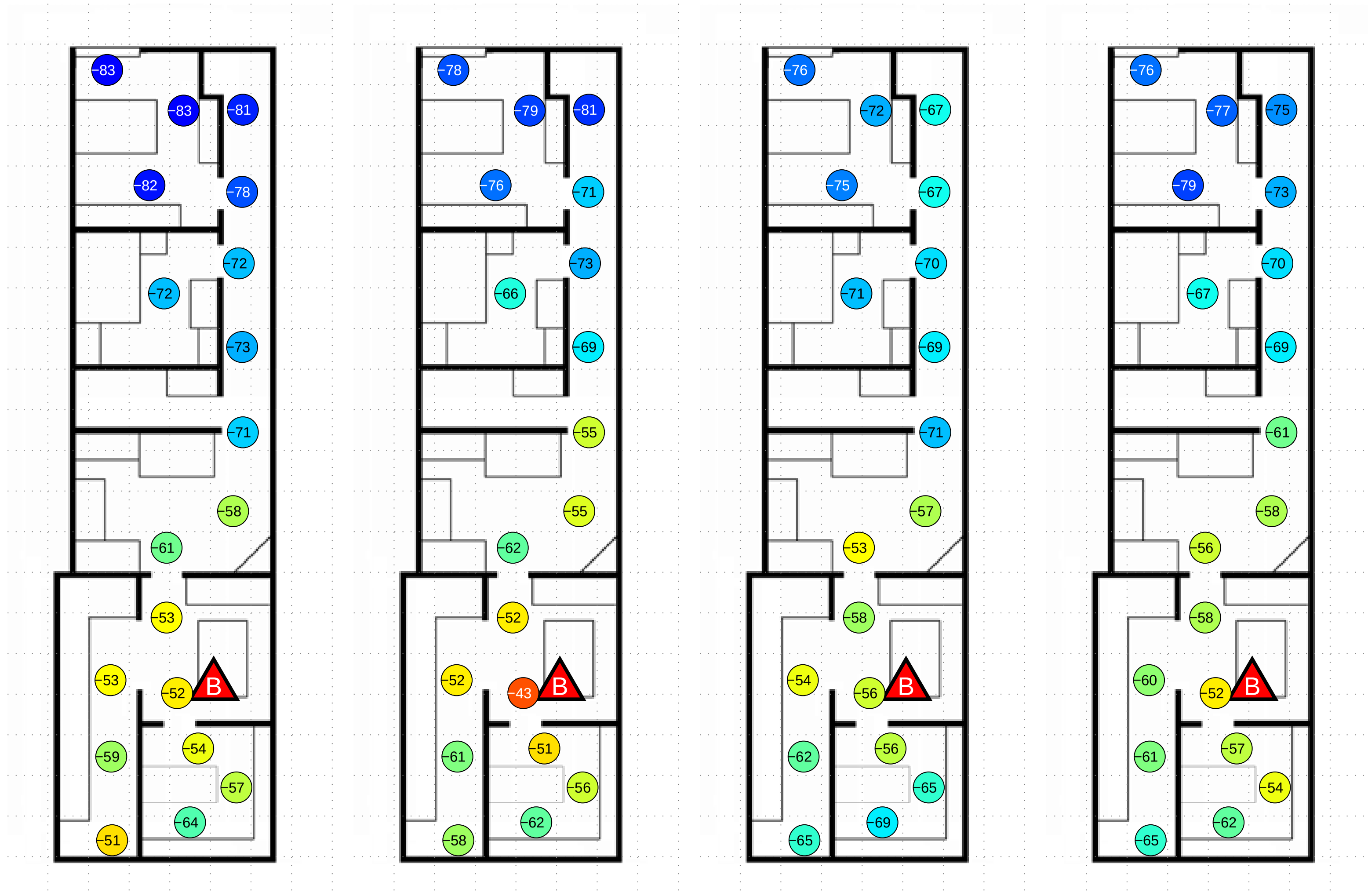
In the directional map for access point C, (Figure C.3) there is a missing point facing south, to the immediate west of the access point. For an unknown reason, no packets were received for this access point during that particular surveying interval. We guessed that this could have been due to either a temporary malfunction on the part of the access point, or noise on the channel - although the signal strengths for the other access points, recorded at the same frequency, appear normal. Packets from the access point were visible when we faced the next direction a few seconds later. This is a good example of visualization picking up an error in the surveying process. In this case we chose not to rerecord the data because we estimated it would have very little impact on the final results; the remaining data from the other directions and access points would have been sufficient to fingerprint that particular location.

The sector map values generally agree with those in the point maps, with the expected property that they usually fall somewhere in between the highest and lowest average signal strengths of the point map.



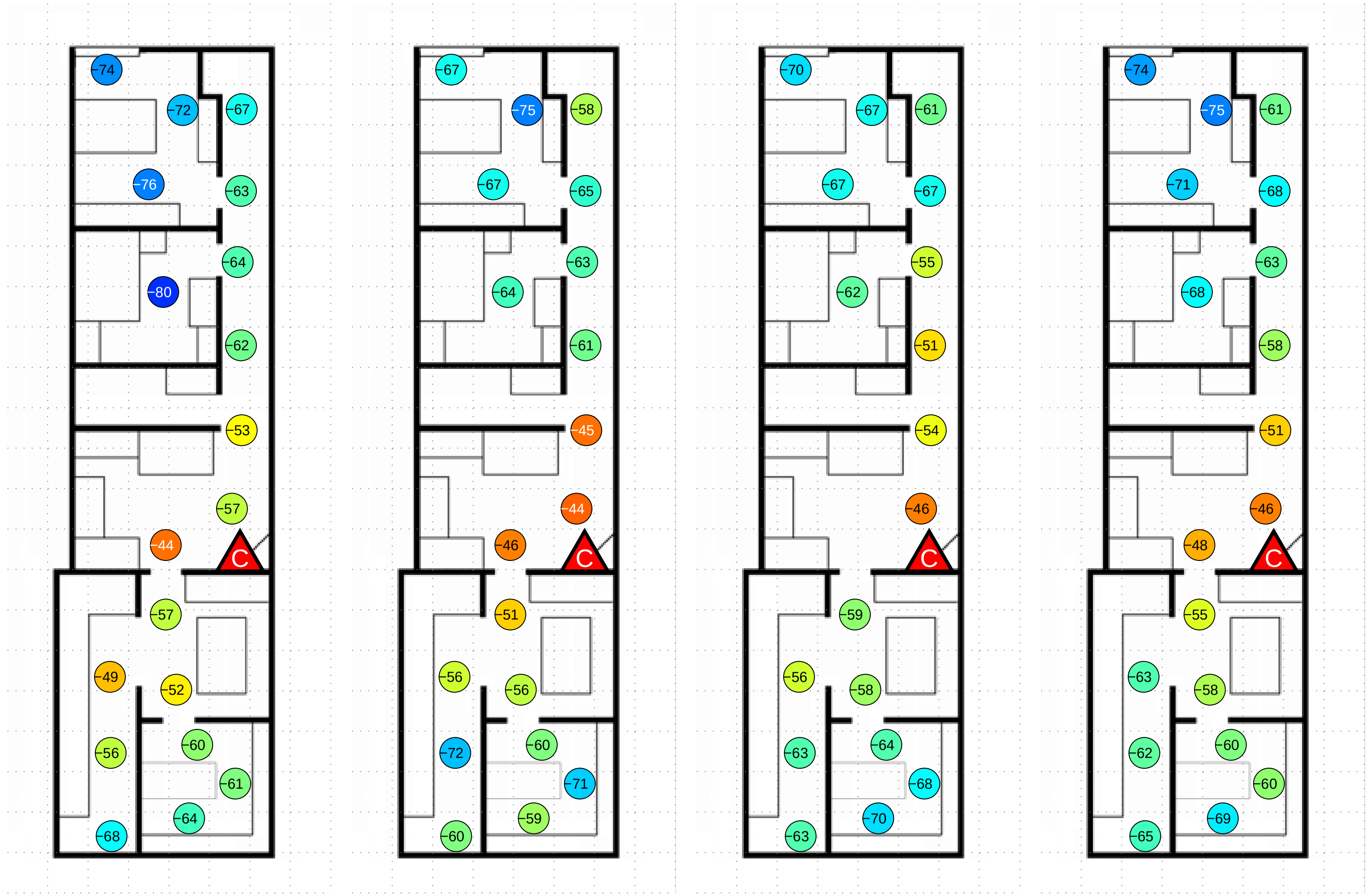
(a) *North* (b) *East* (c) *South* (d) *West*

Figure C.1: *Average RSSIs (dbm) for access point A.*



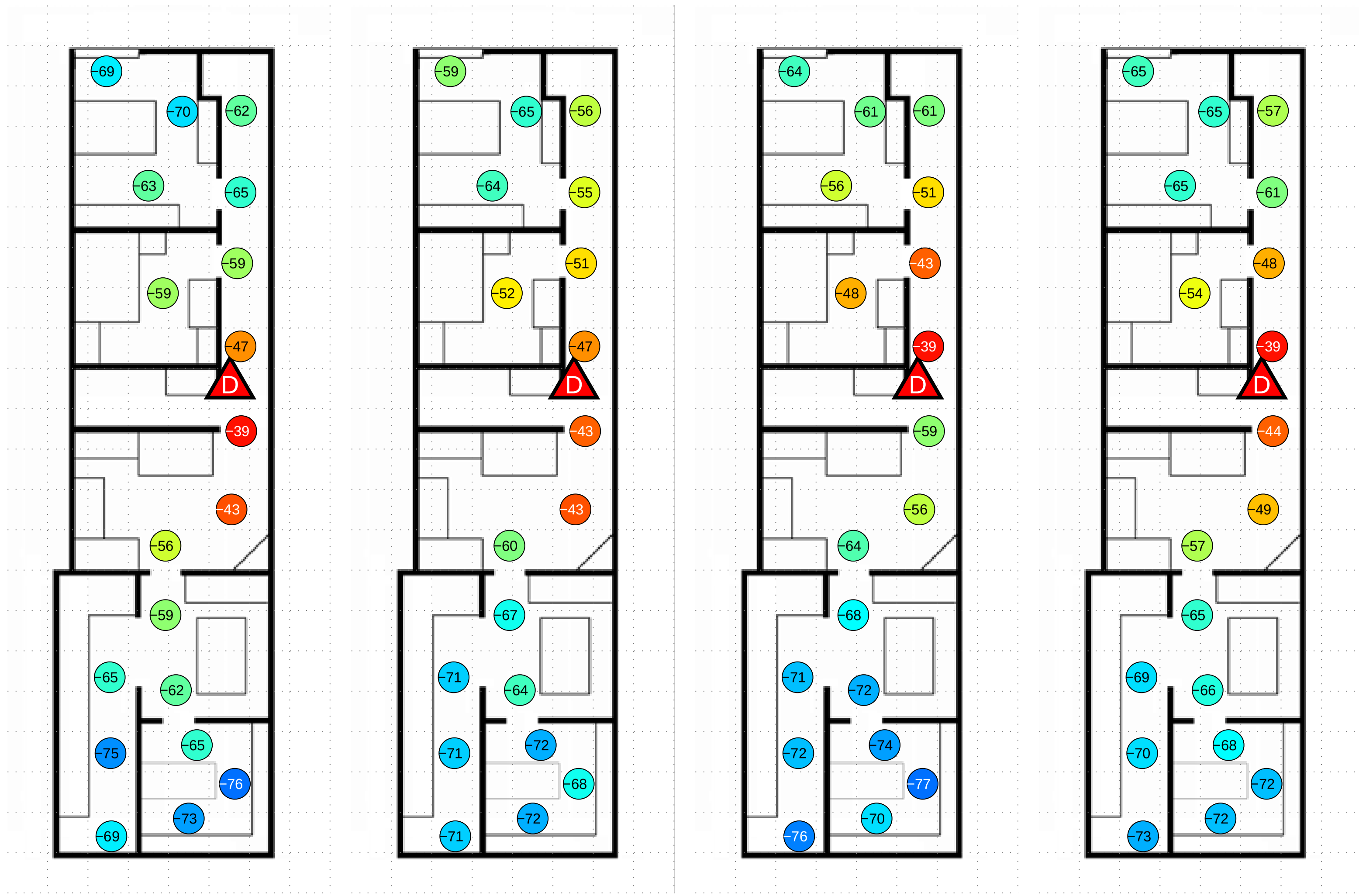
(a) *North* **(b)** *East* **(c)** *South* **(d)** *West*

Figure C.2: *Average RSSIs (dbm) for access point B.*



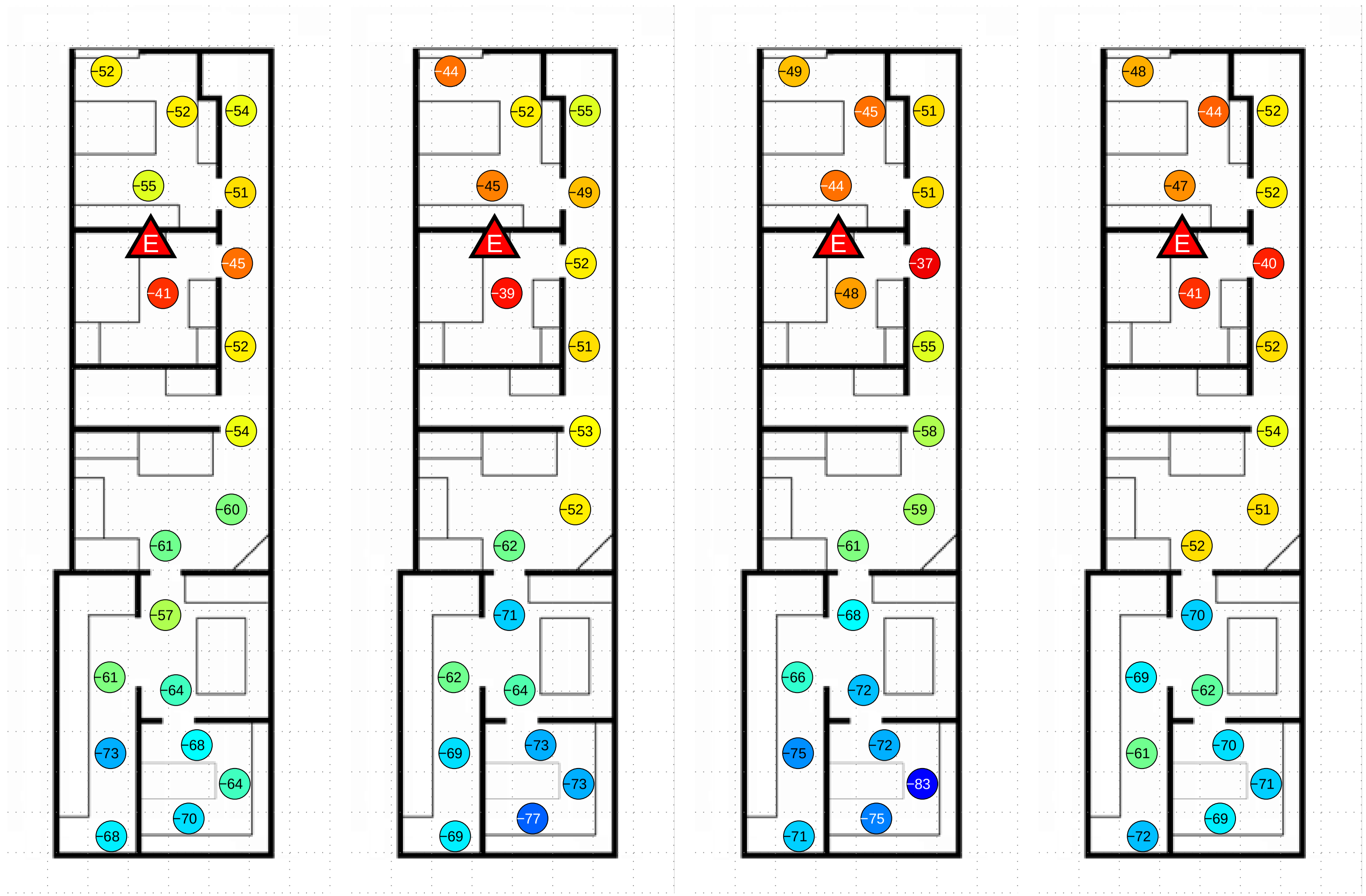
(a) *North* (b) *East* (c) *South* (d) *West*

Figure C.3: *Average RSSIs (dbm) for access point C.*



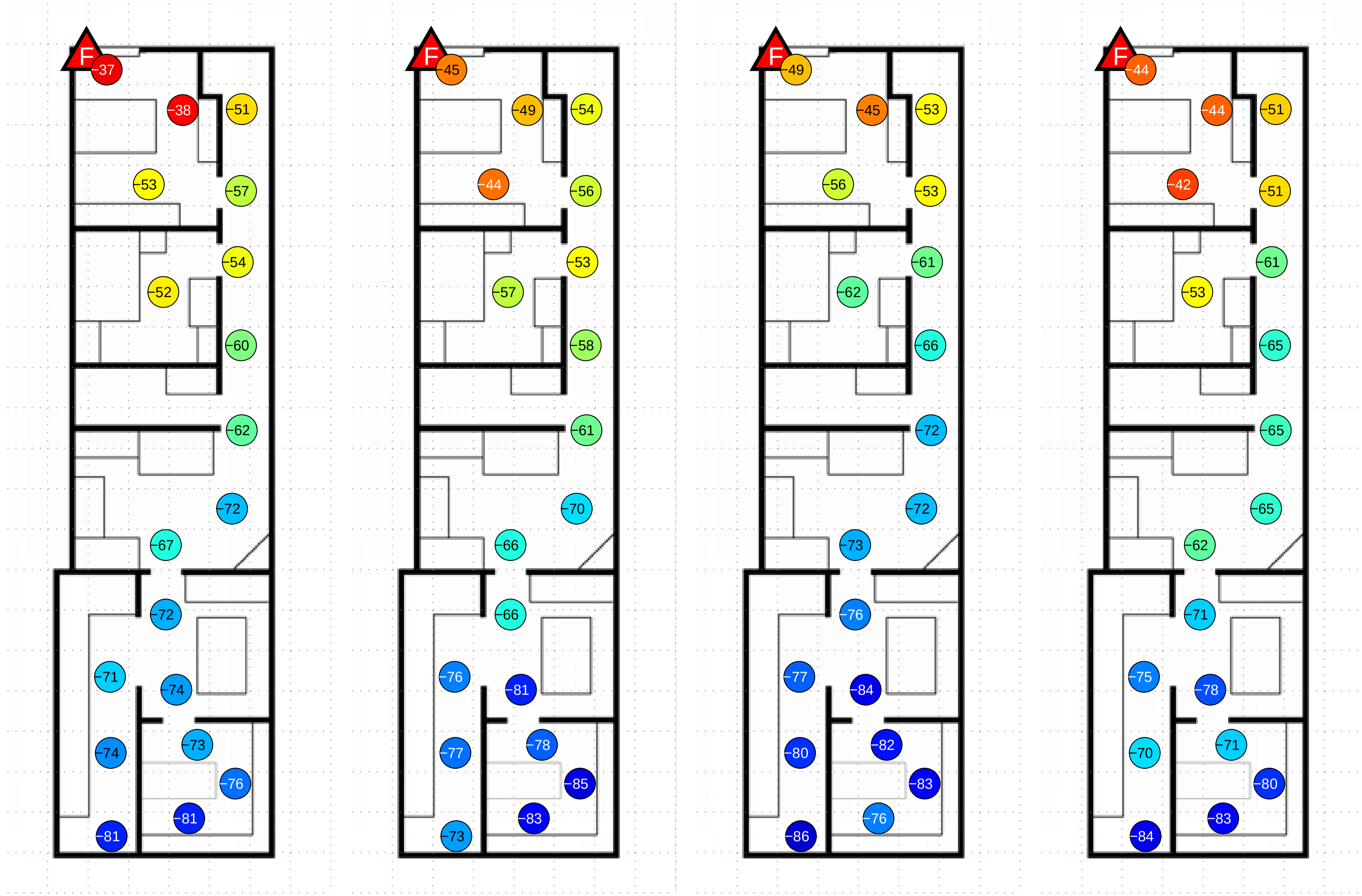
(a) *North* **(b)** *East* **(c)** *South* **(d)** *West*

Figure C.4: *Average RSSIs (dbm) for access point D.*



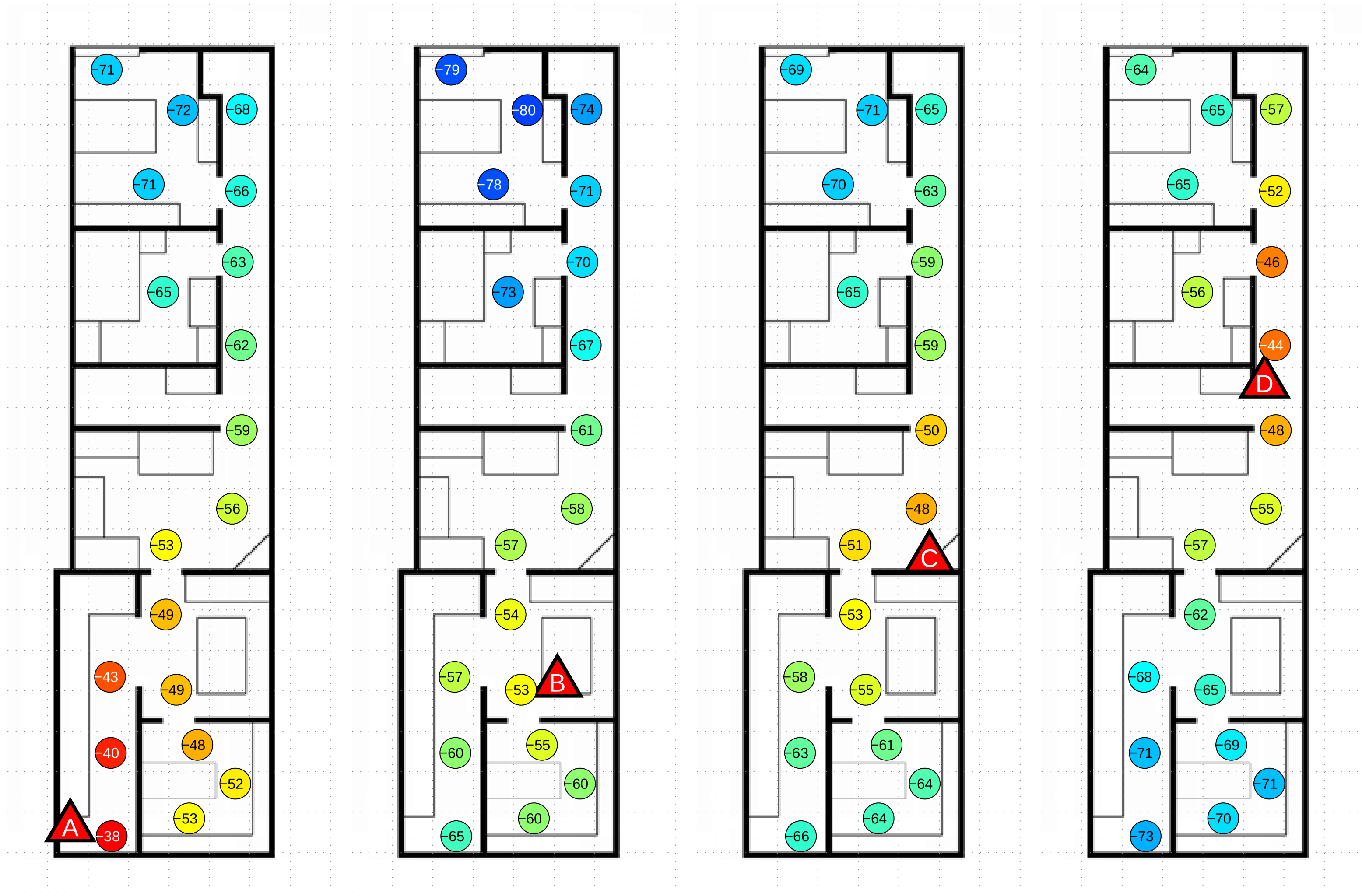
(a) *North* **(b)** *East* **(c)** *South* **(d)** *West*

Figure C.5: *Average RSSIs (dbm) for access point E.*



(a) *North* **(b)** *East* **(c)** *South* **(d)** *West*

Figure C.6: *Average RSSIs (dbm) for access point F.*



(a) *Access point A.* **(b)** *Access point B.* **(c)** *Access point C.* **(d)** *Access point D.*

Figure C.7: *Average RSSIs (dBm) for the house sector map.*

(a) *Access point E.* **(b)** *Access point F.*

Figure C.8: *Average RSSIs (dBm) for the house sector map.*

C.2 CSE Environment

C.2.1 Photos

Photos of the CSE environment are include here for clarity. We did not include photos of the house environment for privacy reasons. The locations at which these photos were taken are indicated in figure C.10.

C.2.2 Signal Strength Maps

Figures C.11-C.15 show the directional signal strength maps, and figures C.16-C.18 shows the sector map. The maps for the CSE environment show similar trends to those of the house environment. There is some directional sensitivity, in particular at locations close to the access points, and the sector maps contain values generally distributed in between the extremes of the point maps.

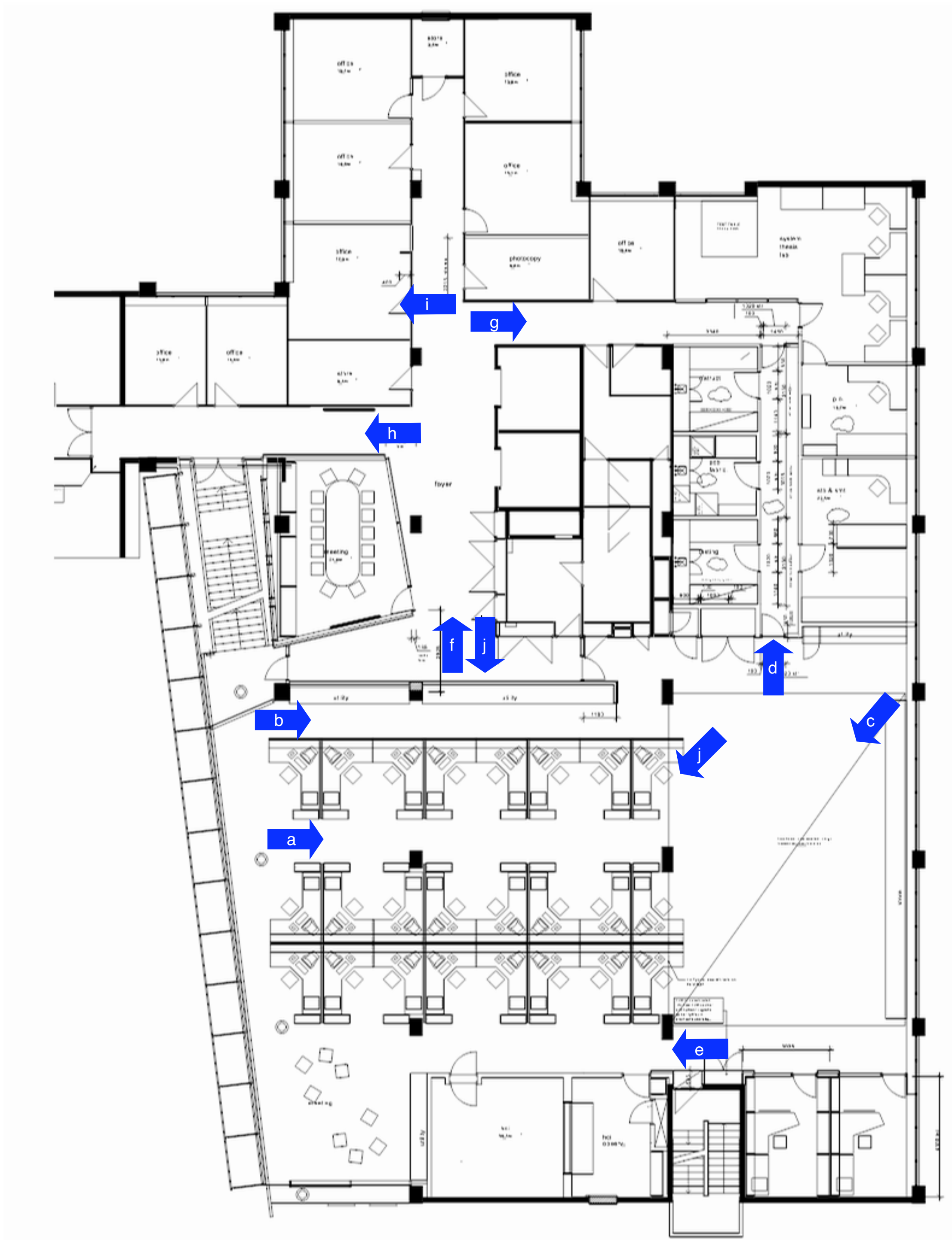
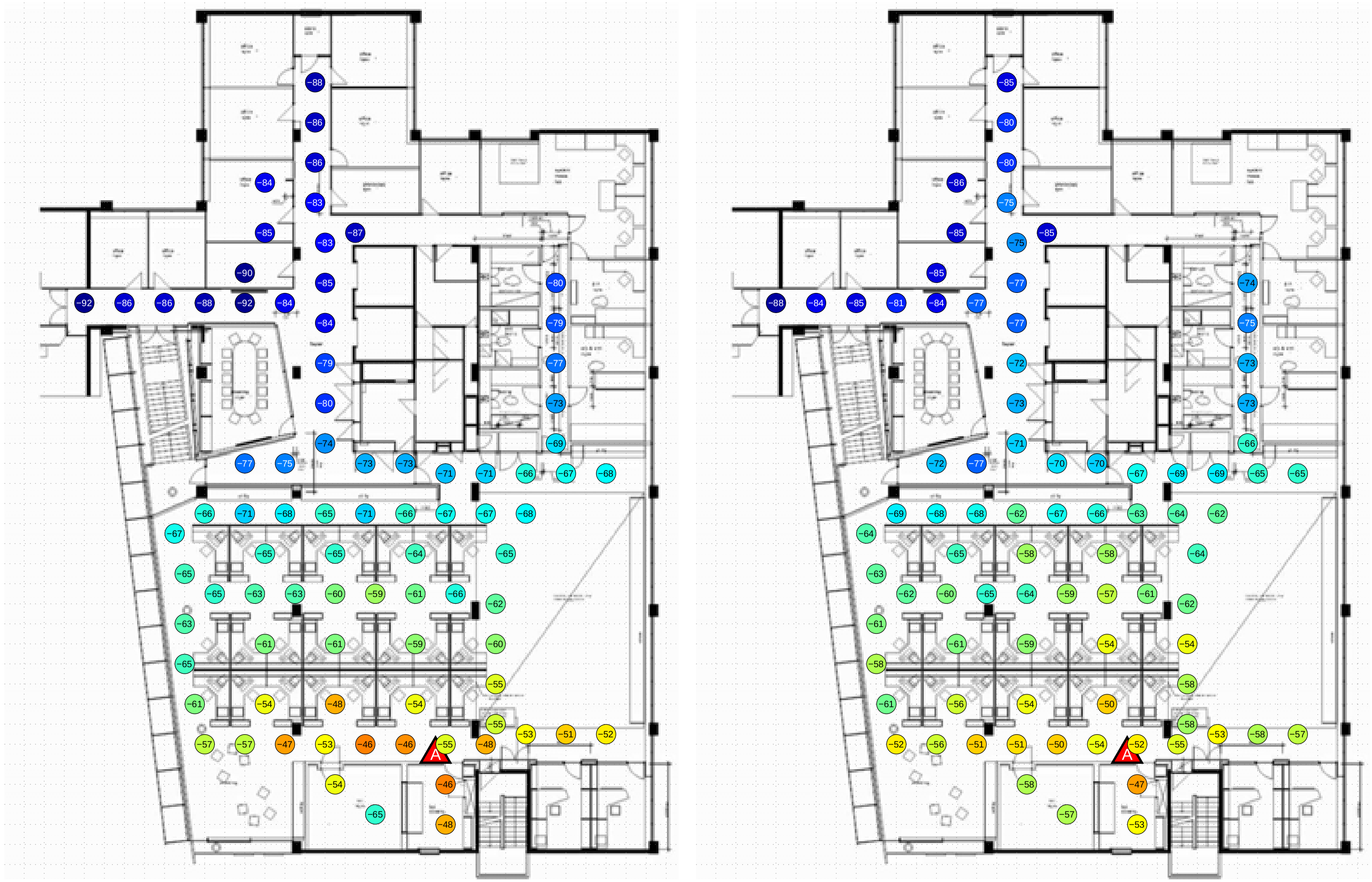


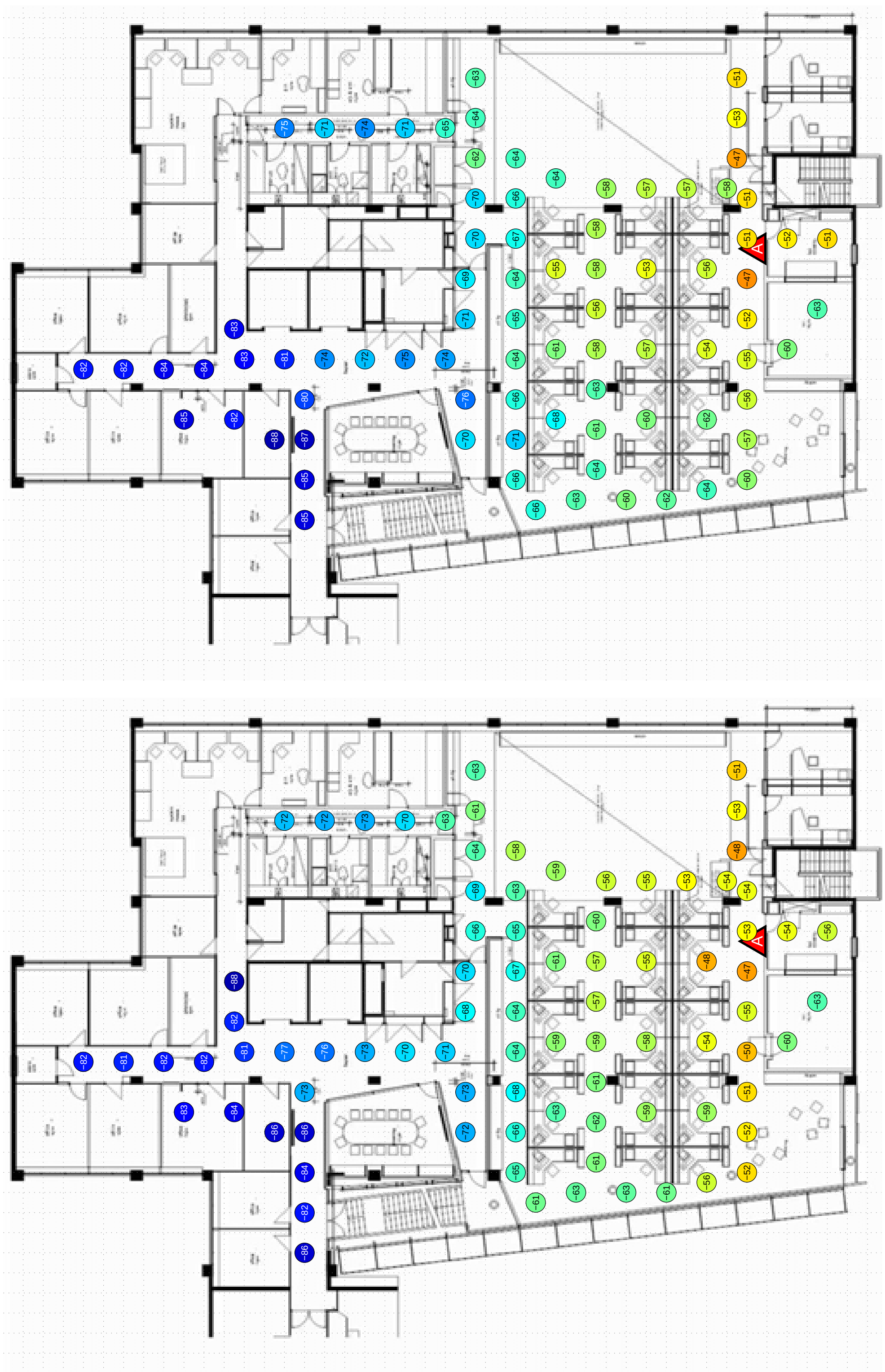
Figure C.9: *Level 3 floor plan, with arrows indicating the location of photos in figure C.10. TODO: there are two Js in this diagram! and might want to edit in the new room on the lower left*

Figure C.10: *Pictures of CSE Level 3.*



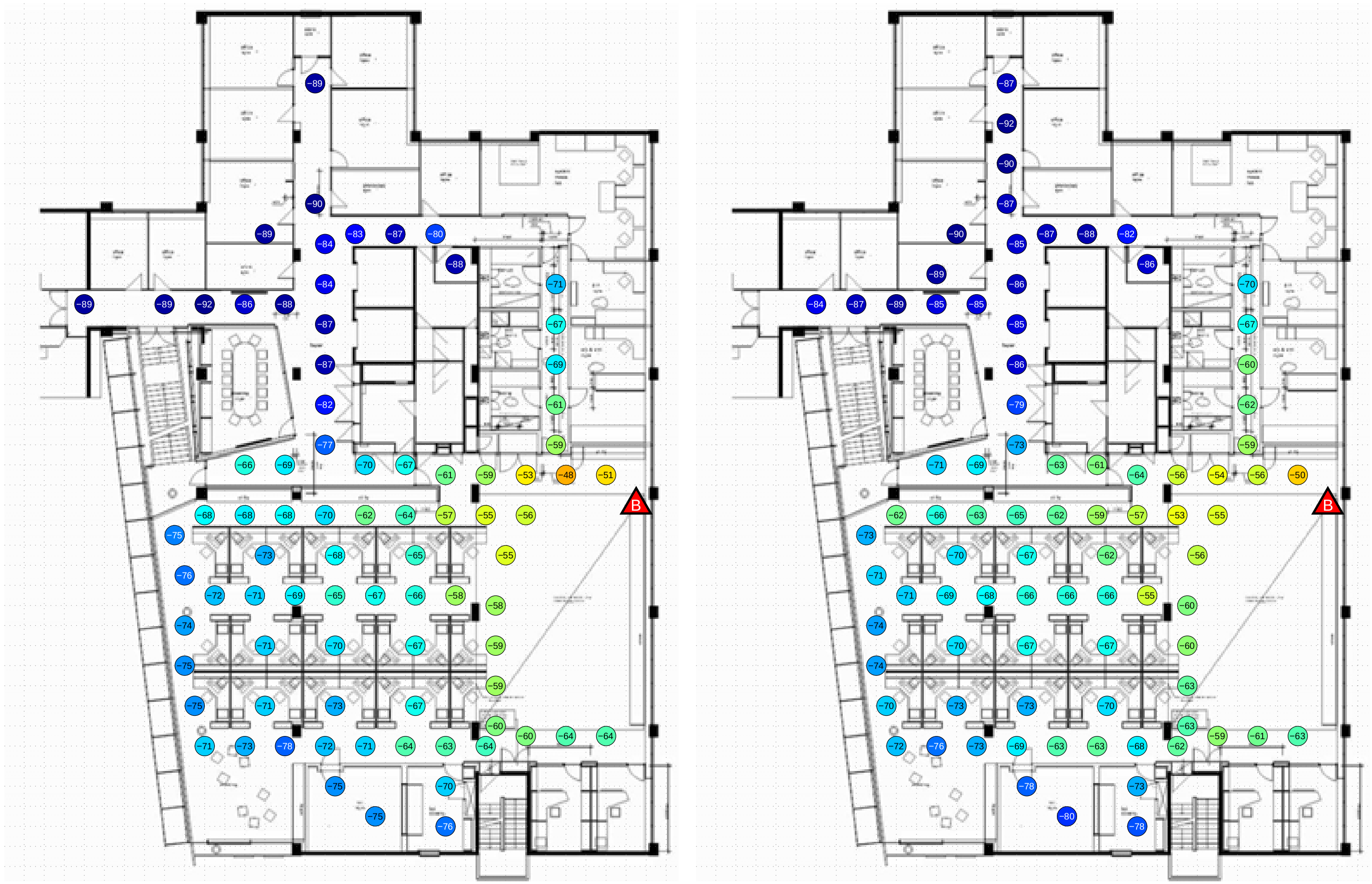
(a) *North* **(b)** *East*

Figure C.11: *Average RSSIs (dBm) for access point A.*



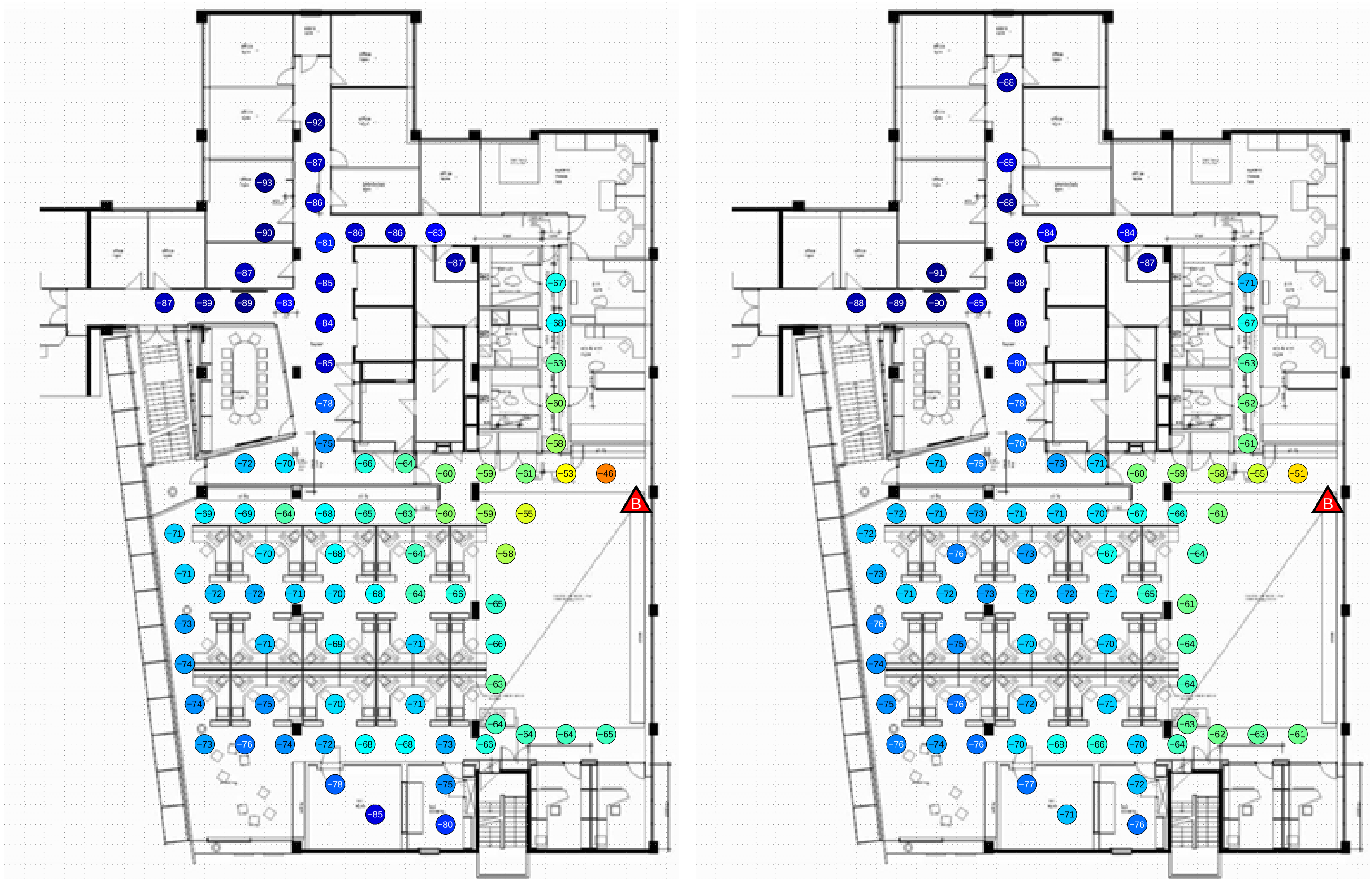
(c) *South* (d) *West*

Figure C.11: *Average RSSIs (dbm) for access point A.*



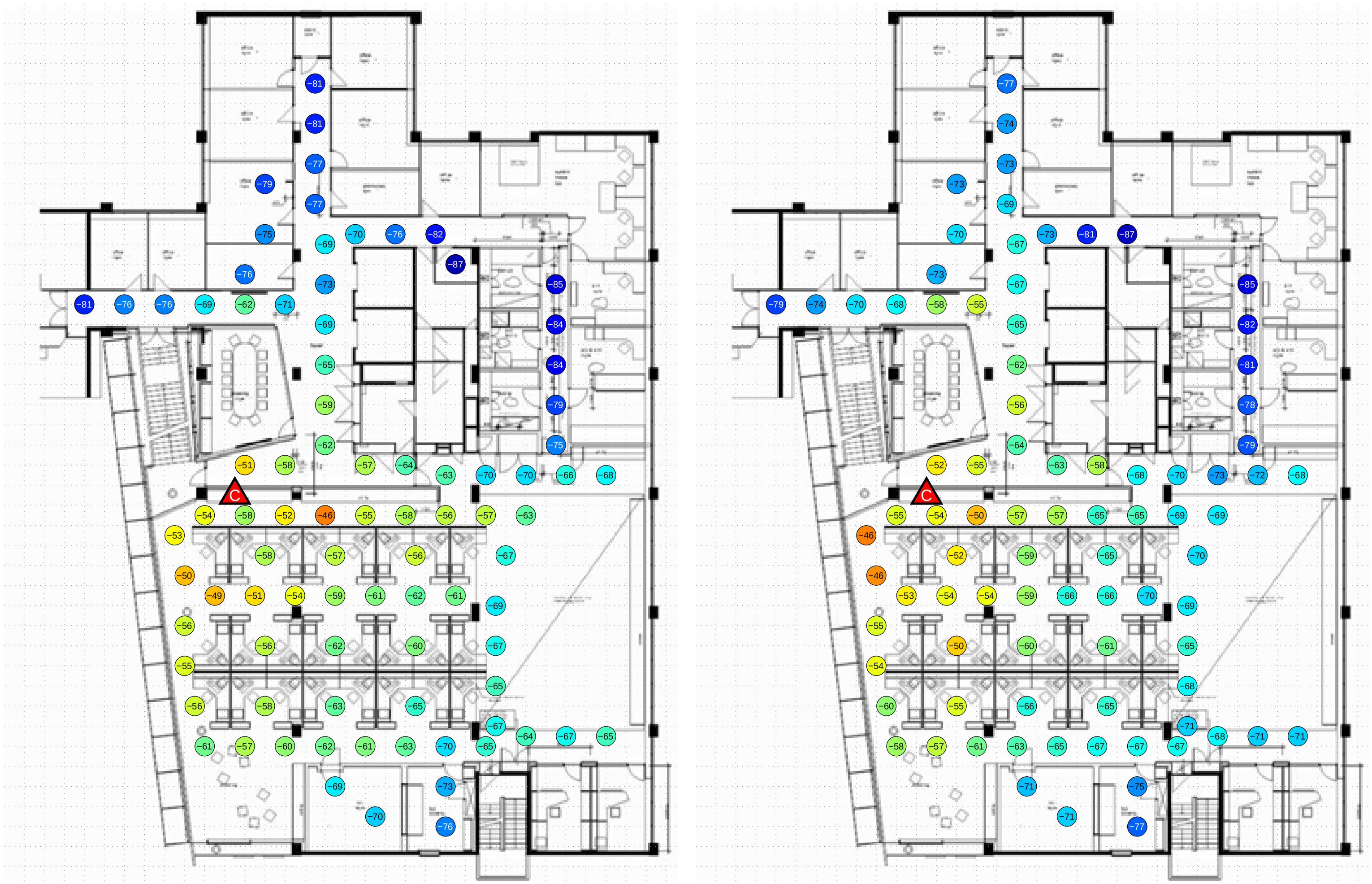
(a) *North* **(b)** *East*

Figure C.12: *Average RSSIs (dBm) for access point B.*



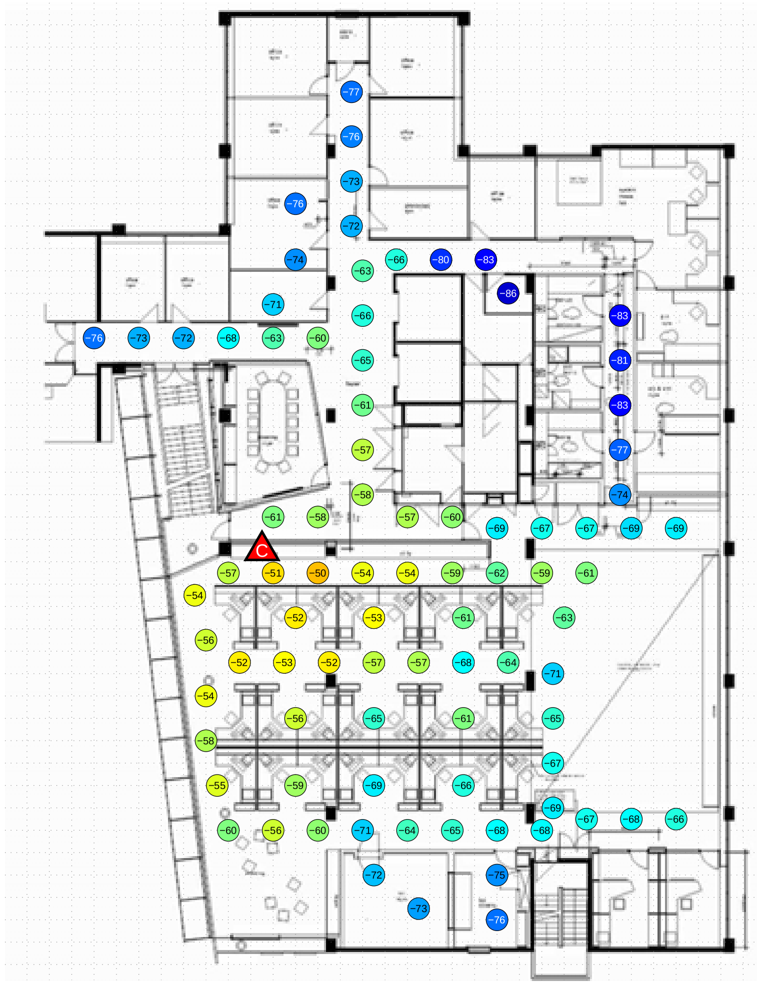
(c) *South* **(d)** *West*

Figure C.12: *Average RSSIs (dbm) for access point B.*

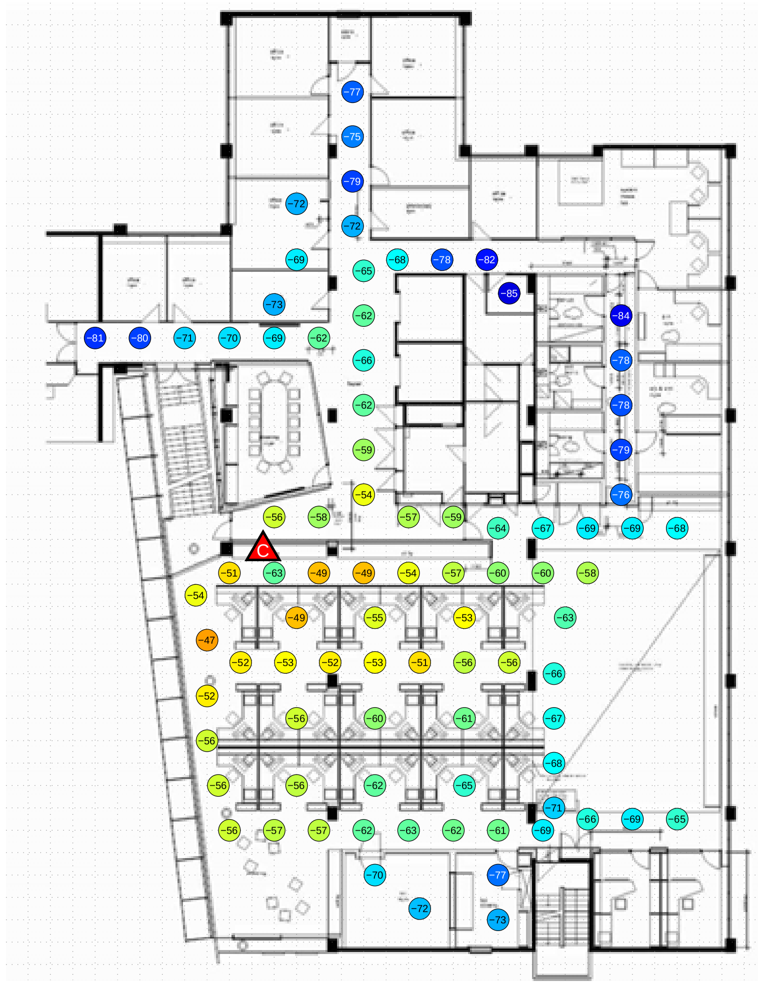


(a) *North* **(b)** *East*

Figure C.13: *Average RSSIs (dcm) for access point C.*
21/2007elfig:lab-rssi-c-ne

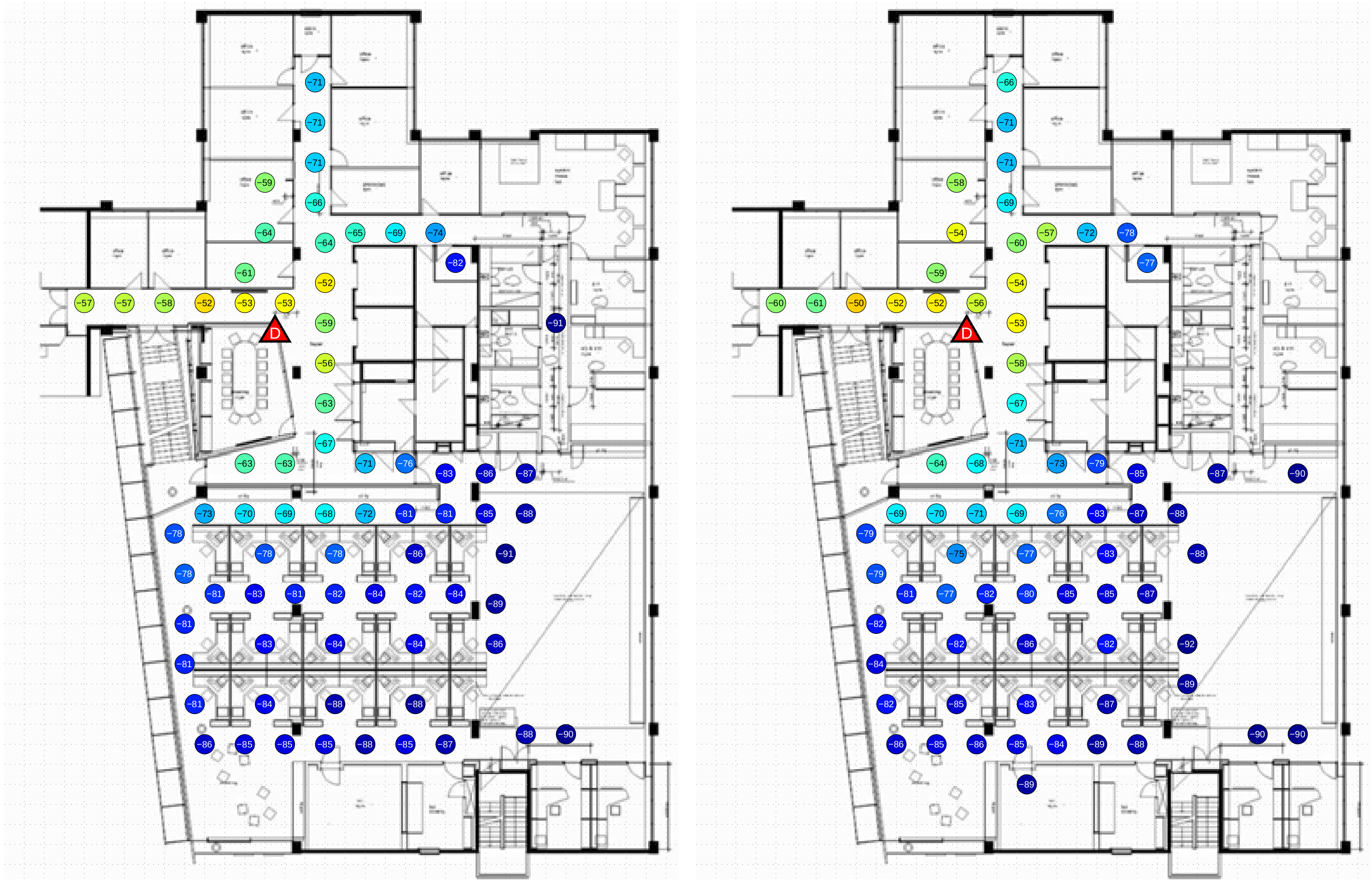


(c) *South*



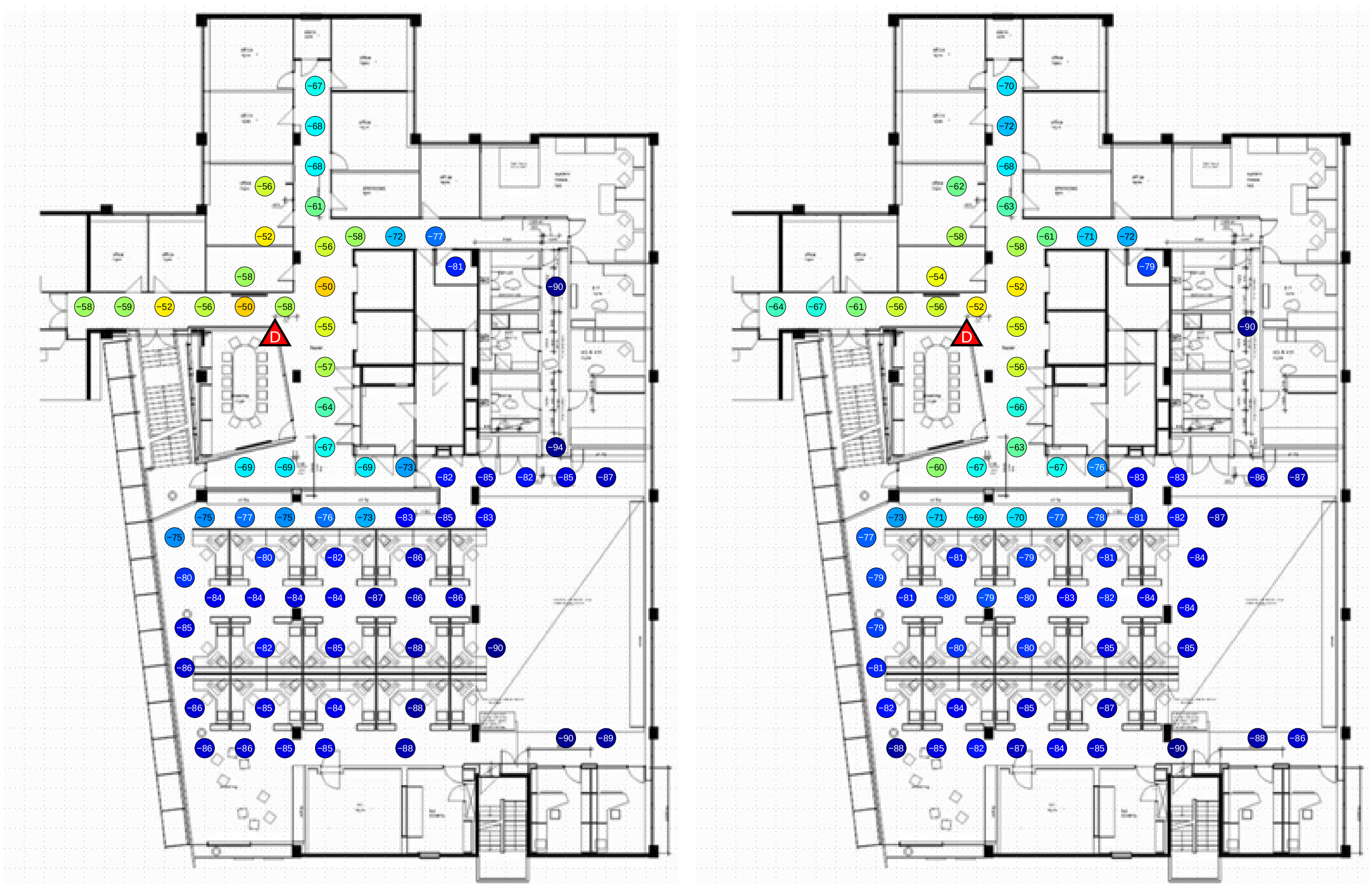
(d) *West*

Figure C.13: *Average RSSIs (dcm) for access point C.*



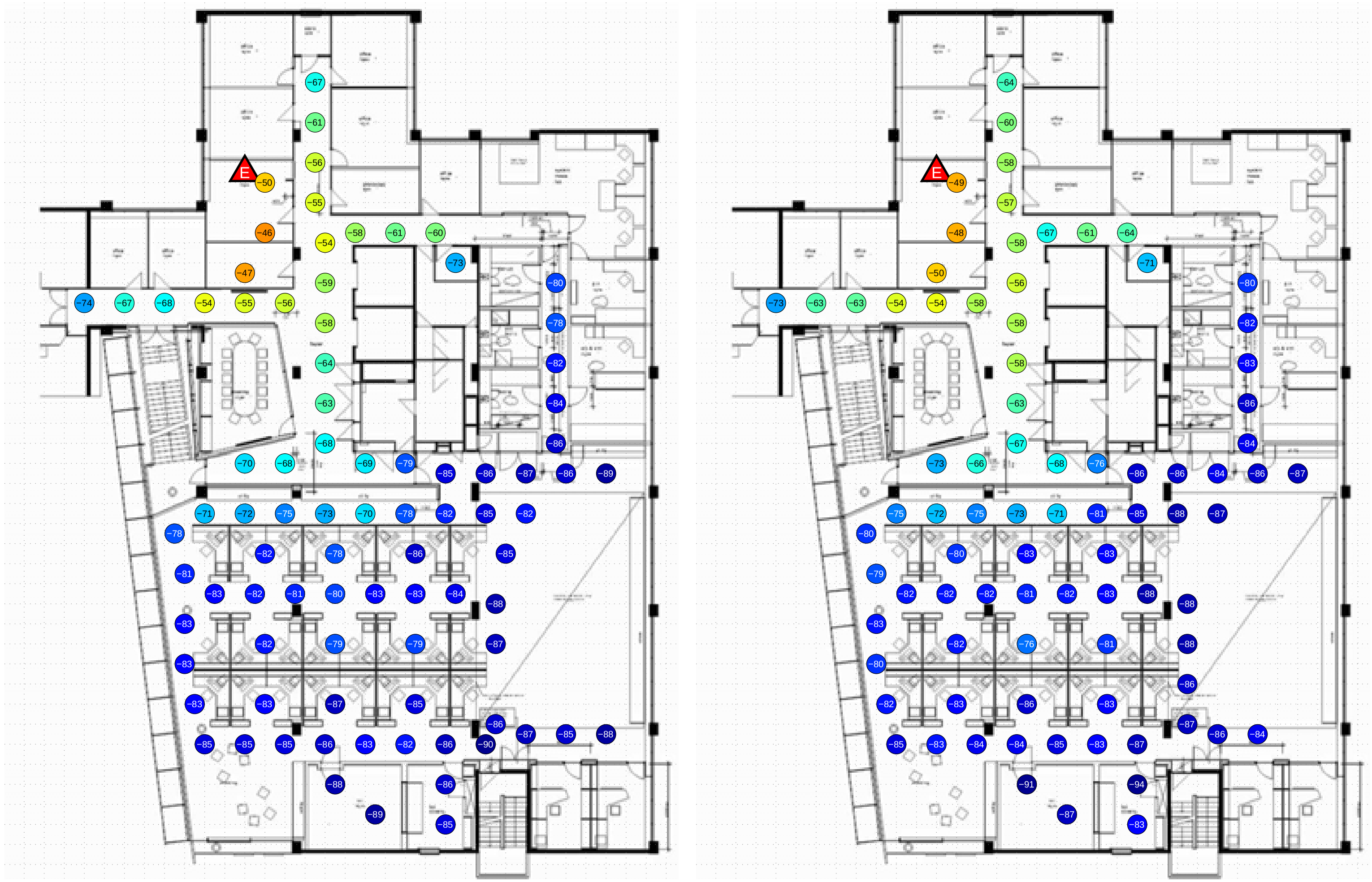
(a) *North* **(b)** *East*

Figure C.14: *Average RSSIs (dBm) for access point D.*



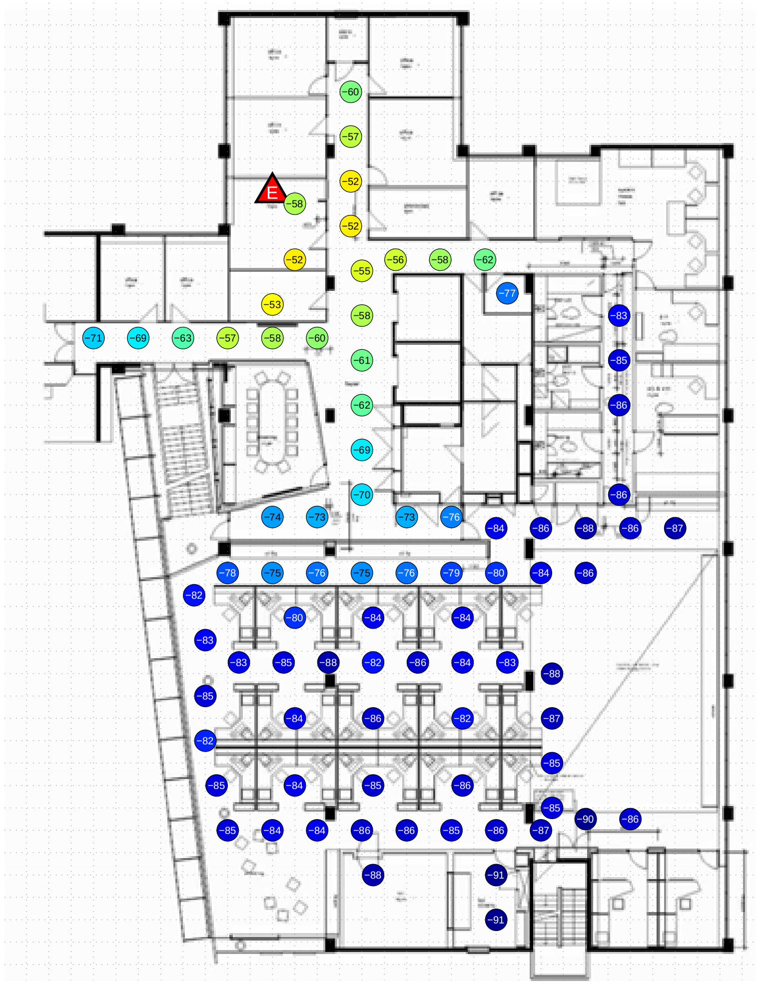
(c) *South* **(d)** *West*

Figure C.14: *Average RSSIs (dbm) for access point D.*

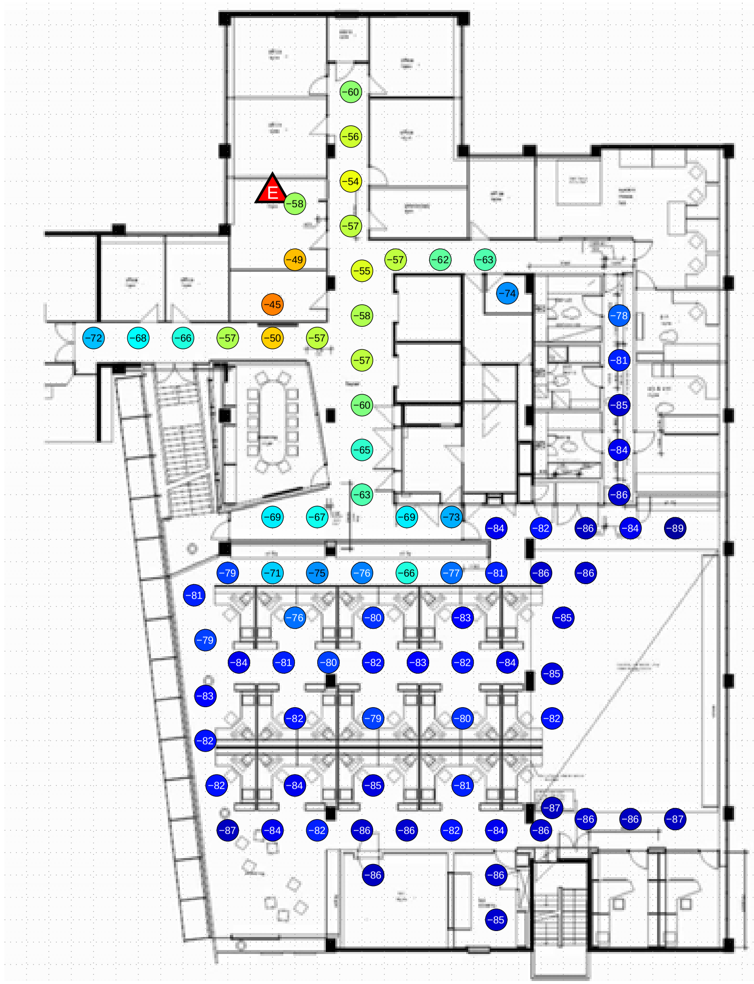


(a) *North* **(b)** *East*

Figure C.15: *Average RSSIs (dBm) for access point E.*

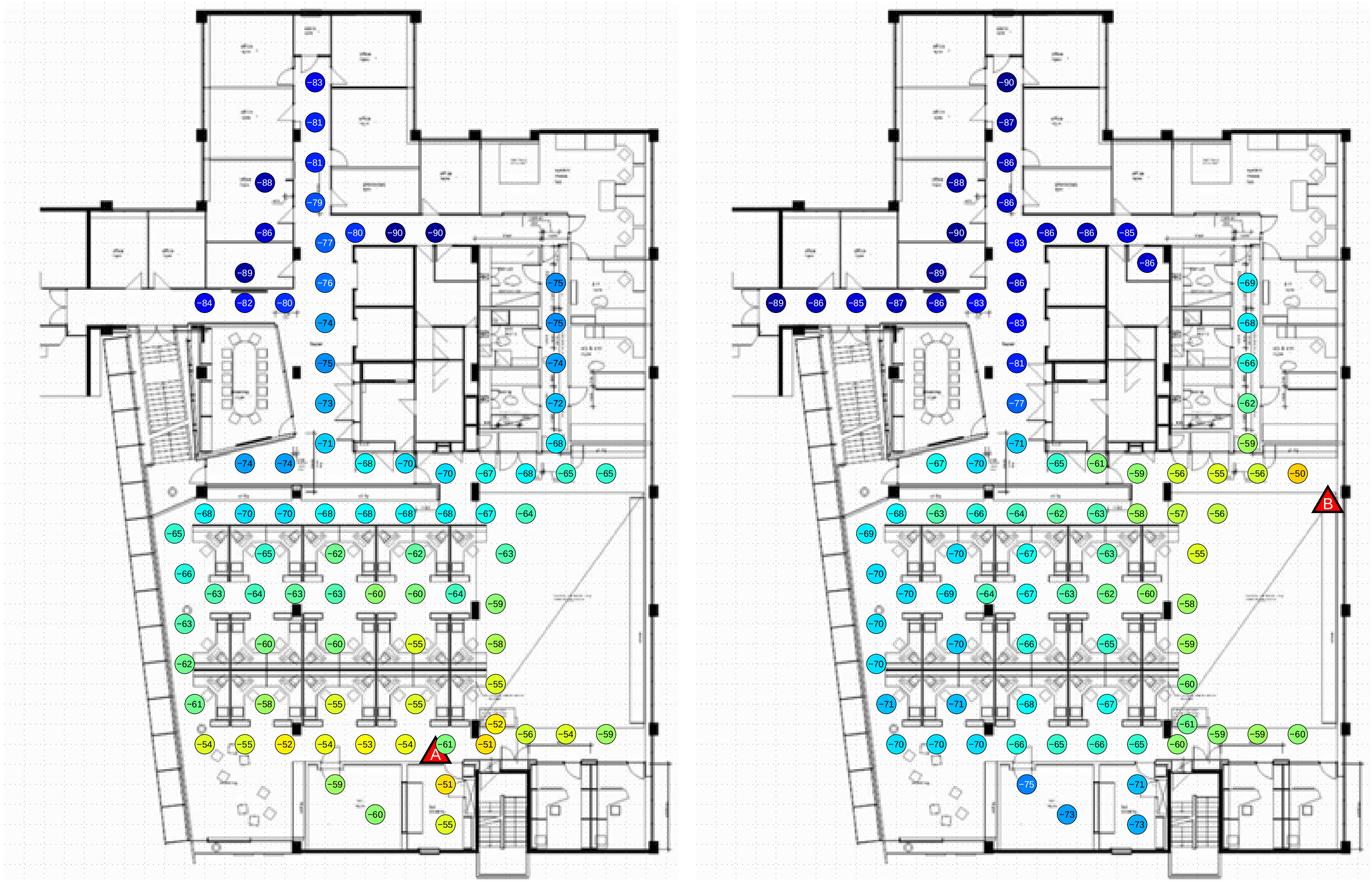


(c) *South*



(d) *West*

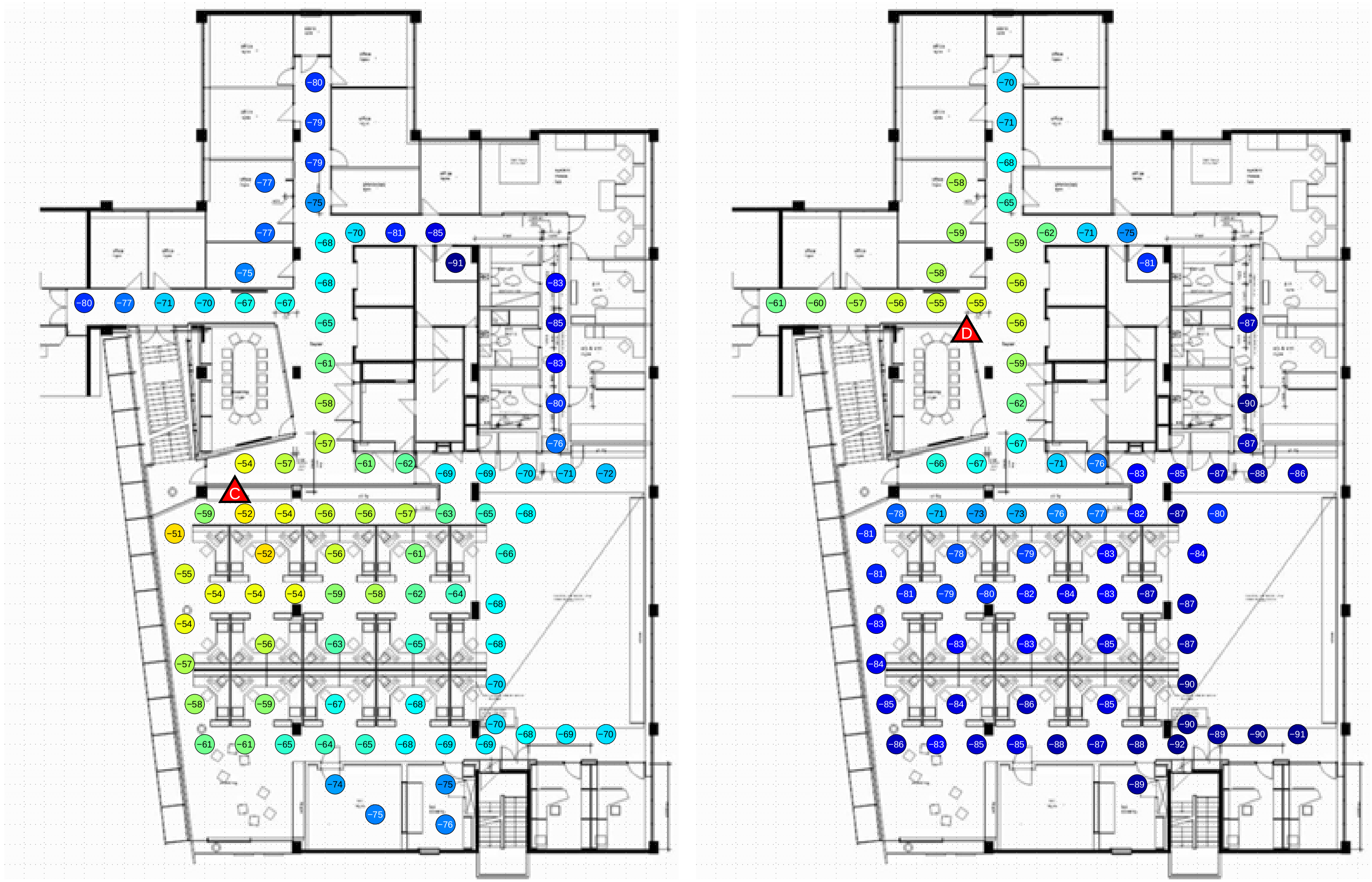
Figure C.15: *Average RSSIs (dbm) for access point E.*



(a) *Access point A.*

(b) *Access point B.*

Figure C.16: *Average RSSIs (dBm) for the CSE sector map.*



(a) *Access point C.* **(b)** *Access point D.*

Figure C.17: *Average RSSIs (dBm) for the CSE sector map.*

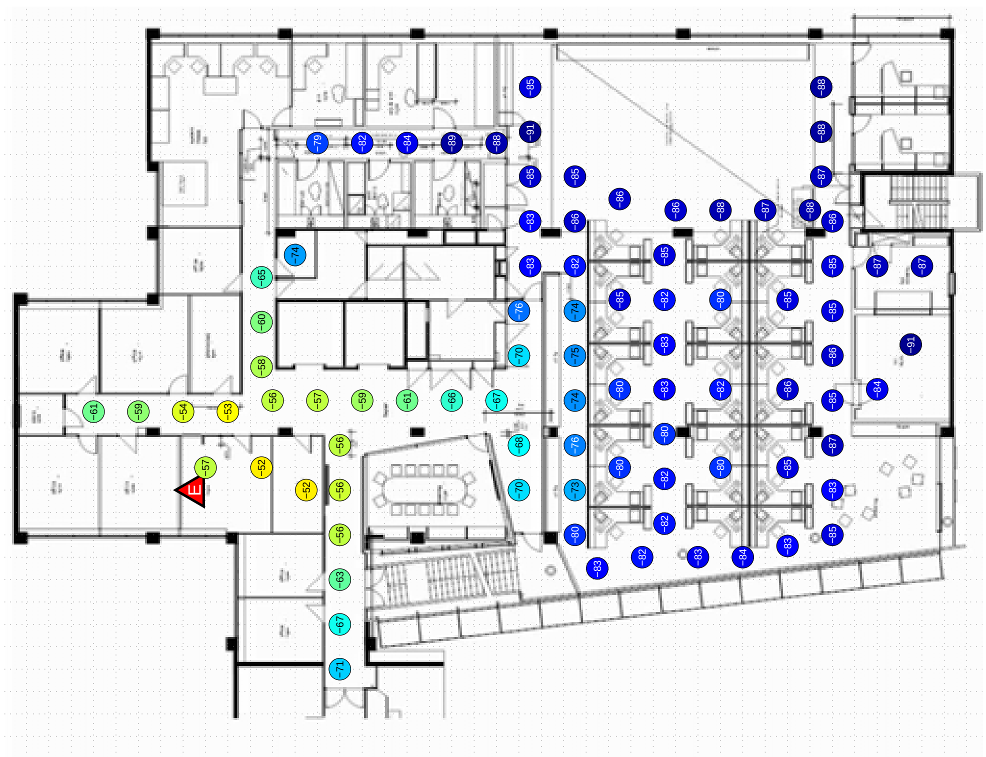


Figure C.18: *Average RSSIs (dBm) for the CSE sector map.*

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